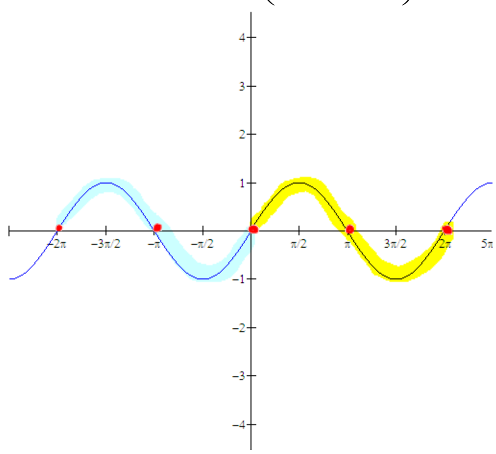
Section 5.3a - Graphs of Secant and Cosecant Functions

Using the identity $\csc(x) = \dfrac{1}{\sin(x)}$, you can conclude that the graph of g will have a vertical asymptote whenever $\sin(x) = 0$. This means that the graph of g will have vertical asymptotes at $x = 0, \pm\pi, \pm 2\pi, \ldots$. The easiest way to draw a graph of $g(x) = \csc(x)$ is to draw the graph of $f(x) = \sin(x)$, sketch asymptotes at each of the zeros of $f(x) = \sin(x)$, then sketch in the cosecant graph.

$g(x) = \csc(x) = \dfrac{1}{\sin(x)}$; if $\sin(x) = 0$, then g(x) has a vertical asymptote.

$f(x) = \csc x = \dfrac{1}{\underbrace{\sin x}_{\text{can be zero}}}$

Here's the graph of $f(x) = \sin(x)$ on the interval $\left(-\dfrac{5\pi}{2}, \dfrac{5\pi}{2}\right)$.



$\sin x = 0$

$x = 0, \pi, 2\pi, 3\pi \ldots$
$-\pi, -2\pi, -3\pi$

$\boxed{x = k\pi}$

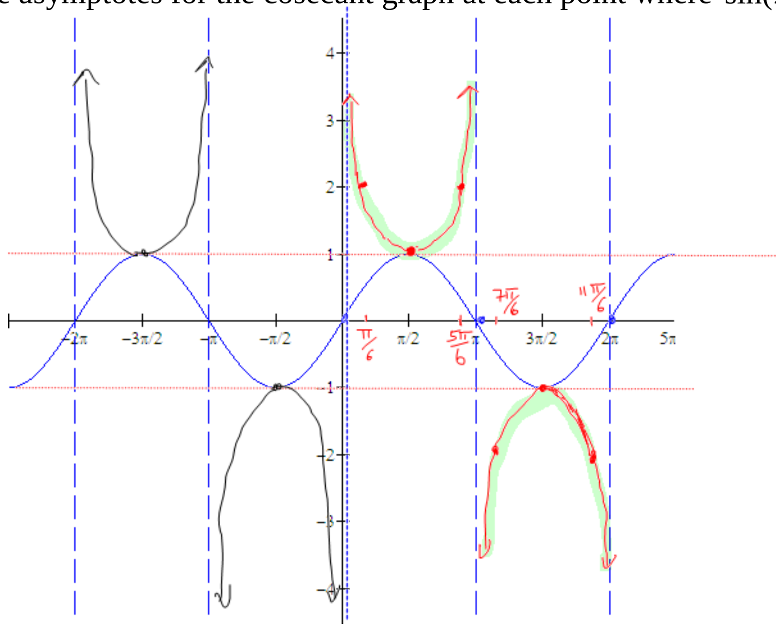
Vertical Asymptotes.

Next, we'll include the asymptotes for the cosecant graph at each point where $\sin(x) = 0$.

Recall:

$\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

$\sin\left(\frac{7\pi}{6}\right) = \sin\left(\frac{11\pi}{6}\right) = -\frac{1}{2}$

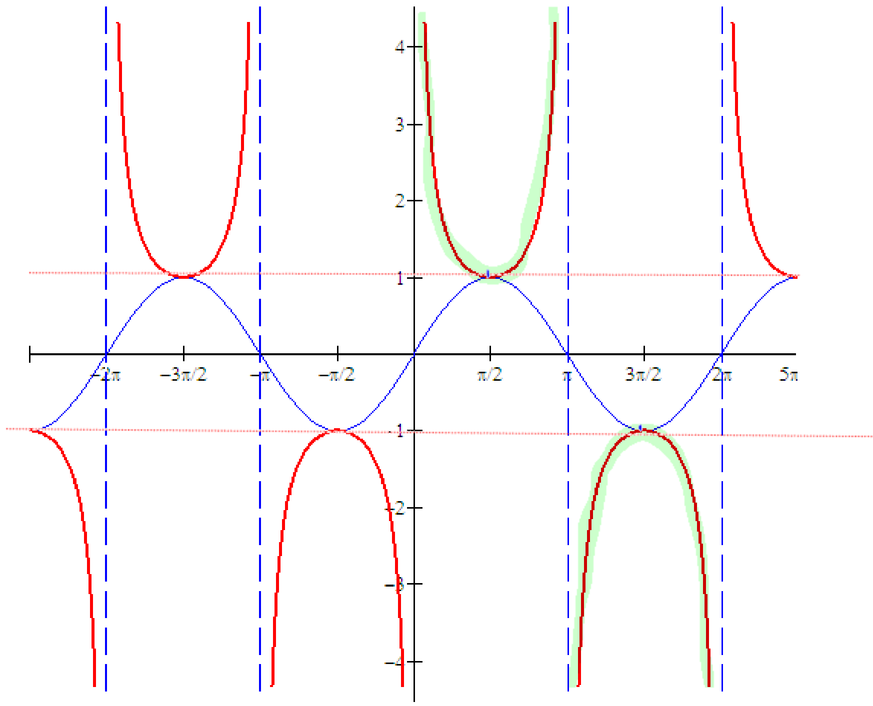


$f(x) = \csc x$

period $= 2\pi$

V.A. $x = k\pi$.

Now we'll include the graph of the cosecant function.



Period: 2π

Vertical Asymptote: $x = k\pi$, k is an integer

$x = 0, \pi, 2\pi, 3\pi, \ldots$
$-\pi, -2\pi, -3\pi, \ldots$ $\Big\}$ $\boxed{x = k\pi}$ V.A.

graph $\Big\{$

x-intercepts: None

y-intercept: None

Domain: $x \neq k\pi$, k is an integer

Range: $(-\infty,-1] \cup [1,\infty)$

Typically, you'll just graph over one period $(0, 2\pi)$.

One period is always enough.

To graph $y = A\csc(Bx - C) + D$, first graph, **THE HELPER GRAPH**: $y = A\sin(Bx - C) + D$.

ex $y = 2\csc(2x - \pi) + 1 \xrightarrow{\text{Always}}$ Graph $y = 2\sin(2x - \pi) + 1$

then put "the parabola" shapes on top of sine function.

Do not forget V.A.

Similarly, we graph $f(x) = sec(x) = \frac{1}{cos(x)}$.

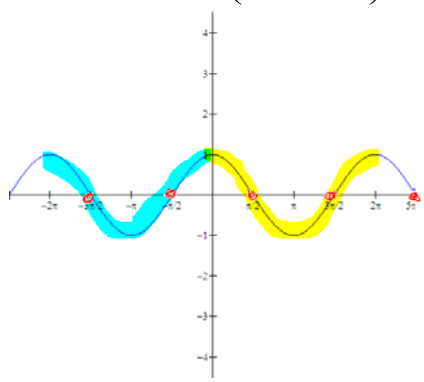
You'll also be able to take advantage of what you know about the graph of $f(x) = \cos(x)$ to help you graph $g(x) = \sec(x)$. Using the identity $\sec(x) = \dfrac{1}{\cos(x)}$, you can conclude that the graph of g will have a vertical asymptote whenever $\cos(x) = 0$.

This means that the graph of g will have vertical asymptotes at $x = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \ldots$. The easiest way to draw a graph of $g(x) = \sec(x)$ is to draw the graph of $f(x) = \cos(x)$, sketch asymptotes at each of the zeros of $f(x) = \cos(x)$, then sketch in the secant graph.

$g(x) = \sec(x) = \dfrac{1}{\cos(x)}$; if $\cos(x) = 0$, then g(x) has a vertical asymptote.

Here's the graph of $f(x) = \cos(x)$ on the interval $\left(-\dfrac{5\pi}{2}, \dfrac{5\pi}{2}\right)$.

$f(x) = sec x = \underbrace{\frac{1}{cos x}}_{\text{can be zero}}$



$cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$

These are V.A. for $f(x) = sec x$

Next, we'll include the asymptotes for the secant graph.

$cos\, 0 = 1$

$cos \frac{\pi}{3} = \frac{1}{2}$

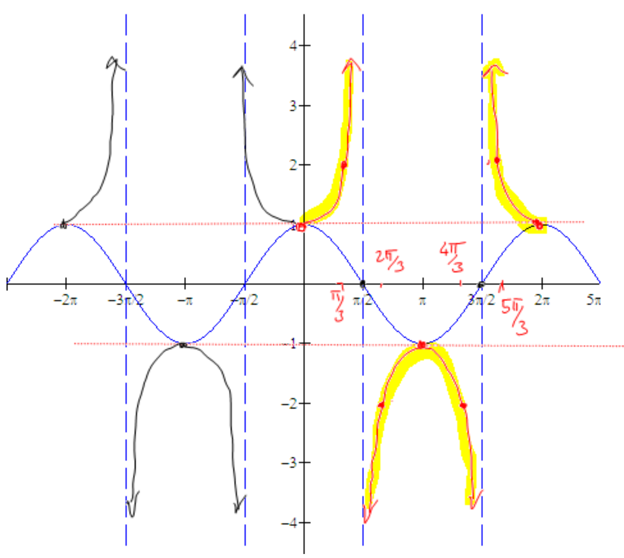
$cos \frac{2\pi}{3} = -\frac{1}{2}$

$cos \frac{4\pi}{3} = -\frac{1}{2}$

$cos \frac{5\pi}{3} = \frac{1}{2}$

$cos\, \pi = -1$

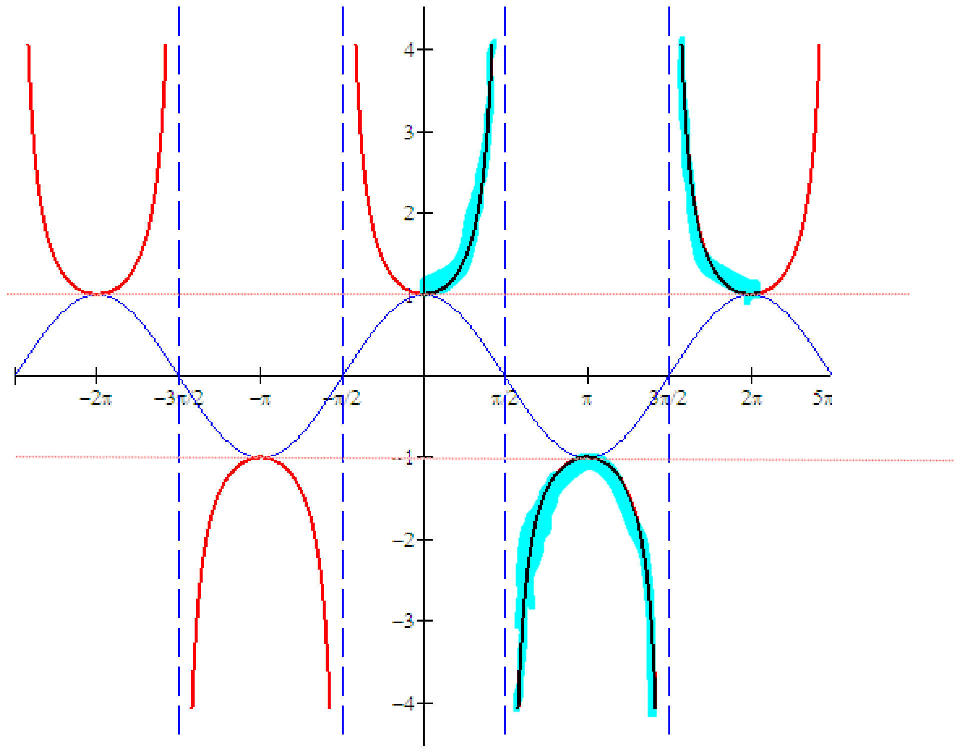
$cos\, 2\pi = 1$



$x = \frac{\pi}{2}, \frac{3\pi}{2}$ V.A.

$-\frac{\pi}{2}, -\frac{3\pi}{2}$

Now we'll include the graph of the secant function.



Period: 2π
Vertical Asymptote: [redacted]
$x = k\pi/2$ is an odd integer, k is odd

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$
$-\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, \ldots$
$\boxed{x = \frac{k\pi}{2},\ k \text{ odd}}$

graph
x-intercepts: None
y-intercept: (0, 1)
Domain: $x \neq k\pi/2$, k is an odd integer
Range: $(-\infty,-1] \cup [1,\infty)$

Typically, you'll just graph over one period $(0, 2\pi)$.

To graph $y = A\sec(Bx - C) + D$, first graph, **THE HELPER GRAPH**: $y = A\cos(Bx - C) + D$.

$y = A\sec(Bx-C)+D \xrightarrow{\text{Always}}$ graph $y = A\cos(Bx-c)+D$ and then put "parabola" shapes on top of cosine!

Do not forget V.A.

Example 1: Sketch $f(x) = \overset{A}{4}\sec\left(\overset{B}{\frac{x}{2}}\right)$

Helper graph:

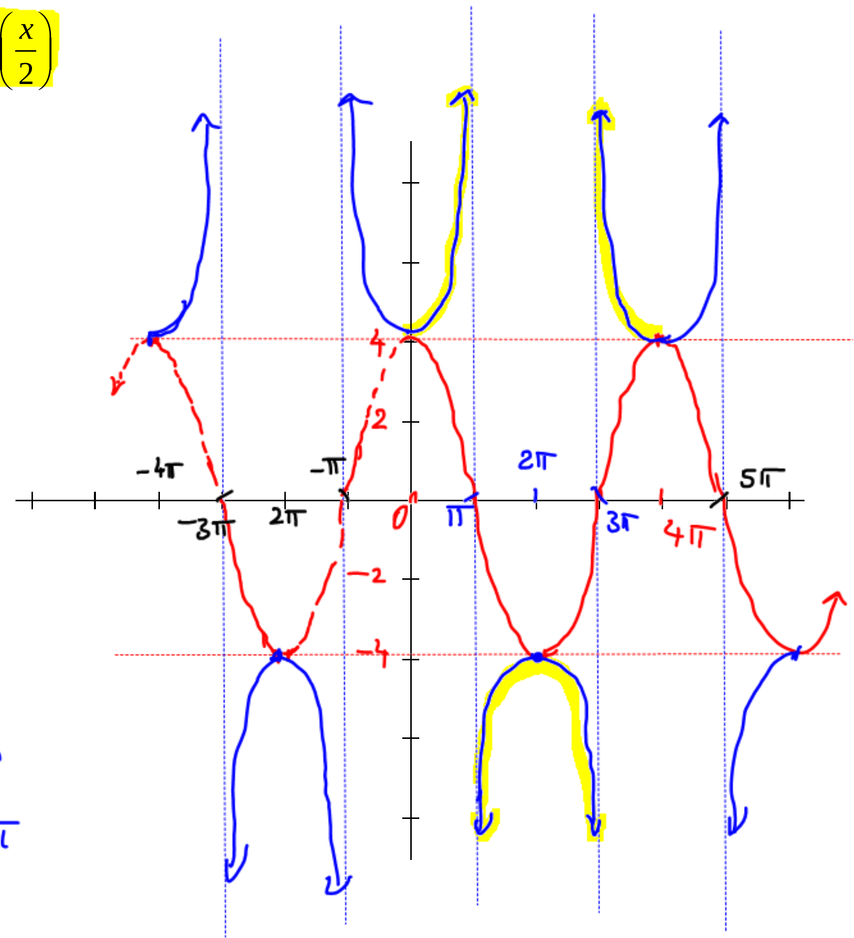
$y = 4\cos\left(\frac{x}{2}\right)$

Vertical Stretch

$B = \frac{1}{2}$

Horizontal Strech

$\text{period} = \frac{2\pi}{1/2} = 4\pi$



Note $f(x) = 4\sec\left(\frac{x}{2}\right)$ has V.A at

$x = \pi, 3\pi, 5\pi \ldots$

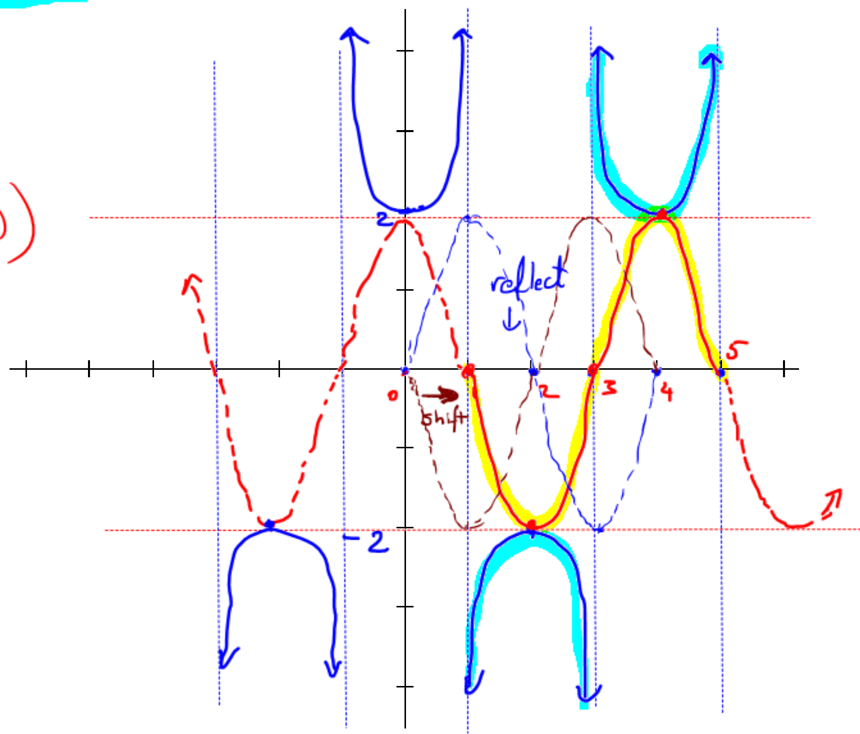
$-\pi, -3\pi, -5\pi$

because $\cos\left(\frac{x}{2}\right) = 0$ at those points!!!

Example 2: Sketch $f(x) = -2\csc\left(\frac{\pi x}{2} - \frac{\pi}{2}\right)$

Helper graph:

$y = \overset{A}{-2}\sin\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) = -2\sin\left(\frac{\pi}{2}(x-1)\right)$



- $A = -2 \to$ vertical stretching and reflection
- $B = \frac{\pi}{2} \to$ horizontal shrinking

  period $= \frac{2\pi}{\pi/2} = 4$

- $C = \frac{\pi}{2} \to$ horizontal shift by $\frac{C}{B} = \frac{\pi/2}{\pi/2} = 1$ unit right.

$\Rightarrow$ Then put $y = -2\csc\left(\frac{\pi}{2}x - \frac{\pi}{2}\right)$.

Note $f(x) = -2\csc\left(\frac{\pi}{2}x - \frac{\pi}{2}\right)$ has V.A.

at $x = 1, 3, 5, 7 \ldots$

$-1, -3, -5, -7 \ldots$

because $\sin\left(\frac{\pi}{2}x - \frac{\pi}{2}\right) = 0$ at these values.

To be continued on Friday. 03/25

Example 3: Give an equation of the form $y = A\csc(Bx - C) + D$ and $y = A\sec(Bx - C) + D$ that could describe the following graph.

Complete the helper graph!

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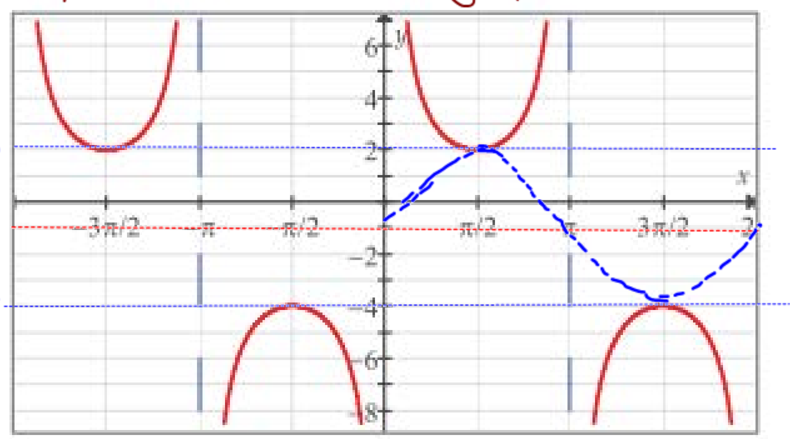

$$\boxed{f(x) = 3\csc(x) - 1}$$

Let's find helper:

$y = A\sin(Bx - C) + D$

- $A = \frac{6}{2} = 3 \Rightarrow \boxed{A = 3}$
- $\boxed{B = 1}$ since period $= 2\pi$.
- $\boxed{C = 0}$ since no shift
- $\boxed{D = -1}$, 1 unit down

$D = \frac{-4+2}{2} = -1$

Exercise: Give an equation of the form $y = A\csc(Bx - C) + D$ and $y = A\sec(Bx - C) + D$ that could describe the following graph.

Viewing as a secant function :

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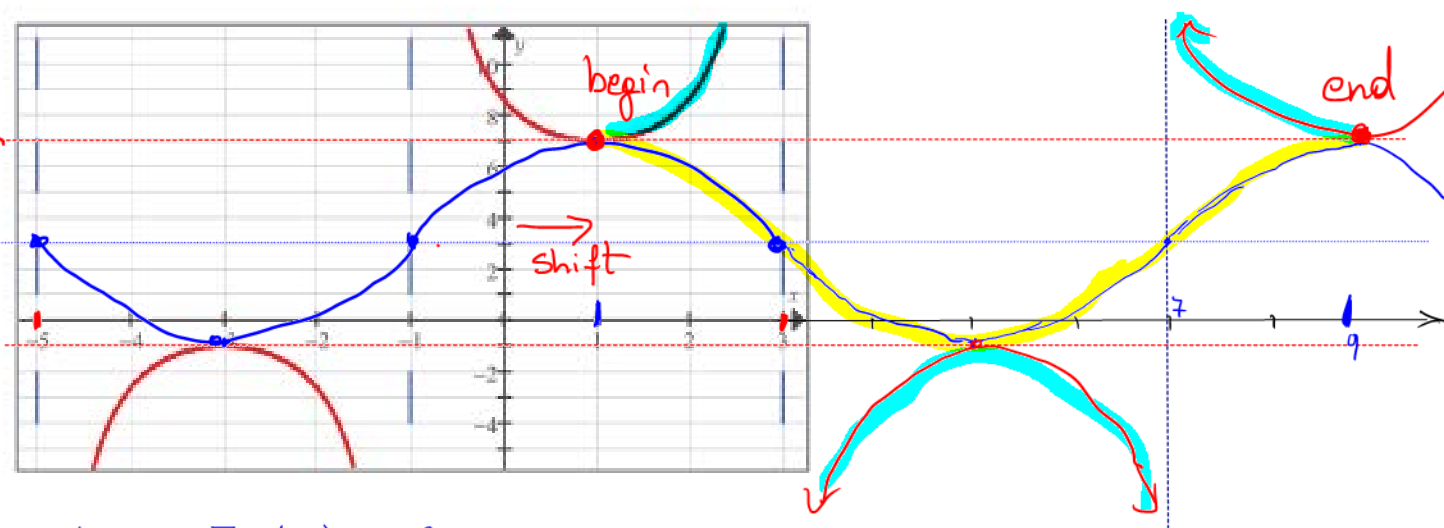


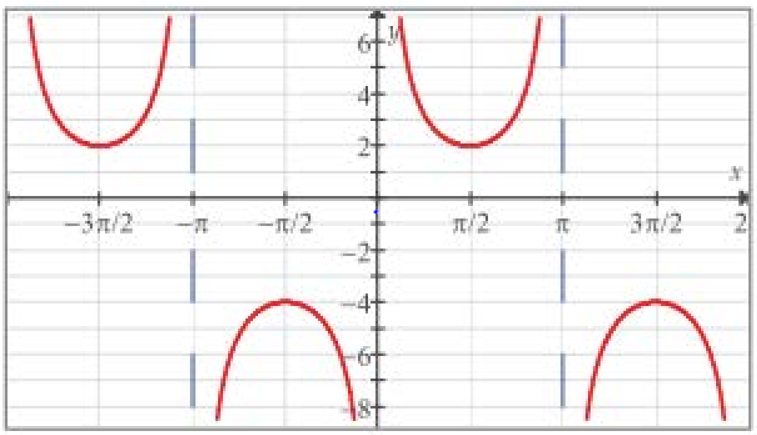
- amplitude $= \frac{7-(-1)}{2} = \frac{8}{2} = 4 \Rightarrow A = 4$
- period $= 8 = \frac{2\pi}{B} \Rightarrow B = \frac{\pi}{4}$
- shift 1 to right, $\frac{C}{B} = 1 \Rightarrow C = B = \frac{\pi}{4}$
- Vertical shift $= \frac{(-1)+7}{2} = 3 \Rightarrow D = 3$

$$y = 4\cos\left(\frac{\pi}{4}(x-1)\right) + 3$$

$$= 4\cos\left(\frac{\pi x}{4} - \frac{\pi}{4}\right) + 3$$

$$\Rightarrow \boxed{y = 4\sec\left(\frac{\pi}{4}x - \frac{\pi}{4}\right) + 3}$$

Example 3: Give an equation of the form $y = A\csc(Bx - C) + D$ and $y = A\sec(Bx - C) + D$ that could describe the following graph.

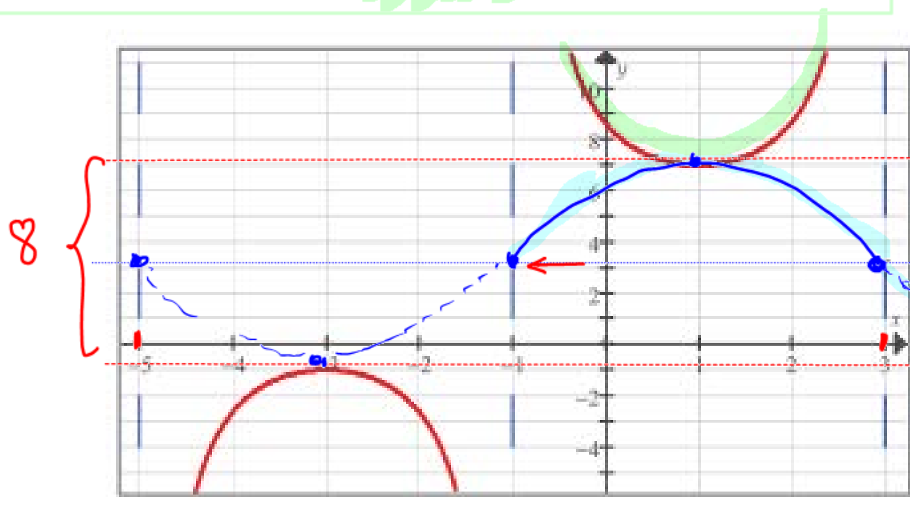


Exercise: Give an equation of the form $y = A\csc(Bx - C) + D$ and $y = A\sec(Bx - C) + D$ that could describe the following graph.

Viewed as Cosecant Function

Let's find helper:

$y = A\sin(Bx - C) + D$



- amplitude $= \frac{8}{2} = 4$ $\boxed{A = 4}$
- period $= 3 - (-5) = 8$

  $\frac{2\pi}{B} = 8 \Rightarrow \boxed{B = \frac{\pi}{4}}$
- shifted 1 unit left

  $\frac{C}{B} = -1 \Rightarrow \boxed{C = -\frac{\pi}{4}}$
- $\boxed{D = 3}$ units up!

$f(x) = 4\sin\left(\frac{\pi}{4}x + \frac{\pi}{4}\right) + 3$

$$\boxed{\Rightarrow f(x) = 4\csc\left(\frac{\pi}{4}x + \frac{\pi}{4}\right) + 3}$$

Exercise: Find the vertical asymptotes of:

a) $\boldsymbol{f(x) = 2\sec\left(\frac{x}{2} - \pi\right)} = 2 \cdot \frac{1}{\cos\left(\frac{x}{2} - \pi\right)}$

Vertical Asymptotes:

$\cos\left(\frac{x}{2} - \pi\right) = 0 \implies$

$\implies \boxed{x = k\pi, \ k \text{ odd}}$

$\frac{x}{2} - \pi = \frac{\pi}{2} \implies \frac{x}{2} = \frac{3\pi}{2} \implies x = 3\pi$

$\frac{x}{2} - \pi = \frac{3\pi}{2} \implies \frac{x}{2} = \frac{5\pi}{2} \implies x = 5\pi$

$\frac{x}{2} - \pi = -\frac{\pi}{2} \implies \frac{x}{2} = \frac{\pi}{2} \implies x = \pi$

$\frac{x}{2} - \pi = -\frac{3\pi}{2} \implies \frac{x}{2} = \frac{-\pi}{2} \implies x = -\pi$

b) $\boldsymbol{f(x) = 2\csc\left(x - \frac{\pi}{4}\right)}$

$= 2 \cdot \frac{1}{\sin\left(x - \frac{\pi}{4}\right)}$

Vertical asymptotes : $\sin\left(x - \frac{\pi}{4}\right) = 0$

$x - \frac{\pi}{4} = 0 \implies x = \frac{\pi}{4}$

$x - \frac{\pi}{4} = \pi \implies x = \frac{5\pi}{4}$

$x - \frac{\pi}{4} = 2\pi \implies x = \frac{9\pi}{4}$

$x - \frac{\pi}{4} = -\pi \implies x = \frac{-3\pi}{4}$

$x - \frac{\pi}{4} = -2\pi \implies x = \frac{-7\pi}{4}$

$\boxed{x = \frac{k\pi}{4}, \quad k = \dots -7, -3, 1, 5, 9, \dots}$

or $x = \frac{\pi}{4} + k\pi$, k integer.