Popper 04

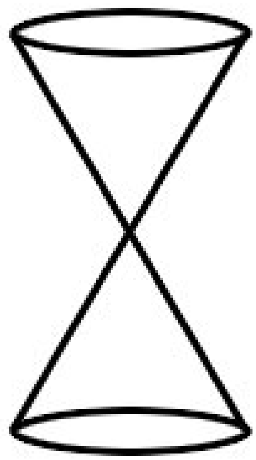
Because we ran out of time,
we have a question-free popper.

1. A
2. A
3. A

Math 1330 – Conic Sections

In this chapter, we will study conic sections (or conics). It is helpful to know exactly what a conic section is. This topic is covered in Chapter 8 of the online text.

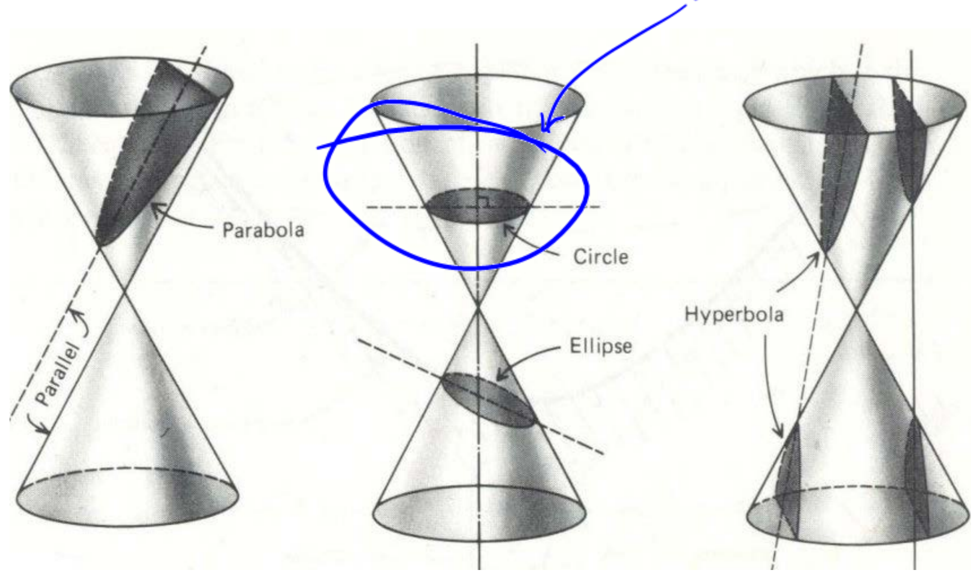
We start by looking at a double cone. Think of this as two "pointy" ice cream cones that are connected at the small tips:



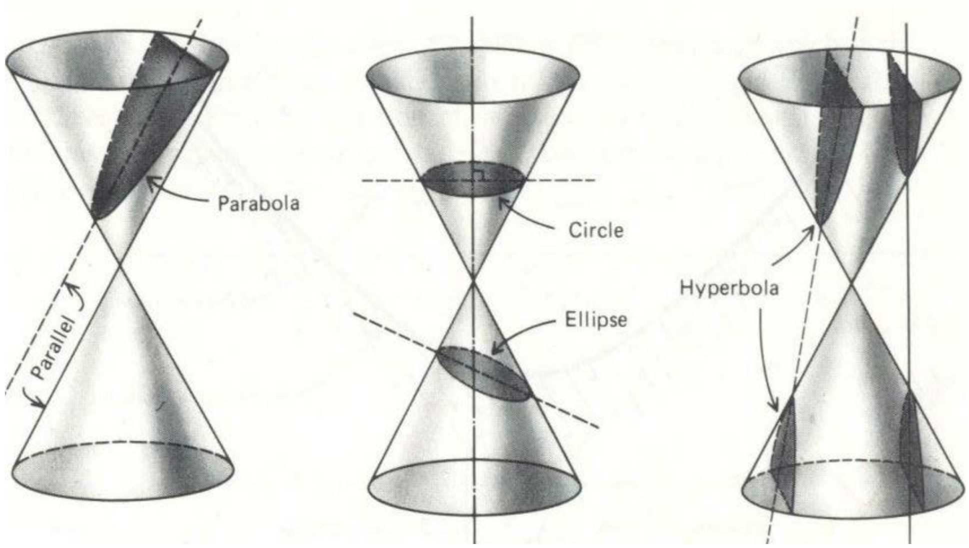
To form a conic section, we'll take this double cone and slice it with a plane. When we do this, we'll get one of several different results.

1. Parabola

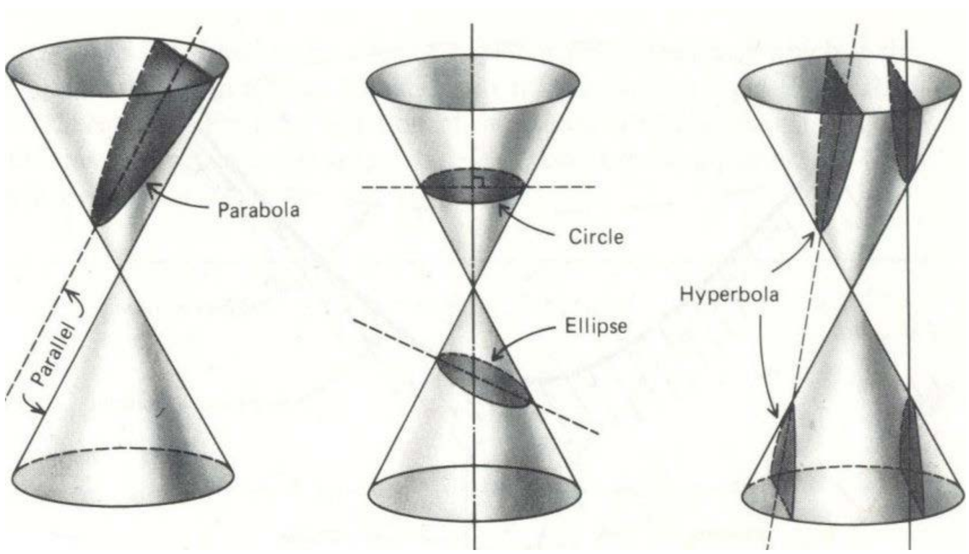
horizontal cut = Circles
Section
8.2A



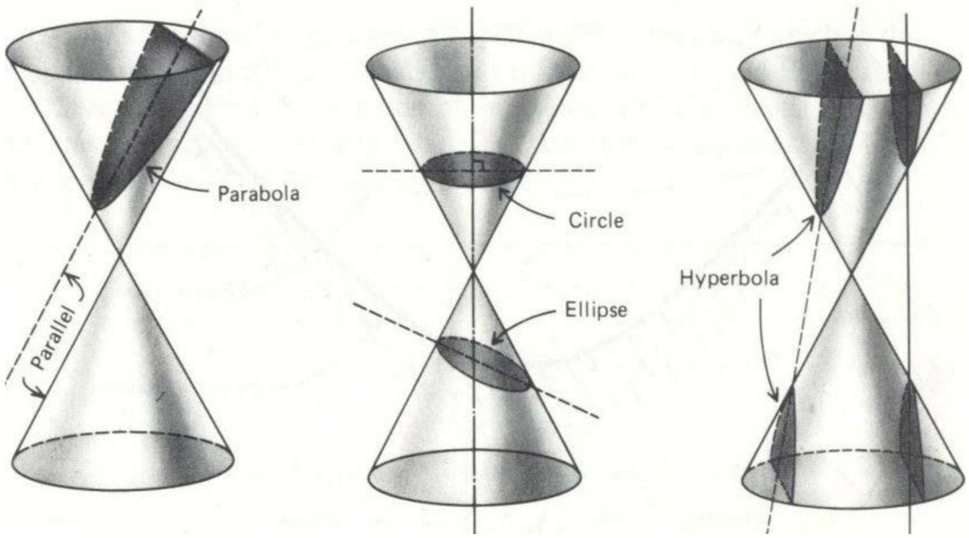
2. Ellipse



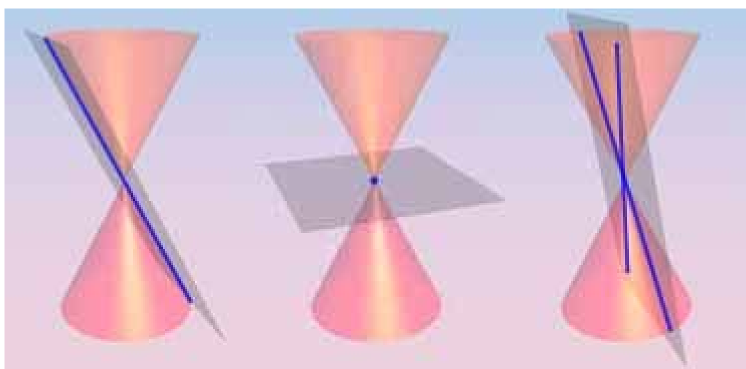
3. Circle



4. Hyperbola



5. Degenerate conic sections



As we study conic sections, we will be looking at special cases of the general second-degree equation: $\underbrace{Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0}$.

all conic sections' general equation.

$(x-h)^2+(y-k)^2=r^2$

(h, k) r (x, y)

Section 8.2a: The Circle

Center

Definition: A circle is the set of all points that are equidistant from a fixed point. The fixed point is called the *center* and the distance from the center to any point on the circle is called the *radius*.

$r=\sqrt{(x-h)^2+(y-k)^2}$

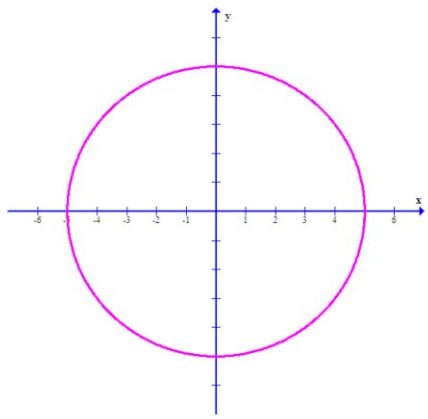
An equation of a circle whose center is at the origin will be $x^2+y^2=r^2$, where r is the radius of the circle.

(0,0)

$(x-0)^2+(y-0)^2=r^2$

$\to x^2+y^2=r^2$

So $x^2+y^2=25$ is an equation of a circle with center (0, 0) and radius 5. Here's the graph of this circle:



circle: $x^2+y^2=25$

center (0,0)

radius $r=\sqrt{25}=5$

Example 1: State the center and the radius of the circle and then graph it: $x^2+y^2-16=0$.

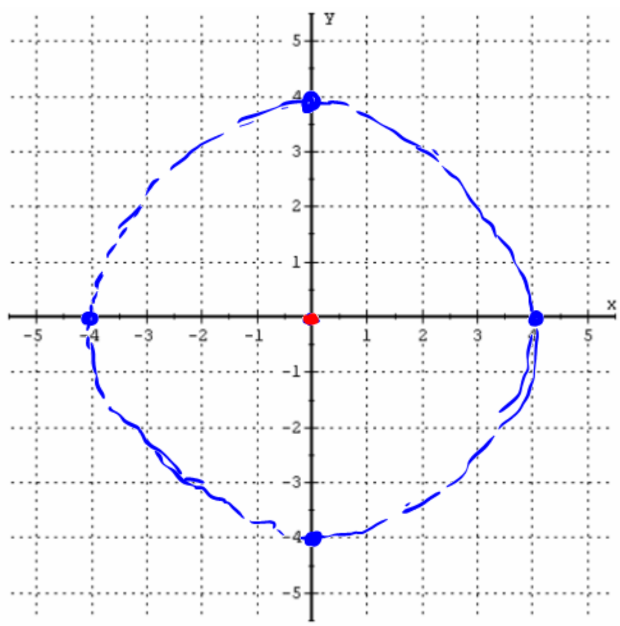
$x^2+y^2-16=0$

$\Rightarrow x^2+y^2=16$

Circle: center (0,0)

radius $r=\sqrt{16}$

$r=4$



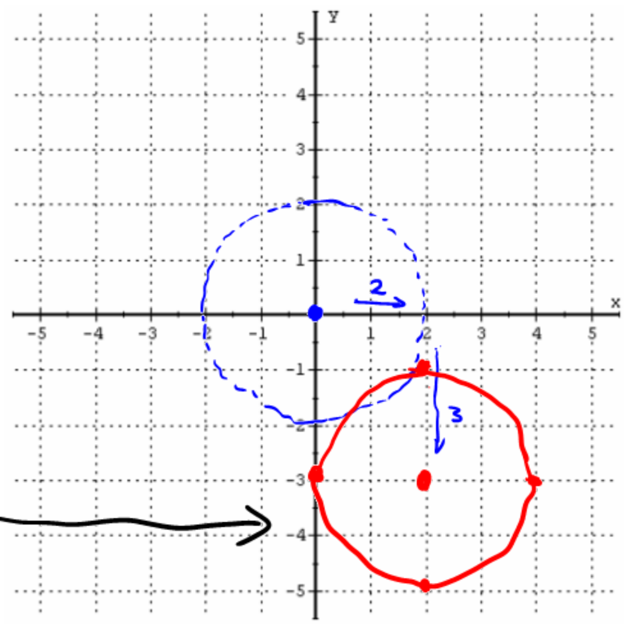
standard form → (h, k) = center

The standard form of the equation of a circle is $(x-h)^2+(y-k)^2=r^2$, where the center of the circle is the point (h,k) and the radius is *r*. Notice that if the center of the circle is (0, 0) you'll get the equation we saw earlier.

Example 2: State the center and the radius of the circle and then graph it:
$(x-2)^2+(y+3)^2=4$

↳ What it means:
You take the circle with $r=\sqrt{4}=2$ and move the center from origin to 2 units right and 3 units down

We get a circle with center (2, −3) and r = 2



Sometimes the equation will be given in the general form, and your first step will be to rewrite the equation in the standard form. You'll need to **complete the square** to do this.

ex The above circle equation $\boxed{(x-2)^2+(y+3)^2=4}$ Standard form – we like it.

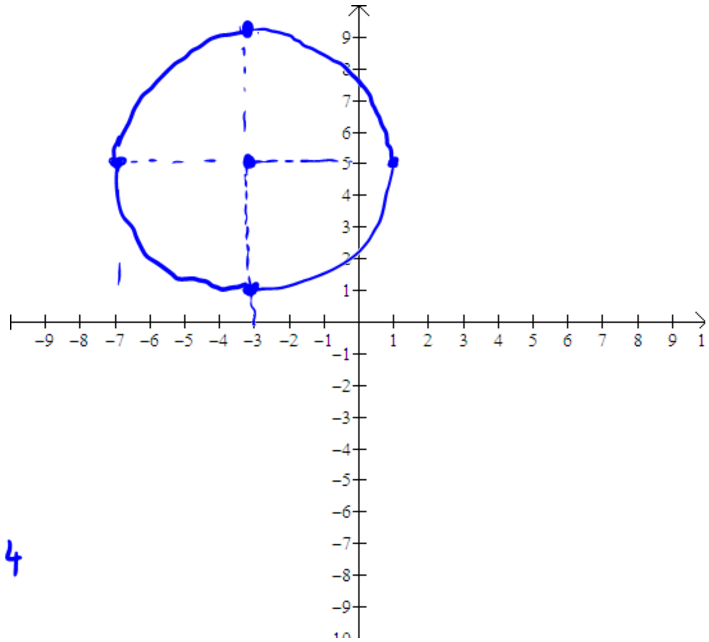
could look as:

$$\boxed{x^2+y^2-2x+6y+9=0}$$ general form

They are equivalent.

To be continued on Wednesday, 02/10.

Example 3: Write the equation in standard form, find the center and the radius and then graph the circle: $x^2 + y^2 + 6x - 10y + 44 = 26$



Need to bring in form:

$(x-h)^2 + (y-k)^2 = r^2$

$x^2 + y^2 + 6x - 10y + 44 = 26$

→ group "x" terms, "y" terms, "constant" terms

$(x^2 + 6x \quad) + (y^2 - 10y \quad) = 26 - 44$

→ look for the missing "magic term"

$(x^2 + 6x + 9) + (y^2 - 10y + 25) = -18 + 9 + 25$

$\left(\frac{6}{2} = 3\right)^2 = 9 \qquad \left(\frac{10}{2} = 5\right)^2 = 25$

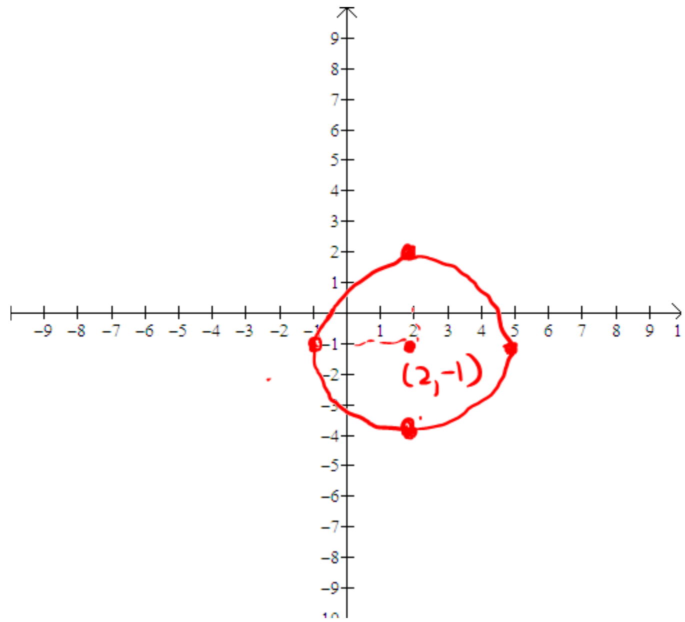
→ rewrite as standard form

$(x+3)^2 + (y-5)^2 = 16 \implies r = \sqrt{16} = 4$

center $= (-3, 5)$

Example 4: Write the equation in standard form, find the center and the radius and then graph the circle:

$5x^2 + 5y^2 - 20x + 10y = 20$



Always keep in mind!

- For circles, x^2 and y^2 should be "free" terms.
- If not free, factor the coefficient.

$$\frac{5(x^2+y^2-4x+2y)}{5} = \frac{20}{5}$$

$$\Rightarrow x^2+y^2-4x+2y = 4$$

follow steps as in previous question.

$$(x^2-4x+4)+(y^2+2y+1) = 4+4+1$$

$$\left(\frac{4}{2}=2\right)^2 \qquad \left(\frac{2}{2}=1\right)^2=1$$

$$\boxed{(x-2)^2+(y+1)^2=9}$$

Center $(2,-1)$

$r = \sqrt{9} = 3$

We can also write the equation of a circle, given appropriate information.

Example 5: Write the equation of a circle with center (2, 5) and radius $2\sqrt{5}$.

Need $(x-h)^2+(y-k)^2=r^2$

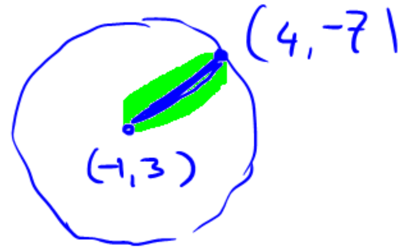
$\Rightarrow$ $(x-2)^2+(y-5)^2=(2\sqrt{5})^2$

i.e $\boxed{(x-2)^2+(y-5)^2=20}$

Example 6: Write an equation of a circle with center (-1, 3) which passes through the point (4, -7).

Center = (-1, 3)

radius $\rightarrow$ find it



= distance btw given points

$r=\sqrt{(-1-4)^2+(3+7)^2} = \sqrt{125}$

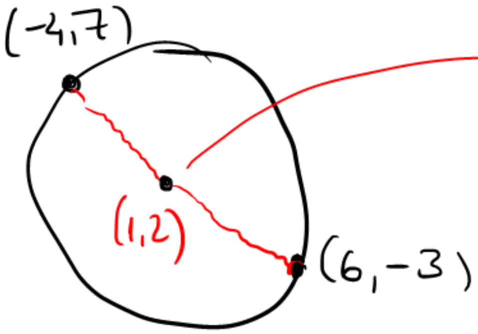
$\Rightarrow$ $(x+1)^2+(y-3)^2=(\sqrt{125})^2$

i.e. $\boxed{(x+1)^2+(y-3)^2=125}$

Finish on your own! Exercise

Example 7: Write an equation of a circle if the endpoints of the diameter of the circle are (6, -3) and (-4, 7).

Sketch : Diameter passes through center.



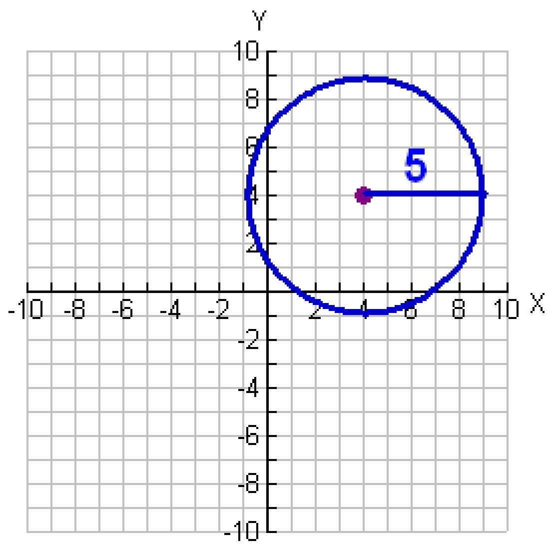
$\Rightarrow$ center $=$ midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{-4+6}{2}, \frac{7+(-3)}{2}\right) = (1,2)$

$\Rightarrow$ radius $= \sqrt{(6-1)^2 + (-3-2)^2} = \sqrt{50}$

$$\Rightarrow \boxed{(x-1)^2 + (y-2)^2 = 50}$$

exercise **Example 8:** What is the equation of the given circle?



center = (4,4)
radius = 5
from graph

$$\Rightarrow \boxed{(x-4)^2 + (y-4)^2 = 25}$$

exercise

Sometimes, you'll need to be able to manipulate an equation of a circle:

(Extra) Example 9: Suppose $(x-2)^2+(y+1)^2=9$. Solve the equation for x. Then solve the equation for y.

- Solve for y: $(y+1)^2 = 9-(x-2)^2$

$$\Rightarrow y+1 = \pm\sqrt{9-(x-2)^2}$$

$$\Rightarrow y = \pm\sqrt{9-(x-2)^2} - 1$$

two solutions – all time.

- Solve for x: $(x-2)^2 = 9-(y+1)^2$

$$\Rightarrow x-2 = \pm\sqrt{9-(y+1)^2}$$

$$x = 2 \pm \sqrt{9-(y+1)^2}$$

two solutions – all time.