Popper #16 — 1. A, 2. A, 3. A.

Popper # 17 ← Bubble

From unit circle, $\theta = \cancel{\frac{\pi}{6}}, \frac{5\pi}{6}$

① Given $\sin\theta = \frac{1}{2}$ and θ is in QII, find θ.

A. $\frac{\pi}{6}$ B. $\frac{5\pi}{6}$ C. $\frac{\pi}{3}$ D. $\frac{2\pi}{3}$ E. none

$\theta = \cancel{\frac{\pi}{4}}, \frac{7\pi}{4} = -\frac{\pi}{4}$

② Given $\cos\theta = \frac{\sqrt{2}}{2}$ and θ is in QIV, find θ.

A. $\frac{\pi}{4}$ B. $\frac{3\pi}{4}$ C. $-\frac{\pi}{4}$ D. none

③ $\sin^{-1}\left(\underbrace{-\frac{1}{2}}_{\text{negative}}\right) = \theta \Leftarrow$ Quadrant $\underline{IV}$ $\Rightarrow \theta = -\frac{\pi}{6}$

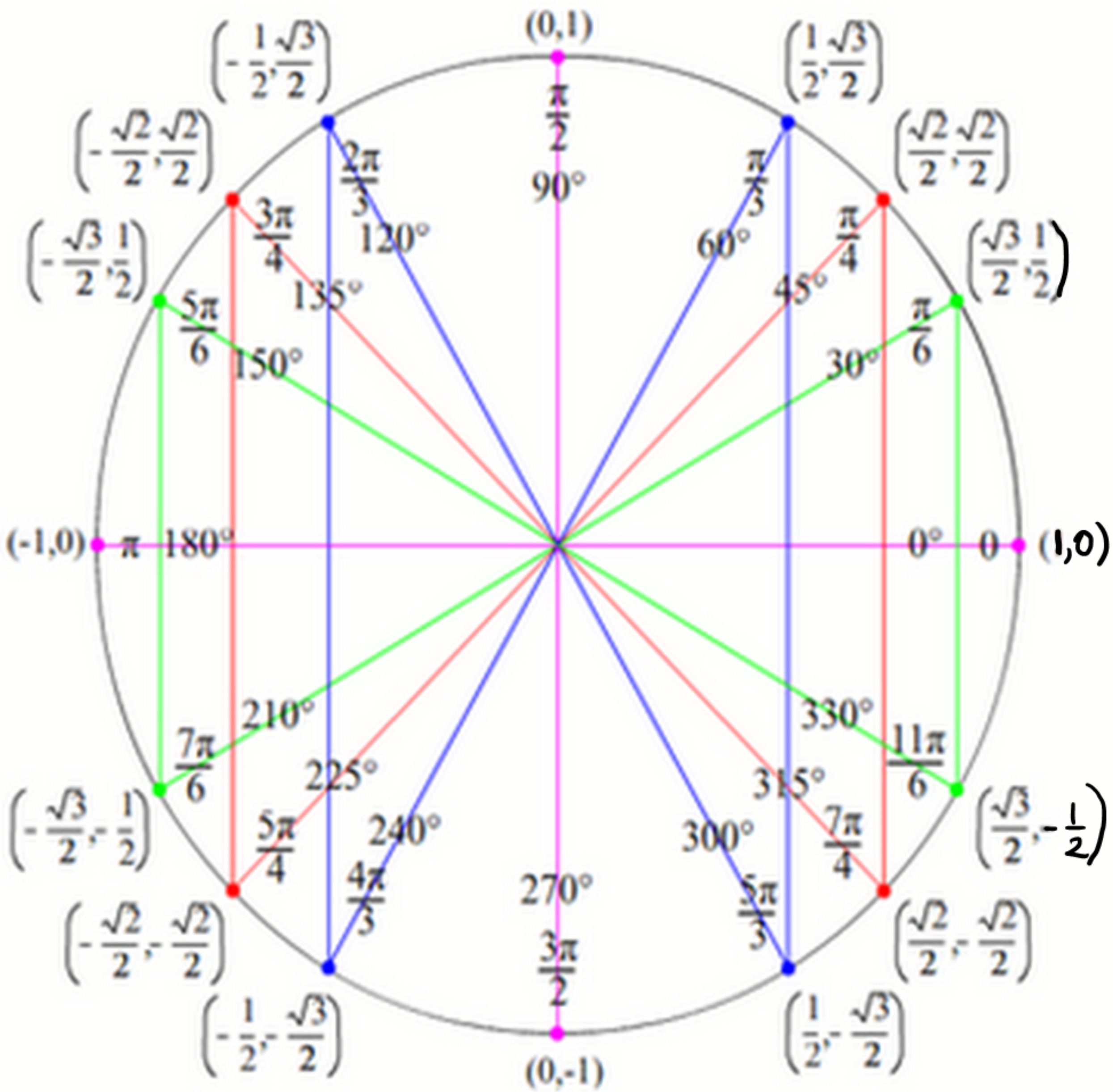
A. $\frac{\pi}{6}$ B. $-\frac{\pi}{6}$ C. $\frac{\pi}{3}$ D. $\frac{2\pi}{3}$

④ $\cos^{-1}\left(\underbrace{-\frac{\sqrt{2}}{2}}_{\text{negative}}\right) = \theta \Leftarrow$ quadrant $\underline{II}$ $\Rightarrow \theta = \frac{3\pi}{4}$

A. $\frac{\pi}{4}$ B. $-\frac{\pi}{4}$ C. $\frac{3\pi}{4}$ D. $\frac{-3\pi}{4}$

Unit Circle : $x^2+y^2=1$, any point $(x,y) = (\cos\theta, \sin\theta)$

Never forget : $\boxed{x=\cos\theta,\ y=\sin\theta}$

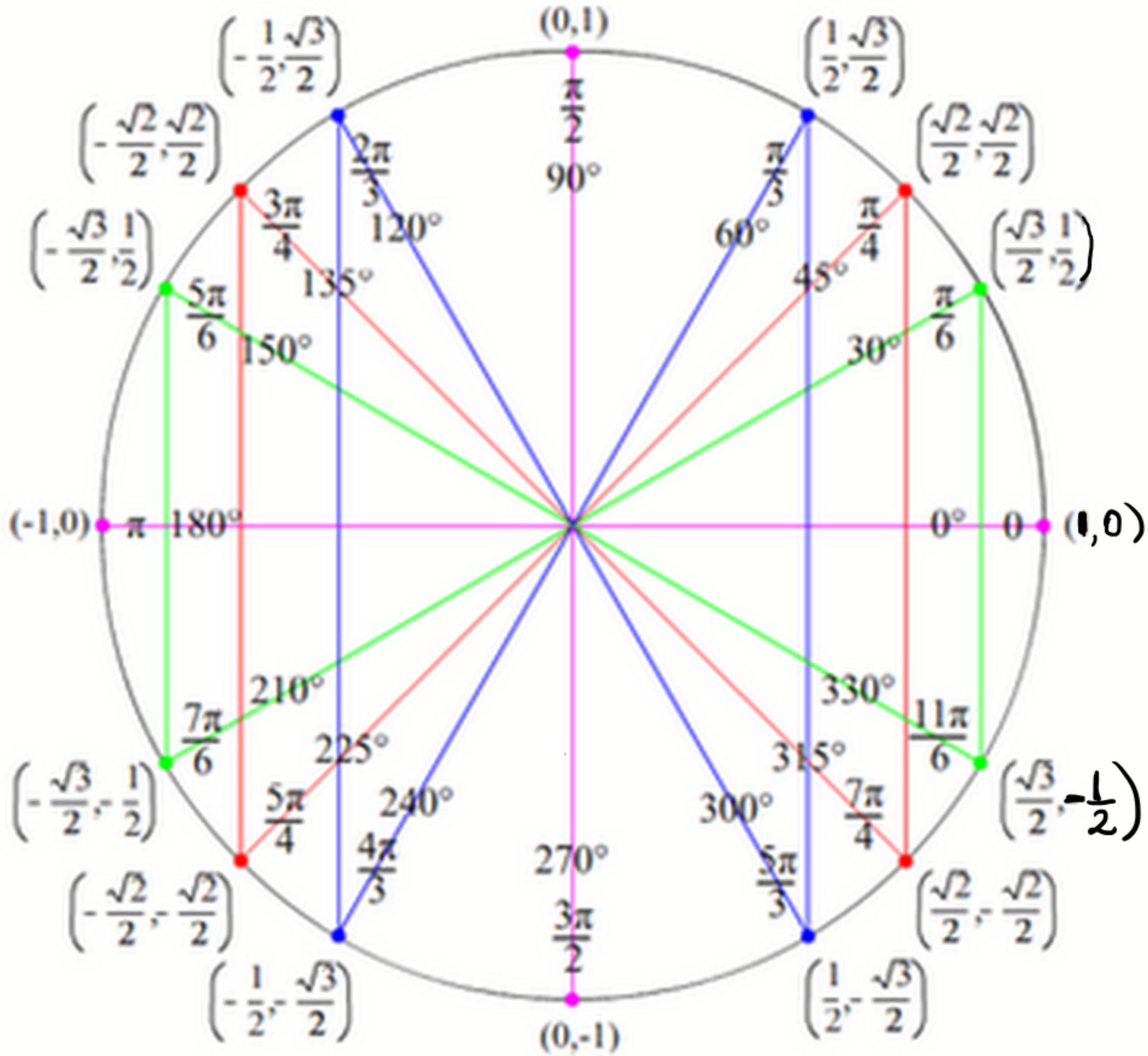


ex. Find all angles in unit circle for which $\cos\theta = \frac{1}{2}$

→ From unit circle , we can see

$\theta = \frac{\pi}{3}$

Two Answers or $\theta = \frac{5\pi}{3} = \frac{-\pi}{3}$ (same position)



Using unit circle, we plot $\boxed{f(x) = \cos x}$.

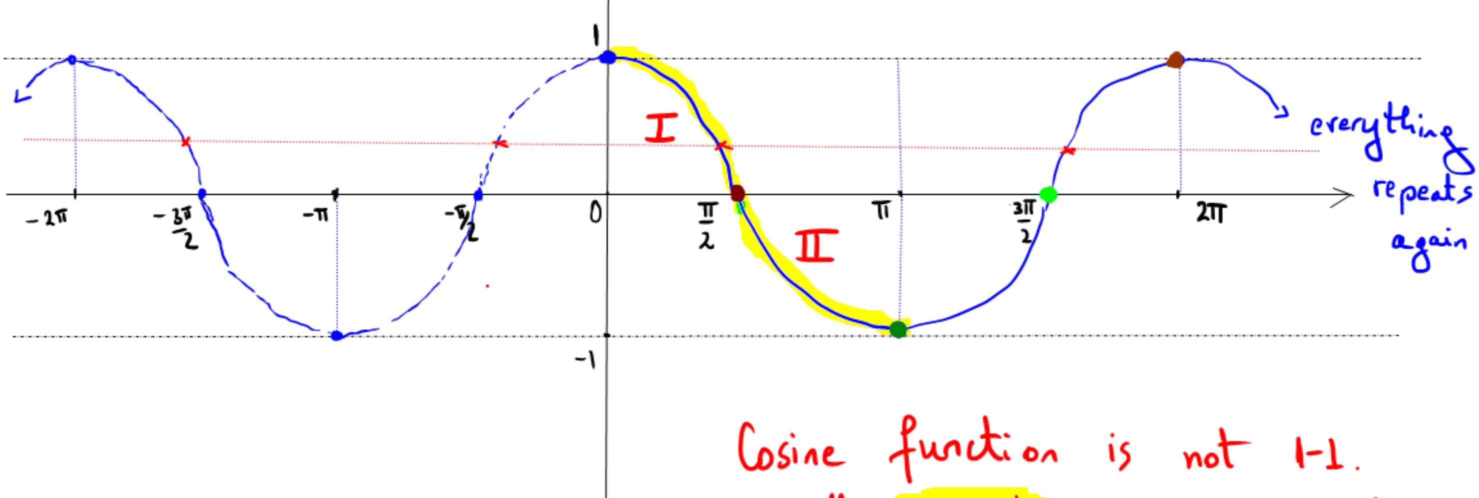
$\theta = 0$ $\cos 0 = 1$

$\theta = \frac{\pi}{2}$ $\cos\frac{\pi}{2} = 0$

$\theta = \pi$ $\cos\pi = -1$

$\theta = \frac{3\pi}{2}$ $\cos\frac{3\pi}{2} = 0$

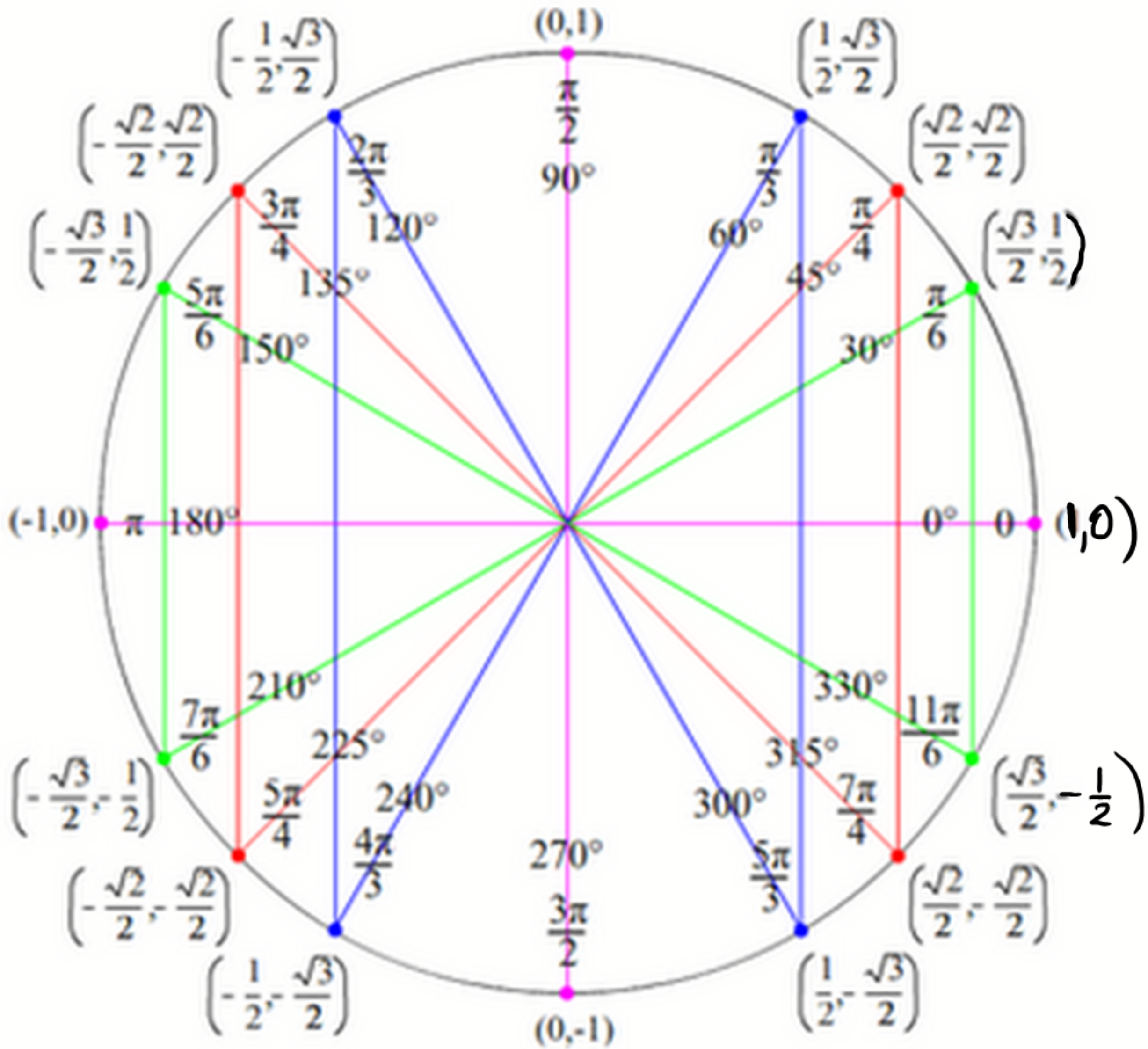
$\theta = 2\pi$ $\cos 2\pi = 1$



Cosine function is not 1-1.

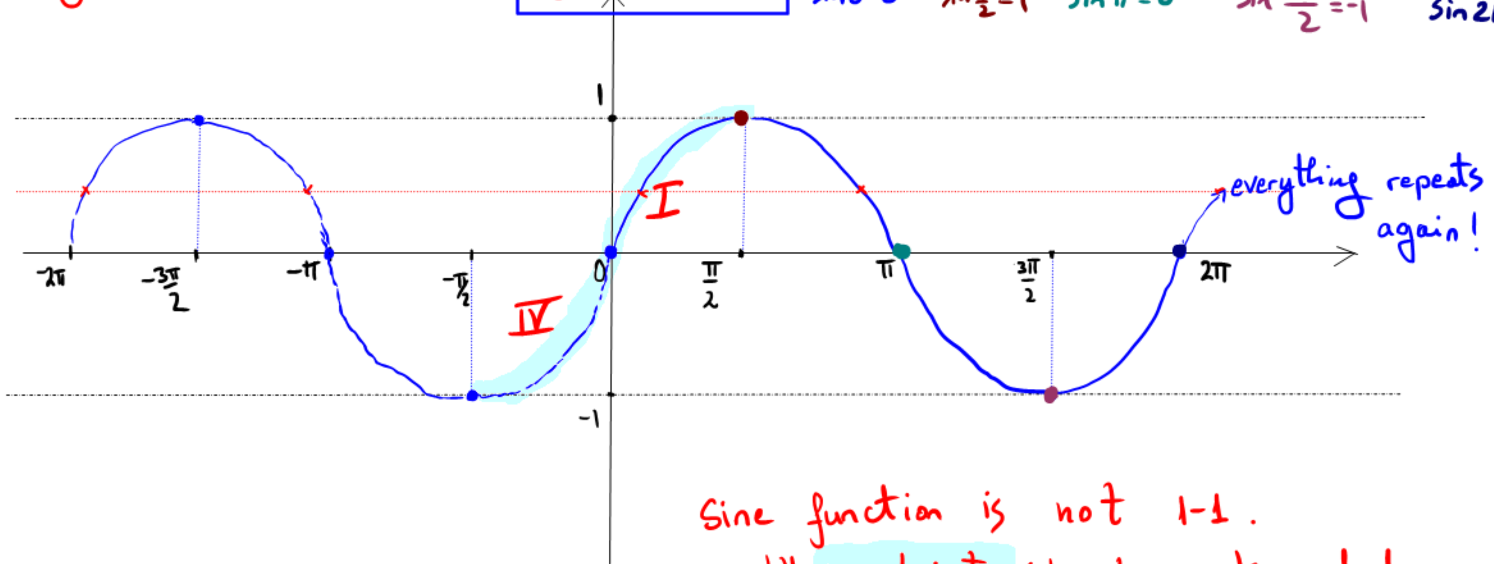
We'll restrict it to make it 1-1.

I, II very important



Using unit circle, we plot $\boxed{f(x) = \sin x}$

$\theta = 0$ $\sin 0 = 0$ | $\theta = \frac{\pi}{2}$ $\sin\frac{\pi}{2} = 1$ | $\theta = \pi$ $\sin\pi = 0$ | $\theta = \frac{3\pi}{2}$ $\sin\frac{3\pi}{2} = -1$ | $\theta = 2\pi$ $\sin 2\pi = 0$



Sine function is not 1-1.

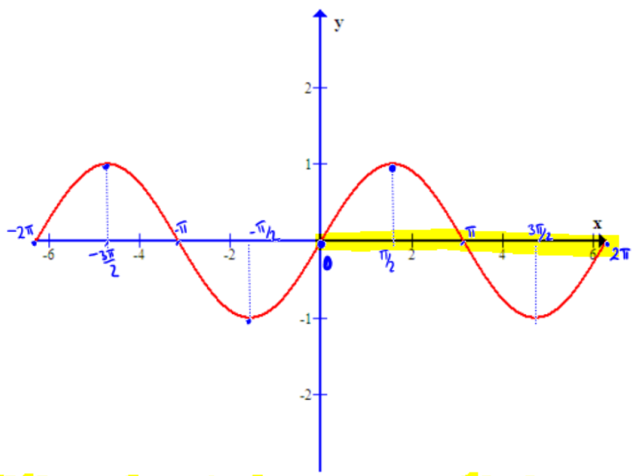
We'll restrict it to make 1-1.

IV, I very important

Section 5.4 - Inverse Trigonometry

We have not yet studied the graphs of the sine and cosine functions, but we are going to take a quick look at them before we cover inverse trigonometry.

Here's the graph of $f(x) = \sin(x)$.



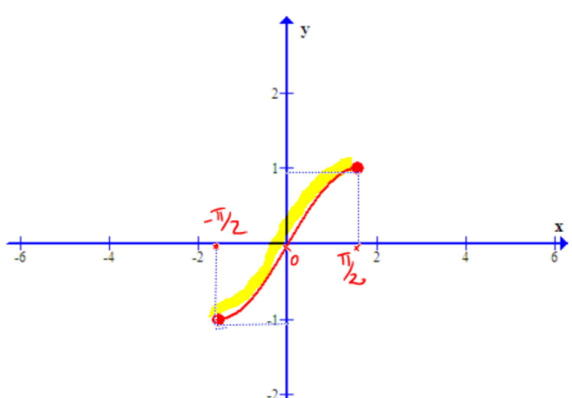
The function is a periodic function. That means that the functions repeats its values in regular intervals, which we call the period. The period of sine function $= 2\pi$.

Is it one to one? $f(x) = \sin(x)$ is not one-to-one! We can make it one-to-one. Restrict the domain!!!

If the function is not one-to-one, we run into problems when we consider the inverse of the function. What we want to do with the sine function is to restrict the values for sine. When we make a careful restriction, we can get something that IS one-to-one.

If we limit the function to the interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$, the graph will look like this:

In this interval, sine function gets all its possible values. Hence, the restriction defines the function correctly.



Restricted Sine function

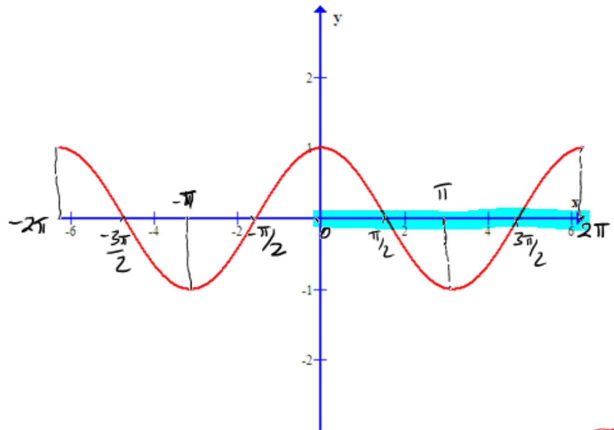
Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ← I, IV

Range: $[-1,1]$

On this limited interval, we have a one-to-one function.

$$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \xrightarrow[\text{1-1}]{f(x)=\sin x} [-1,1]$$

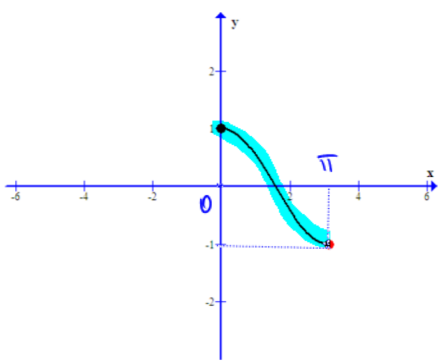
Let’s do the same thing with $f(x)=\cos(x)$. Here’s the graph of $f(x)=\cos(x)$.



$f(x) = \cos x$,
periodic,
period $= 2\pi$,
not 1-1.
Restrict the domain.

It’s also not one-to-one. If we limit the function to the interval $[0,\pi]$, however, the function IS one-to-one.

Here’s the graph of the restricted cosine function. In this interval, $f(x)=\cos x$ gets all its possible values, hence restriction defines the function correctly.



Restricted Cosine function

Domain: $[0,\pi]$ ← I, II

Range: $[-1,1]$

$$[0,\pi] \xrightarrow[1-1]{f(x)=\cos x} [-1,1]$$

To be continued on Wednesday, 03/09.

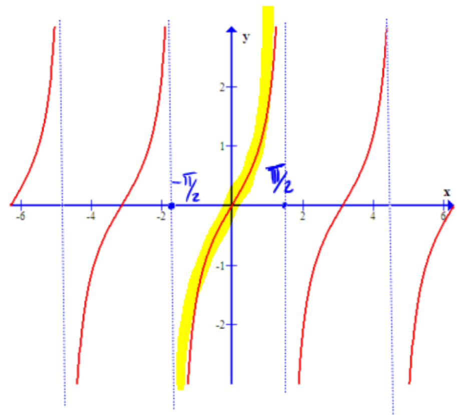
$\boxed{\text{Period} = \pi}$ ⇐ Length of interval where it repeats.

Here's the graph of $f(x) = \tan(x)$. Is it one-to-one?

$\tan x = \dfrac{\sin x}{\cos x}$

$\cos x = 0$

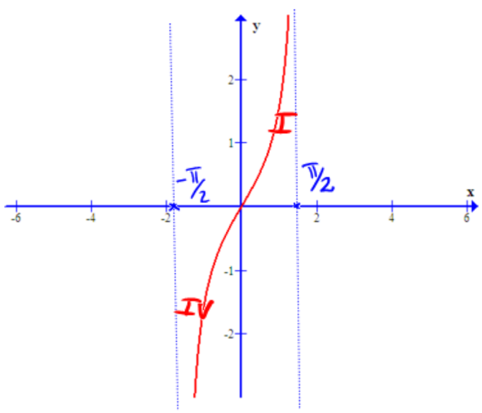
V.A ⇔ $x = \frac{\pi}{2}, -\frac{\pi}{2}, \ldots$



The highlighted shape repeats forever.

If we restrict $f(x) = \tan x$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$, it becomes 1-1.

If we restrict the function to the interval $\left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$, then the restricted function IS one-to-one.

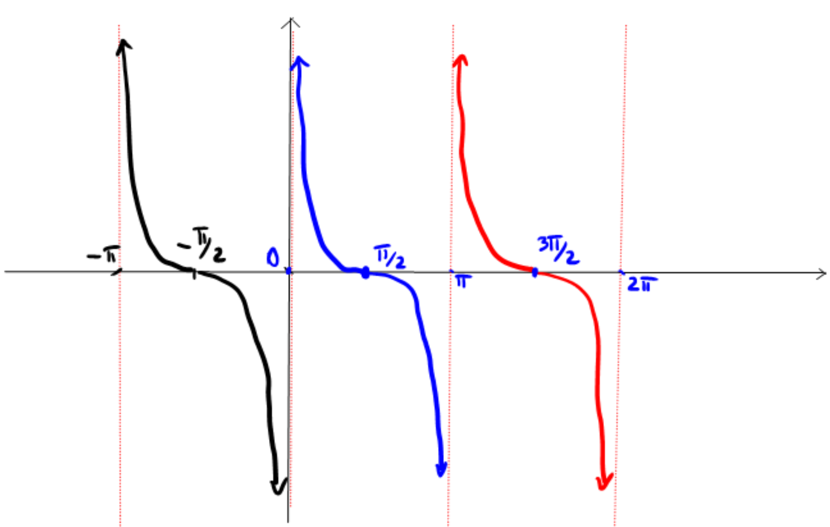


Restricted Tangent function

Domain: $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ ← I, IV

Range: $(-\infty, \infty)$

Graph for $\cot x = \dfrac{\cos x}{\sin x}$, $\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$ ⇔ V.A.



Similar as $\tan x$, $\cot x$ gets repeated in every π-interval

⇒ period $= \pi$.

Restrict on $(0, \pi)$ ⇔ I, II

then

$\cot x : \underset{\text{I, II}}{(0, \pi)} \xrightarrow{1-1} (-\infty, \infty)$

$\cot^{-1}(x)$ exists

We now want to evaluate inverse trig functions. With these problems, instead of giving you the angle and asking you for the value, I'll give you the value and ask you what angle gives you that value.

Important: When we covered the unit circle, we saw that there were two angles that had the same value for most of our angles. With inverse trig, we can't have that. We need a unique answer, because of our need for 1-to-1 functions. We'll have one quadrant in which the values are positive and one quadrant where the values are negative. The restricted graphs we looked at can help us know where these values lie. We'll only state the values that lie in these intervals (same as the intervals for our graphs):

Restricted Sine function

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\leftrightarrow$ IV , I

Range: $[-1,1]$

Inverse Sine Function:

Domain: $[-1,1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (quadrants 1 and 4)

Notation: $\sin^{-1}(x)$ or $\arcsin(x)$

$\sin(x)$: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \xrightarrow{1-1} [-1,1]$

$\sin^{-1}(x)$: $[-1,1] \xrightarrow{1-1} \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\leftrightarrow$ quadrant I, IV

Example: $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \;\rightarrow\; \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ ← use unit circle to evaluate.

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ ← What is the angle whose sine is $\frac{1}{2}$?

(Looking at unit circle, you find $\frac{\pi}{6}$ or $\cancel{\frac{5\pi}{6}}$ since

$\sin^{-1}(x)$: $[-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ belongs does not belong)

- When asked to find all angles in unit circle for which

$$\sin\theta = \frac{1}{2} \Rightarrow \text{Answer: } \boxed{\theta = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$$

- Evaluate $\sin^{-1}\left(\frac{1}{2}\right)$:

$\sin^{-1} : [-1, 1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ = Quadrant I or IV

Think $\sin^{-1}\left(\frac{1}{2}\right) = \theta$ in quadrant I or IV

$$\Updownarrow$$

$$\sin\theta = \frac{1}{2} \Rightarrow \theta = \underset{\text{I}}{\frac{\pi}{6}} \text{ or } \underset{\text{II}}{\cancel{\frac{5\pi}{6}}} \text{ No}$$

$$\boxed{\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}}$$

- Short cut:

Evaluate $\sin^{-1}(x)$ $\longrightarrow$
- if x positive $\Rightarrow$ Q I
- if x negative $\Rightarrow$ Q IV

$-1 \le x \le 1$.

Restricted Cosine function

Domain: $[0, \pi]$ $\leftrightarrow$ I, II

Range: $[-1, 1]$

Inverse Cosine Function:

Domain: $[-1, 1]$

Range: $[0, \pi]$ (quadrants 1 and 2)

Notation: $\cos^{-1}(x)$ or $\arccos(x)$

$\cos x : [0, \pi] \xrightarrow{1\text{-}1} [-1, 1]$

$\cos^{-1}(x) : [-1, 1] \xrightarrow{1\text{-}1} [0, \pi] \leftrightarrow$ quadrant I, II

Example: $\cos\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2} \;\rightarrow\; \cos^{-1}\left(\dfrac{\sqrt{2}}{2}\right) = \dfrac{\pi}{4}$

$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\leftarrow$ look at unit circle

$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$ $\leftarrow$ What is the angle whose cosine is $\frac{\sqrt{2}}{2}$?

(Looking at unit circle, you find $\frac{\pi}{4}$ and $\cancel{\frac{7\pi}{4}}$ does not belong
belongs
$\cos^{-1}(x) : [-1, 1] \rightarrow [0, \pi]$)

- When asked find all angles in unit circle for which

$$\cos\theta = \frac{\sqrt{2}}{2} \implies \boxed{\text{Answer: } \theta = \frac{\pi}{4} \text{ or } \frac{7\pi}{4} = -\frac{\pi}{4}}$$

- Evaluate $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\cos^{-1} : [-1,1] \longrightarrow [0,\pi]$ = quadrant I or II

Think $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \theta$ in quadrant I or II.

$$\Updownarrow$$

$$\cos\theta = \frac{\sqrt{2}}{2} \implies \theta = \underset{\text{I}}{\frac{\pi}{4}} \text{ or } \underset{\text{IV}}{\cancel{\frac{7\pi}{4}}} \text{ No}$$

$$\implies \boxed{\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}}$$

- Shortcut:

Evaluate $\cos^{-1}(x)$ $\longrightarrow$
- if x is positive $\implies$ Q I
- if x is negative $\longrightarrow$ Q II

$$\downarrow$$

$$-1 \leq x \leq 1$$

To be continued on Wednesday, 03/09.

Restricted Tangent function

Domain: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Range: $(-\infty, \infty)$

Inverse Tangent Function:

Domain: $(-\infty, \infty)$

Range: $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ (quadrants 1 and 4)

Notation: $\tan^{-1}(x)$ or $\arctan(x)$

$\tan(x) : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \xrightarrow{1\text{-}1} (-\infty, \infty)$

$\tan^{-1}(x) : (-\infty, \infty) \xrightarrow{1\text{-}1} \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ ← quadrant I, IV.

Example: $\tan\left(\frac{\pi}{4}\right) = 1 \;\rightarrow\; \tan^{-1}(1) = \frac{\pi}{4}$

$\tan\left(\frac{\pi}{4}\right) = 1$ ← use unit circle.

$\tan^{-1}(1) = \frac{\pi}{4}$ ← What is the angle whose tangent is 1?

By unit circle, you find $\underbrace{\frac{\pi}{4}}_{\text{belong}}$ or $\cancel{\frac{5\pi}{4}}$, and $\frac{5\pi}{4}$ does not belong!

$\tan^{-1}(x) : (-\infty, +\infty) \xrightarrow[1\text{-}1]{} \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$$\boxed{\begin{array}{l} \text{RULE}: \tan^{-1}(x) \longrightarrow \text{if } x \text{ positive} \Rightarrow \text{Q I} \\ \qquad\qquad\qquad\qquad\quad \text{if } x \text{ negative} \Rightarrow \text{Q IV}. \end{array}}$$

Example 1: Compute each of the following:

a) $\sin^{-1}\left(\frac{1}{2}\right) = \theta \iff \sin\theta = \frac{1}{2} \iff \theta = \frac{\pi}{6}$ or $\cancel{\frac{5\pi}{6}} \implies \boxed{\theta = \frac{\pi}{6}}$

(I, IV; I; II)

b) $\tan^{-1}\left(\sqrt{3}\right) = \theta \iff \tan\theta = \sqrt{3} \iff \theta = \frac{\pi}{3}$ or $\cancel{\frac{4\pi}{3}} \implies \boxed{\theta = \frac{\pi}{3}}$

(I, IV; I; III)

c) $\arccos(0) = \theta \iff \cos\theta = 0 \iff \theta = \frac{\pi}{2}$ or $\cancel{\frac{3\pi}{2}} \implies \boxed{\theta = \frac{\pi}{2}}$

(I, II; I, II)

d) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \theta \iff \sin\theta = -\frac{\sqrt{2}}{2} \iff \theta = \cancel{\frac{5\pi}{4}}$ or $\frac{7\pi}{4} = -\frac{\pi}{4} \implies \boxed{\theta = -\frac{\pi}{4}}$

(I, IV; III; IV)

e) $\sin^{-1}\left(-\frac{1}{2}\right) = \theta \iff \sin\theta = -\frac{1}{2} \iff \theta = \cancel{\frac{7\pi}{6}}$ or $\frac{11\pi}{6} = -\frac{\pi}{6} \implies \boxed{\theta = -\frac{\pi}{6}}$

(I, IV; III; IV)

f) $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \theta \iff \cos\theta = -\frac{\sqrt{3}}{2} \iff \theta = \frac{5\pi}{6}$ or $\cancel{\frac{7\pi}{6}} \implies \boxed{\theta = \frac{5\pi}{6}}$

(I, II; II; III)

g) $\arctan(-1) = \theta \iff \tan\theta = -1 \iff \theta = \cancel{\frac{3\pi}{4}}$ or $\frac{7\pi}{4} = -\frac{\pi}{4} \implies \theta = -\frac{\pi}{4}$

(I, IV; II; IV)

Never forget: If the angle is in quadrant IV, write its negative equivalent.

$\mathrm{I,II}$

h) $\sec^{-1}(2). = \theta \Leftrightarrow \sec\theta = 2$

$\sec\theta = \frac{1}{\cos\theta}$ $\quad$ $\mathrm{I,II}$ $\quad$ I $\quad$ IV

$\sec\theta = \frac{1}{\cos\theta} = 2 \Leftrightarrow \cos\theta = \frac{1}{2} \Leftrightarrow \theta = \frac{\pi}{3}$ or $\cancel{\frac{5\pi}{3}}$

$\Rightarrow \boxed{\theta = \frac{\pi}{3}}$

i) $\csc^{-1}(0) = \theta \Leftrightarrow \csc\theta = 0$

$\Downarrow$

undefined!

$\frac{1}{\sin\theta} = 0 \Leftrightarrow$ there is no angle for which $\frac{1}{\sin\theta} = 0$!

$\uparrow$

$\mathrm{I,IV}$

NOTE: Domains of inverse trig functions: DO NOT FORGET!

$f(x) = \sin^{-1}(x)$; [-1,1] $\longrightarrow [-\frac{\pi}{2}, \frac{\pi}{2}]$ = range = quadrant I & IV

$f(x) = \cos^{-1}(x)$; [-1,1] $\longrightarrow [0, \pi]$ = range = quadrant I & II

$f(x) = \tan^{-1}(x)$; $(-\infty, \infty)$ $\longrightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$ = range = quadrant I & IV

$f(x) = \cot^{-1}(x)$; $(-\infty, \infty)$ $\longrightarrow (0, \pi)$ = range = quadrant I & II

look at the next page for graphs

$f(x) = \sec^{-1}(x)$; $(-\infty, 1] \cup [1, \infty)$ $\longrightarrow [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ = range = quadrant I, II

$f(x) = \csc^{-1}(x)$; $(-\infty, 1] \cup [1, \infty)$ $\longrightarrow [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ = range = quadrant I, IV

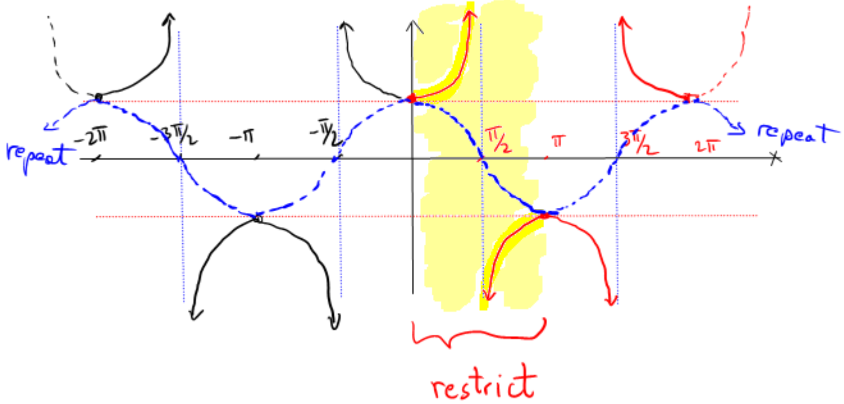
For example; $\sin^{-1}(2)$ or $\cos^{-1}(\sqrt{2})$ are not defined.

$\uparrow$ $\quad$ $\uparrow$

$2 > 1$ $\quad$ $\sqrt{2} > 1$

can't be $\quad$ can't be

Plot $f(x) = \sec x = \frac{1}{\cos x}$ $\Rightarrow$ V.A. $\cos x = 0 \Rightarrow x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$
$-\frac{\pi}{2}, -\frac{3\pi}{2}, \ldots$

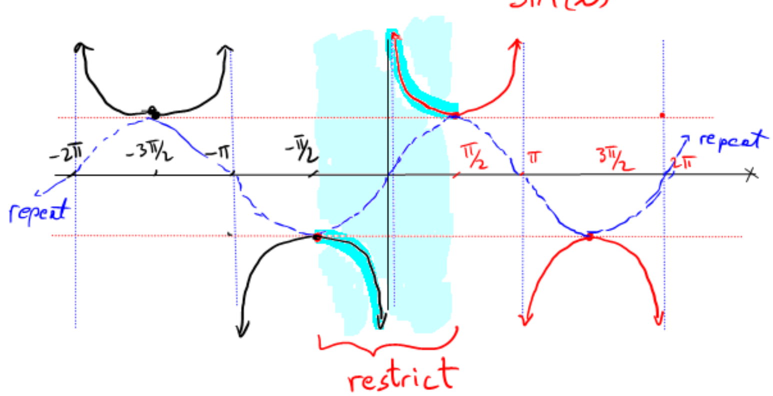


$\sec x : [0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi] \xrightarrow{1-1} (-\infty, -1] \cup [1, \infty)$

$\underline{\sec^{-1}(x)} : \underbrace{(-\infty, -1] \cup [1, \infty)}_{\text{domain}} \longrightarrow \overbrace{[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]}^{\text{range}}$

I & II

Plot $f(x) = \csc x = \frac{1}{\sin(x)}$ $\Rightarrow$ V.A. $\sin x = 0 \Rightarrow x = 0, \pi, 2\pi, 3\pi, \ldots$
$-\pi, -2\pi, -3\pi \ldots$



$\csc x : [-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}] \xrightarrow{1-1} (-\infty, -1] \cup [1, \infty)$

$\underline{\csc^{-1}(x)} : \underbrace{(-\infty, -1] \cup [1, \infty)}_{\text{domain}} \longrightarrow \overbrace{[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]}^{\text{range}}$

IV & I

Never forget the range of inverse trigonometric functions.

Example 2: Find the exact value: $\sin^{-1}\left[\underbrace{\sin\left(\frac{7\pi}{6}\right)}_{\sin\left(\frac{7\pi}{6}\right)=-\frac{1}{2}}\right] = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ ← quadrant IV

Example 3: Find the exact value: $\cos^{-1}\left[\underbrace{\cos\left(\frac{4\pi}{3}\right)}_{\cos\left(\frac{4\pi}{3}\right)=-\frac{1}{2}}\right] = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ ← quadrant II

Note: $\overset{\text{I, II}}{\cos^{-1}}\left(\cos\left(\underbrace{\frac{2\pi}{3}}_{\text{II}}\right)\right) = \underbrace{\frac{2\pi}{3}}_{\text{II}}$ but $\underset{\text{I, II}}{\cos^{-1}}\left(\cos\left(\underbrace{\frac{4\pi}{3}}_{\text{III}}\right)\right) \neq \frac{4\pi}{3}$

Example 4: Find the exact value: $\tan^{-1}\left[\underbrace{\tan\left(\frac{3\pi}{4}\right)}_{\tan\left(\frac{3\pi}{4}\right)=-1}\right] = \tan^{-1}(-1) = -\frac{\pi}{4}$ ← quadrant IV

$\overset{\text{I, IV}}{\tan^{-1}}\left(\tan\left(\underbrace{-\frac{\pi}{4}}_{\text{IV}}\right)\right) = \underbrace{-\frac{\pi}{4}}_{\text{IV}}$ but $\underset{\text{I, IV}}{\tan^{-1}}\left(\tan\left(\underbrace{\frac{3\pi}{4}}_{\text{II}}\right)\right) \neq \frac{3\pi}{4}$

$\overset{\text{I, IV}}{\tan^{-1}}\left(\tan\left(\underbrace{\frac{7\pi}{4}}_{\text{IV}}\right)\right) = -\frac{\pi}{4}$

$\frac{7\pi}{4} \sim -\frac{\pi}{4}$

Note: If a trigonometric function and its inverse are composed, then we have a shortcut. However, we need to be careful about giving an answer that is in the range of the inverse trig function.

$$\cos^{-1}[\cos(\theta)] = \theta \quad \text{if } \theta \in [0, \pi]$$

$$\sin^{-1}[\sin(\theta)] = \theta \quad \text{if } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\tan^{-1}[\tan(\theta)] = \theta \quad \text{if } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

Examples:

Be careful!!!

$$\sin^{-1}\left[\sin\left(\frac{\pi}{8}\right)\right] = \frac{\pi}{8} \quad \text{but} \quad \sin^{-1}\left[\sin\left(\frac{7\pi}{8}\right)\right] \neq \frac{7\pi}{8}$$ should be $\frac{\pi}{8}$

$$\cos^{-1}\left[\cos\left(\frac{\pi}{8}\right)\right] = \frac{\pi}{8} \quad \text{but} \quad \cos^{-1}\left[\cos\left(\frac{9\pi}{8}\right)\right] \neq \frac{9\pi}{8}$$ should be $-\frac{\pi}{8}$

$$\tan^{-1}\left[\tan\left(\frac{\pi}{8}\right)\right] = \frac{\pi}{8} \quad \text{but} \quad \tan^{-1}\left[\tan\left(\frac{7\pi}{8}\right)\right] \neq \frac{7\pi}{8}$$ should be $-\frac{\pi}{8}$

Example 5: Find the exact value: $\cos\left[\cos^{-1}\left(\frac{1}{6}\right)\right]$. $= \frac{1}{6}$

well-defined

If the inverse trig function is the inner function, then our job is easier.

$\cos[\cos^{-1}(x)] = x$ for any number x such that $-1 \le x \le 1$.
$\sin[\sin^{-1}(x)] = x$ for any number x such that $-1 \le x \le 1$.
$\tan[\tan^{-1}(x)] = x$ for any number x.
as long as it is well-defined!

Examples:

$$\sin\left[\sin^{-1}\left(\frac{1}{5}\right)\right] = \frac{1}{5}$$

$$\cos\left[\cos^{-1}\left(-\frac{2}{7}\right)\right] = -\frac{2}{7}$$

ex. $\sin(\underbrace{\sin^{-1}(2)}_{\text{undefined}}) = \text{undefined}.$

$$\tan[\tan^{-1}\left(\frac{1}{4}\right)] = \frac{1}{4}$$

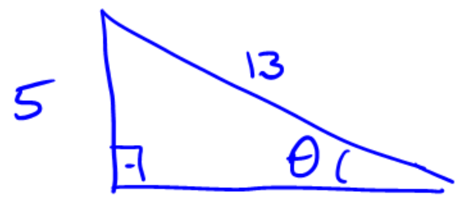
$$\tan[\tan^{-1}(5)] = 5 .$$

Example 6: Find the exact value: $\underbrace{\cos\left[\sin^{-1}\left(\frac{5}{13}\right)\right]}_{\theta}$. $= \cos\theta = ?$

$\sin^{-1}\left(\frac{5}{13}\right) = \theta \iff$ Quadrant I

$\iff \sin\theta = \frac{5}{13}$

$\implies \boxed{\cos\theta = \frac{12}{13}}$

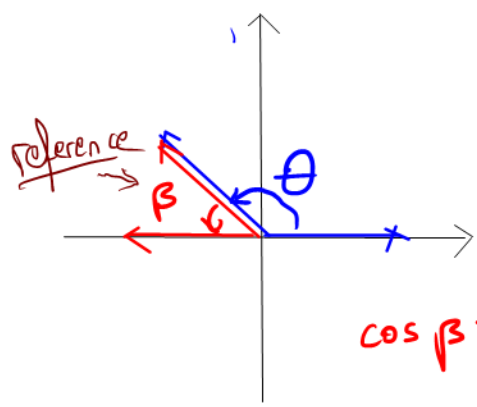


$x = 12 \leftarrow$ Pythagorean

Example 7: Find the exact value: $\tan\left[\underbrace{\cos^{-1}\left(-\frac{2}{5}\right)}_{\theta}\right]$. $= \tan\theta = ?$

$\cos^{-1}\left(-\frac{2}{5}\right) = \theta$

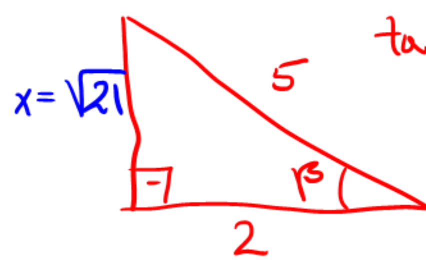
$\iff \cos\theta = \frac{-2}{5}$, quadrant II



$\cos\beta = \frac{2}{5}$

II

$\implies \boxed{\tan\theta = -\frac{\sqrt{21}}{2}}$



$\tan\beta = \frac{\sqrt{21}}{2}$