Popper 01 ← Bubble 01 for popper #.

① Where do you take your tests?

A. in class B. CASA C. online

② How many times can you work on a quiz?

A. once B. 20 times C. infinitely many

③ Bubble A.

④ Bubble C.

Bubble correctly.

Linear function : $f(x) = mx + b$ (m: slope, b: y-intercept)

Math 1330 - Section 2.1
Linear and Quadratic Functions

Recall: Equation of a line: $2x+3y=4 \iff \frac{\cancel{3}y}{\cancel{3}} = \frac{-2x+4}{3} \Rightarrow y = -\frac{2}{3}x + \frac{4}{3}$

General form: $ax + by = c$. (slope is: $m = -\frac{a}{b}$)

Slope-intercept form: $\boxed{y = mx + b}$ (m is the slope and b is the y-intercept)

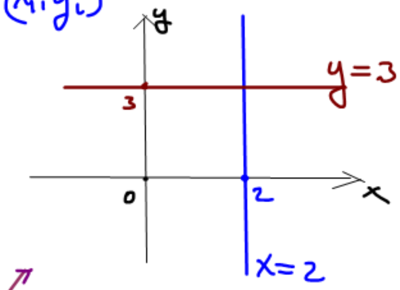
Point-slope form: $y - y_1 = \underset{\text{slope}}{m}(x - x_1)$

If two points on the line are given, then the slope is: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$

(x_1, y_1) (x_2, y_2)

Vertical lines are of the form: $x = c$. ← slope undefined

Horizontal lines are of the form: $y = c$. ← slope = 0



Two lines are parallel if they have the same slope. $m_1 = m_2$
Two lines are perpendicular if their slopes are negative reciprocals of each other. $m_1 = -\frac{1}{m_2}$

Definition: A **linear function** is a function of the form $f(x) = mx + b$,
where m is the **slope** and b is the **y-intercept**.

Example 1: Write an equation of the linear function for which $f(2) = 5$ and $f(-1) = 2$.

Need two points

$(2, 5)$
$(-1, 2)$

slope $= \frac{5-2}{2-(-1)} = \frac{3}{3} = 1$

$y - 5 = 1(x - 2)$

$\boxed{y = x + 3}$

Example 2: Write an equation of the linear function which contains the point (2, -5) and whose inverse contains the point (-1, 6).

$f^{-1}(-1) = 6 \iff f(6) = -1$

Need two points

$(2, -5)$
$(6, -1)$

Slope $m = \frac{-5+1}{2-6} = \frac{-4}{-4} = 1 \Rightarrow y + 5 = 1 \cdot (x - 2)$

$\boxed{y = x - 7}$

Example 3: Write an equation of the linear function which is parallel to the line $\underbrace{2x - 5y = 10}_{\text{gives the slope}}$ and which passes through the point (-1, -4).

Being parallel, we get slope $= \frac{2}{5}$.

$2x - 5y = 10$

$\frac{\cancel{5}y}{\cancel{5}} = \frac{2x - 10}{5}$

$y = \frac{2x}{5} - \frac{10}{5}$

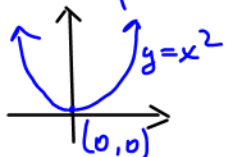
- $m = \frac{2}{5}$
- point $(-1, -4)$

$\Rightarrow y + 4 = \frac{2}{5}(x + 1)$

$\boxed{y = \frac{2}{5}x - \frac{18}{5}}$

Basic quadratic function:

$f(x) = x^2$



$y = 2(x-1)^2 + 3$

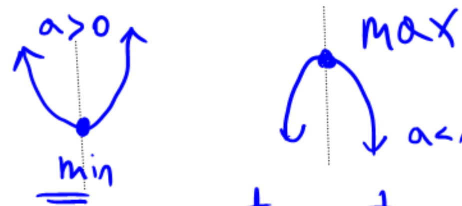
– stretch, shift 1 right, and 3 up.

easy to graph

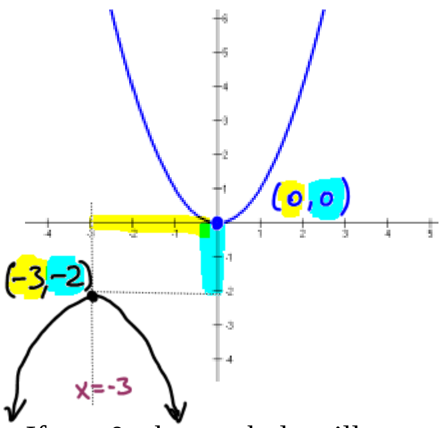
A **quadratic function** is a function of the form $f(x) = ax^2 + bx + c,\ a \neq 0.$

The graph of a quadratic function is called a parabola. You should be able to identify the following features of the graph of a quadratic function:

- direction the graph opens (upward or downward) $a>0$ $a<0$
- whether the function has a maximum or a minimum
- y intercept ($f(0)$.) $y = f(0)$
- coordinates of the vertex
- equation of the axis of symmetry
- maximum or minimum value



occur at vertex.



ex $f(x) = -(x+3)^2 - 2$

(downward, shift 3 left, 2 down)

vertex is $(-3, -2)$

→ $\boxed{\text{maximum value } f(-3) = -2}$

If $a > 0$, the parabola will open upward. In this case, the function has a minimum value.

If $a < 0$, the parabola will open downward. In this case, the function has a maximum value.

} at vertex.

The standard form of a quadratic function:

We like this form,

$\boxed{f(x) = a(x-h)^2 + k \text{ is in the standard form.}}$

Vertex (h, k)

The vertex is (h,k) and the axis of symmetry is the line $x = h.$ axis of symmetry

The maximum or minimum value of the function is the number k (the y-coordinate of the vertex). $f(h) = k.$

→ because we can find vertex, axis of symmetry $x=h$ and max/min value of function.

$f(h) = k$

What if it is not in standard form?

example:

$f(x) = 2x^2 + 4x - 5$

- upward
- what is vertex ???

"Complete square Method"

$(x+a)^2 = \underbrace{x^2 + 2ax + a^2}$,

hence

$\underbrace{x^2 + 2a\,x} + \boxed{a^2} = (x+a)^2$

using these two terms, we'll find the $\underline{\underline{a^2}}$ term.

Note $\boxed{a = \frac{(2a)}{2} = \text{half}.}$

$f(x) = (2x^2 + 4x) - 5$

factor 2

$= 2(x^2 + 2x) - 5$

$= 2(x^2 + \underset{\sim}{2}x + 1^2) - 5 - 2 \cdot 1^2$

find $a = \frac{2}{2} = 1$

$f(x) = 2(x+1)^2 - 7$ $\Rightarrow$ easy to visualize:

- upward stretched out
- vertex $(-1, -7)$
  (shift 1 left, 7 down).

Recall : $(x+a)^2 = x^2 + 2a\,x + a^2$ → half gives "a".

$(x-a)^2 = x^2 - 2a\,x + a^2$

Example 4: Given the function $f(x) = -2x^2 - 20x + 6$.

Find the standard form: $f(x) = -2x^2 - 12x + 6 = -2(x^2 + 6x + 3^2) + 6 + 2\cdot 3^2$ $\quad -2\cdot 3^2$

half $= \frac{6}{2} = 3 = a$

$\boxed{f(x) = -2(x+3)^2 + 24}$

Find the vertex: $(-3, 24)$

Find the axis of symmetry: $x = -3$

State the max/min value: since downward, f has a maximum value at vertex, $f(-3) = 24$

To be continued on Wednesday

02/03 **NOTE:** If you are not asked to write the function in standard form, you can find the vertex using a different method. The coordinates of the vertex of the graph of the function

$f(x) = ax^2 + bx + c,\ a \neq 0$ is the ordered pair $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

You don't have to use "complete the square method" all the time.

$\boxed{\text{Vertex formula: } x = -\frac{b}{2a},\ y = f(x).}$

If you are given the vertex of the graph of a function and another point, you can find the quadratic function equation.

→ look at next page for an application.

Example 5: Write the equation of the quadratic function which passes through the point (0, 7) and whose vertex is (-2, 10).

→ Quadratic function vertex $(-2, 10)$ ⟹ Standard form $f(x) = a(x-h)^2 + k$

⟹ $f(x) = a(x+2)^2 + 10$ ← find "a"

?

→ The function passes through $(0, 7)$ ⟹ $f(0) = a(0+2)^2 + 10 = 7$

$4a + 10 = 7 \Rightarrow 4a = -3$

$a = -\frac{3}{4}$

⟹ $\boxed{f(x) = -\frac{3}{4}(x+2)^2 + 10}$

Let's apply vertex formula: (example 4)

$$f(x) = -2x^2 - 12x + 6$$

$\Longrightarrow$ $a = -2$, $b = -12$, $c = 6$

Vertex: • x-coordinate $= h$

$$x = -\frac{b}{2a} = \frac{-(-12)}{2(-2)} = \frac{12}{-4} = -3$$

$(-3, 24)$

$\Longrightarrow$ $\boxed{x = -3}$ exactly same as we found by the other method.

• y-coordinate $= \boxed{k = f(h)}$

$y = f(-3)$ $\leftarrow$ substitute "-3" in f.

$$= -2(-3)^2 - 12\cdot(-3) + 6$$

$$= -18 + 36 + 6 = 24$$

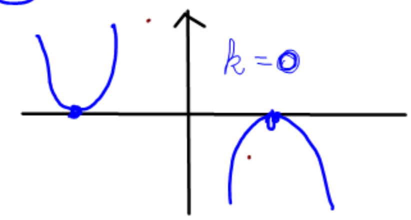
$\boxed{y = 24}$ exactly same.

standard form $\rightarrow f(x) = a(x-h)^2 + k$

axis of sym. $x = h$

Example 6: Find the quadratic function that satisfies:

1. The axis of symmetry is: $x = -4$ $\Rightarrow f(x) = a(x+4)^2 + k$ using 1
2. The y-intercept is: (0,80)
3. There is only one x-intercept. No vertical shift $\Rightarrow k = 0 \Rightarrow f(x) = a(x+4)^2$



2 $(0,80) \Rightarrow f(0) = a(0+4)^2 = 80 \Rightarrow a = 5$

$\Rightarrow \boxed{f(x) = 5(x+4)^2}$

Exercise

Example 7: A rocket is fired directly upwards with a velocity of 80 ft/sec. The equation for its height, H, as a function of time, t, is given by the function $\underbrace{H(t) = -16t^2 + 80t}_{\text{parabola, downward}}$.

a. Find the time at which the rocket reaches its maximum height.

$\Rightarrow$ reaches maximum at vertex x-coordinate:

$$t = \frac{-b}{2a} = \frac{-80}{2(-16)} = 2.5 \text{ seconds}$$

b. Find the maximum height of the rocket.

maximum height:

$$H(2.5) = -16 \cdot (2.5)^2 + 80 \cdot (2.5)$$

$$= 100 \text{ ft}.$$