Popper \# $18<$ Bubble

(A.) $[0, \pi]$ B. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \quad$ C. $[0,2 \pi]$

I, (iI)
(2) $\cos ^{-1}\left(-\frac{\sqrt{2}}{2}\right)=\theta=\frac{3 \pi}{4} \quad$ i, $Q \pi$
A. $\frac{\pi}{4}$
(B.) $\frac{3 \pi}{4}$
$\begin{array}{ll}\text { C. } \frac{5 \pi}{4} & \text { D. }-\frac{\pi}{4}\end{array}$
(3) $f(x)=\sin x$ is $1-1 / 0$
A. $[0, \pi]$
(B. $\left[\frac{\pi}{2} \cdot \frac{\pi}{2}\right]$
C. $[0,2 \pi]$
(4) $\sin ^{-1}\left(-\frac{\sqrt{2}}{2}\right)=\theta=-\frac{\pi}{4} \rightarrow Q$ IT
A. $\frac{\pi}{4}$
B. $\frac{3 \pi}{4}$
C. $\frac{5 \pi}{4}$
(D.) $-\frac{\pi}{4}$

Section 5.1 Trigonometric Functions of Real Numbers
In calculus and in the sciences many of the applications of the trigonometric functions require that the inputs be real numbers, rather than angles. By making this small but crucial change in our viewpoint, we can define the trigonometric functions in such a way that the inputs are real numbers. The definitions of the trig functions, and the identities that we have already met (and will meet later) will remain the same, and will be valid whether the inputs are angles or real numbers.

Here are some identities you need to know:

$$
\begin{aligned}
& \tan (t)=\frac{\sin (t)}{\cos (t)} \\
& \cot (t)=\frac{\cos (t)}{\sin (t)}=\frac{1}{\tan t}
\end{aligned}
$$

That's why we use radian measure. They are "the real numbers" where we define the trigonometric functions.

Reciprocal Identities

$$
\left.\begin{array}{ll}
\sec (t)=\frac{1}{\cos (t)}, & \cos (t) \neq 0 \\
\csc (t)=\frac{1}{\sin (t)}, & \sin (t) \neq 0 \\
\cot (t)=\frac{1}{\tan (t)}, & \tan (t) \neq 0
\end{array}\right\} \quad \begin{gathered}
\text { when becomes "O" } \\
\text { you get V.A. } \\
\text { for each function. }
\end{gathered}
$$

Pythagorean Identities

$$
\sin ^{2}(t)+\cos ^{2}(t)=1 \rightarrow \sin c e \quad x=\cos t, y=\sin t \text { and } x^{2}+y^{2}=1 \text {, wit circle. }
$$

$$
\left[\begin{array}{l}
1+\tan ^{2}(t)=\sec ^{2}(t) \\
1+\cot ^{2}(t)=\csc ^{2}(t)
\end{array} \rightarrow 1+\tan ^{2} t=1+\frac{\sin ^{2} t}{\cos ^{2} t}=\frac{\cos ^{2} t}{\cos ^{2} t}+\frac{\sin ^{2} t}{\cos ^{2} t}=\frac{1}{\cos ^{2} t}=\sec ^{2} t\right.
$$

Opposite Angle Identities

them!!



III
Example 1: Given that $\cos (t)=-\frac{1}{4}$ and $\pi<t<\frac{3 \pi}{2}$; find $\sin (t)$ and $\tan (t)$.

$$
\begin{array}{ll}
\cos ^{2} t+\sin ^{2} t=1 \\
\left(-\frac{1}{4}\right)^{2}+\sin ^{2} t=1 \\
\sin ^{2} t=1-\frac{1}{16}=\frac{15}{16}
\end{array} \quad \Rightarrow \sin t=-\sqrt{\frac{15}{16}}=-\frac{\sqrt{15}}{4}, \frac{\pi}{\cos t}=\frac{-\sqrt{15} / 4}{-1 / 4}=\sqrt{15}
$$

$$
\sin t=\frac{1}{4}>0
$$

$$
\sin ^{2} t+\cos ^{2} t=1
$$

$$
\tan t=\frac{\sin t}{\cos t}<0 \Rightarrow \cos t<0:\left(\frac{1}{4}\right)^{2}+\cos ^{2} t=1
$$

Example 3: Use the opposite-angle identities to find:

$$
\begin{aligned}
\cos ^{2} t & =\frac{15}{16} \\
\Rightarrow \cos t & =-\sqrt{\frac{15}{16}}=-\frac{\sqrt{15}}{4}
\end{aligned}
$$

$$
\text { Sect }=\frac{1}{\cos t}=-\frac{4}{\sqrt{15}}
$$

b. $\cos \left(-\frac{5 \pi}{6}\right)=\cos \left(\frac{5 \pi}{6}\right)=-\frac{\sqrt{3}}{2}$

$$
\text { c. } \begin{aligned}
\tan \left(-\frac{\pi}{4}\right)+\cot \left(-\frac{3 \pi}{4}\right) & =-\tan \left(\frac{\pi}{4}\right)-\cot \left(\frac{3 \pi}{4}\right) \\
& =-1-(-1) \\
& =0
\end{aligned}
$$

Here's another set of identities:
Periodicity (the interval where the function gets repeated)
The sine and cosine functions are periodic functions. That means that there is some number p such that $f(x+p)=f(x)$. The number p is the period of the function. So

$$
\begin{array}{lll}
\sin (t+2 \pi)=\sin (t) & \text { more generally } & \sin (t+2 k \pi)=\sin (t) \\
\cos (t+2 \pi)=\cos (t) & & \cos (t+2 k \pi)=\cos (t)
\end{array}
$$

After a full
for all real numbers t and all integers k .
revolution, sine and cosine repeat!

The tangent and cotangent functions are also periodic functions. However, these functions repeat themselves when $\mathbf{p}=\pi$. So

$$
\begin{array}{lrr}
\tan (t+\pi)=\tan (t) & \text { more generally } & \tan (t+k \pi)=\tan (t) \\
\cot (t+\pi)=\cot (t) & & \cot (t+k \pi)=\cot (t)
\end{array}
$$

for all real numbers t and all integers k .
Tangent and cotangent repeat in every $\pi$-lengthened interval
Note: the period for the sine and cosine functions is $2 \pi$ while the period for the tangent and cotangent functions is $\pi$.

The secant and cosecant functions are the reciprocal functions, so they will follow the same periodicity rules as sine and cosine.

$$
\left\{\begin{array}{l}
\sec (t+2 \pi k)=\sec (t) \\
\csc (t+2 \pi k)=\csc (t)
\end{array} \quad \text { for all real numbers } t \text { and all integers } k\right.
$$

They depend on cosine and sine, respectively. We will use the identities and periodicity to evaluate trig functions of real numbers.
So the periods coincide.

Example 4: Evaluate $\tan \left(\frac{15 \pi}{4}\right)=\tan \left(3 \pi+\frac{3 \pi}{4}\right)=\tan \left(\frac{3 \pi}{4}\right)=-1$

$$
\frac{15 \pi}{4}=3 \pi+\frac{3 \pi}{4}
$$

Example 5: Evaluate

$$
\cos \left(\frac{25 \pi}{6}\right)=\cos (\underbrace{\left(\frac{n}{6} \pi\right.}_{\text {2 rotations }}+\frac{\pi}{6})=\cos \left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}
$$

$$
\frac{25 \pi}{6}=4 \pi+\frac{\pi}{6}
$$

$$
\frac{19 \pi}{3}=6 \pi+\frac{\pi}{3}
$$

Example 7: Evaluate

$$
\frac{20 \pi}{3}=6 \pi+\frac{2 \pi}{3}
$$

$$
\begin{aligned}
\sin \left(\frac{-20 \pi}{3}\right)=-\sin \left(\frac{20 \pi}{3}\right) & =-\sin \left(6 \pi+\frac{2 \pi}{3}\right) \\
& =-\sin \left(\frac{2 \pi}{3}\right) \\
& =-\frac{\sqrt{3}}{2}
\end{aligned}
$$

extra example: mot full rotation

$$
\text { - } \begin{aligned}
\cos \left(\frac{16 \pi}{3}\right)=\cos \left(\frac{5 \pi}{?}+\frac{\pi}{3}\right) & =\cos \left(4 \pi+\pi+\frac{\pi}{3}\right) \\
& =\frac{16 \pi}{3}=5 \pi+\frac{\pi}{3}
\end{aligned}
$$

Example 8: Evaluate

$$
\begin{aligned}
& \frac{19 \pi}{3}=6 \pi+\frac{\pi}{3} \\
& \frac{211 \pi}{4}=5 \pi+\frac{\pi}{4} \\
& 8 \pi=8 \pi+0 \\
& \frac{25 \pi}{6}=4 \pi+\frac{\pi}{6}
\end{aligned}
$$

Example 9: Evaluate

$$
\begin{aligned}
& \frac{21 \pi}{4}=5 \pi+\frac{\pi}{4} \\
& \frac{12 \pi}{4}=4 \pi+\frac{\pi}{4} \\
& 11 \pi=100 \pi+\pi \\
& \frac{17 \pi}{6}=2 \pi+\frac{5 \pi}{6}
\end{aligned}
$$

Example 10: Simplify: $\quad \cot (-t) \sec (-t)$

$$
\begin{aligned}
& =-\cot (t) \cdot \sec (t) \\
& =-\frac{\cos t}{\sin t} \cdot \frac{1}{\cos t} \\
& =-\frac{1}{\sin t}=-\csc (t)
\end{aligned}
$$

