

## Section 5.1 Trigonometric Functions of Real Numbers

Here are some identities you need to know:

**definition**

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

$$\cot(t) = \frac{\cos(t)}{\sin(t)}$$

### Reciprocal Identities

**reciprocal**

$$\csc(t) = \frac{1}{\sin(t)}, \sin(t) \neq 0 \iff \sin t \cdot \csc t = 1$$

$$\sec(t) = \frac{1}{\cos(t)}, \cos(t) \neq 0 \iff \cos t \cdot \sec t = 1$$

$$\cot(t) = \frac{1}{\tan(t)}, \tan(t) \neq 0 \iff \tan t \cdot \cot t = 1$$

### Opposite Angle Identities

**opposite**

$$\begin{aligned} \sin(-t) &= -\sin(t) \\ \cos(-t) &= \cos(t) \\ \tan(-t) &= -\tan(t) \\ \csc(-t) &= -\csc(t) \\ \sec(-t) &= \sec(t) \\ \cot(-t) &= -\cot(t) \end{aligned}$$

### Pythagorean Identities

**identities**

$$\begin{aligned} \sin^2(t) + \cos^2(t) &= 1 \iff \cos^2 t = 1 - \sin^2 t \quad \text{or} \quad \sin^2 t = 1 - \cos^2 t \\ 1 + \tan^2(t) &= \sec^2(t) \iff \tan^2 t = \sec^2 t - 1 \quad \text{or} \quad \sec^2 t - \tan^2 t = 1 \\ 1 + \cot^2(t) &= \csc^2(t) \iff \cot^2 t = \csc^2 t - 1 \quad \text{or} \quad \csc^2 t - \cot^2 t = 1 \end{aligned}$$

### Periodicity

**period**

$$\begin{aligned} \sin(t + 2k\pi) &= \sin(t) & \tan(t + k\pi) &= \tan(t) \\ \cos(t + 2k\pi) &= \cos(t) & \cot(t + k\pi) &= \cot(t) \\ \sec(t + 2\pi k) &= \sec(t) \\ \csc(t + 2\pi k) &= \csc(t) \end{aligned} \quad (\text{for all real numbers } t \text{ and all integers } k.)$$

Sine, cosine  $\iff$  period =  $2\pi$   $\iff$  multiple of  $\underline{\underline{2\pi}}$ .

Tangent, cotangent  $\iff$  period =  $\pi$   $\iff$  multiple of  $\pi$ .

**Example 1 :** Simplify:

$$\begin{aligned}
 \frac{\cot(-t)}{\cos(-t)} &= \frac{-\cot(t)}{\cos(t)} = \frac{-\frac{\cos t}{\sin t}}{\cos t} \\
 &= -\frac{\cancel{\cos t}}{\sin t} \cdot \frac{1}{\cancel{\cos t}} \\
 &= -\frac{1}{\sin t} = -\csc t = \csc(-t)
 \end{aligned}$$

**Example 2 :** Simplify:

$$\begin{aligned}
 &\frac{\sin(t+6\pi)\csc(t-2\pi)}{\cot(t+\pi)\tan(t+2\pi)} \\
 &= \frac{\sin t \cdot \csc t}{\cot t \cdot \tan t} \\
 &= \frac{\frac{\sin t}{1} \cdot \frac{1}{\sin t}}{\frac{\cos t}{\sin t} \cdot \frac{\sin t}{\cos t}} \\
 &= \boxed{1}
 \end{aligned}$$

period for sine =  $2\pi$   
 cosine  
 secant  
 cosecant

period for tangent =  $\pi$   
 cotangent

**Example 3:** Simplify:  $\cos(-t) + \cos(-t) \tan^2(-t)$

$$\begin{aligned} &= \cos t + \cos t \cdot (-\tan t)^2 \\ &= \cos t + \cos t \cdot \tan^2 t \\ &= \cos t + \cos t \cdot \frac{\sin^2 t}{\cos^2 t} = \cos t \cdot \cos t \\ &= \cancel{\cos t} \frac{\cos t}{\cos t} + \frac{\sin^2 t}{\cos t} \\ &= \frac{\cos^2 t + \sin^2 t}{\cos t} = \frac{1}{\cos t} = \boxed{\sec t} \end{aligned}$$

**Example 4:** Simplify:  $\frac{\sec(t+4\pi) + \csc(t+6\pi)}{1 + \tan(t+5\pi)} = \frac{\sec t + \csc t}{1 + \tan t}$

$$\begin{aligned} &= \frac{\frac{\sin t}{\sin t} \cdot \frac{1}{\cos t} + \frac{1}{\sin t} \cdot \frac{\cos t}{\cos t}}{\frac{\cos t}{\cos t} \cdot 1 + \frac{\sin t}{\cos t}} = \frac{\frac{\sin t + \cos t}{\sin t \cdot \cos t}}{\frac{\cos t + \sin t}{\cos t}} \\ &= \frac{\cancel{\sin t + \cos t}}{\cancel{\sin t \cdot \cos t}} \cdot \frac{\cos t}{\cancel{\cos t + \sin t}} = \frac{1}{\sin t} = \boxed{\csc t} \end{aligned}$$