MATH 1330 Review for Test -2

When: 02/25 – 02/27
Where: CASA Testing Center (222 Garrison Gym)
Time: 50 minutes
Number of questions: 14

10 Multiple Choice Questions (total of 60 points)
4 Free Response Questions (total of 40 points)

What is covered: **Chapters 1, 2 and 8.**

Do not forget to reserve a seat for Test – 2.

Do not be late for your test. Plan to be at the testing center 10-15 minutes before your scheduled time. If you miss your reserved seat, log in to your CASA account and try to reschedule; you can do this if there are any available seats.

Remember the make-up policy: NO MAKE-UPS! If you miss your test, you will get a zero. When you take the final, it will replace ONE missed test.

Take Practice Test – 2! 10% of your best score will be added to your test grade.

Do not forget to go to CASA for fingerprint/picture process before your test date.

Do not forget to bring your COUGAR ID when you go to the testing center.

For the free response part, please show your work neatly. Do not skip steps.

When you take the test, you will see a score in your CASA grade sheet right away. That score is for the multiple choice part only. So, **it is out of 60 points.** The grade for the Free Response Part will be posted later, after the papers are graded.

Example 1: $f(x)=\dfrac{x^2-4x+1}{x^2+3x+2} = \dfrac{x^2-4x+1}{(x+1)(x+2)}$

a) Domain: denominator $\neq 0 \Rightarrow x\neq -1,\ x\neq -2 \Rightarrow \boxed{(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)}$

b) Vertical Asymptote(s): The zeros of denominator $\Rightarrow \boxed{x=-1,\ x=-2}$

c) Hole: $\boxed{\text{None}}$ (no common factor)

d) Horizontal Asymptote: $\boxed{y=1}$

e) Does the graph intersect the HA? If so, what is the x-coordinate of the intersection?

$f(x)=\dfrac{x^2-4x+1}{x^2+3x+2}=\dfrac{1}{1} \Rightarrow x^2-4x+1=x^2+3x+2$

$-7x=1 \Rightarrow \boxed{x=-\frac{1}{7}}$

Cross-product

f) x and y-intercepts:

- x-intercept: $f(x)=\dfrac{x^2-4x+1}{x^2+3x+2}=0$

$\Rightarrow x^2-4x+1=0$ Quadratic formula $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\dfrac{4\pm\sqrt{16-4\cdot1\cdot1}}{2\cdot1}$

$x=\dfrac{4}{2}\pm\dfrac{\sqrt{12}}{2}=2\pm\sqrt{3}$

$\Rightarrow x=2+\sqrt{3},\quad x=2-\sqrt{3}$

1

- y-intercept: $f(0)=\dfrac{0^2-4\cdot0+1}{0^2+3\cdot0+2}=\dfrac{1}{2} \Rightarrow \boxed{(0,\frac{1}{2})}$

A. $(0,0)$ B. $(0,1)$ C. $(0,\frac{1}{2})$ D. none

Example 2: $f(x)=\dfrac{x-4}{x^2-3x-4} = \dfrac{\cancel{x-4}}{\cancel{(x-4)}(x+1)} = \dfrac{1}{x+1}, \; x \neq 4$

a) Domain: $x \neq 4, \; x \neq -1 \Rightarrow \boxed{(-\infty,-1)\cup(-1,4)\cup(4,\infty)}$

b) Vertical Asymptote(s): $\boxed{x=-1}$

c) Hole: $\boxed{x=4} \Rightarrow f^*(4) = \dfrac{1}{4+1} = \dfrac{1}{5} \Rightarrow (4,\tfrac{1}{5}) \leftarrow$ position

d) Horizontal Asymptote: $\boxed{y=0}$

$f^*(x) = \dfrac{1}{x+1} = 0 \Longrightarrow$ No solution $\Rightarrow$ No intercept

e) x and y-intercepts:

- No x-intercept
- y-intercept $\Rightarrow f(0) = \dfrac{0-4}{0^2-3\cdot 0-4} = 1 \Rightarrow \boxed{(0,1)}$

(2) Hole

A. $(1,0)$ B. $(4,\tfrac{1}{5})$ C. none

Example 3: $f(x)=10x^2-7x+4$

Calculate $f(x-1) = 10(x-1)^2 - 7(x-1) + 4 = 10(x^2-2x+1) - 7x + 7 + 4$

$= 10x^2 - 20x + 10 - 7x + 7 + 4 = \boxed{10x^2 - 27x + 21}$

What is the y-intercept of $f(x-1)$?

$f(x) = 10x^2 - 7x + 4$

y-int of $f(x-1) = f(0-1) = f(-1) = 10(-1)^2 - 7(-1) + 4 = 21$

$\Longrightarrow \boxed{(0,21)}$

Example 4: Given $f(x) = \dfrac{x+1}{2x-1}$ and $g(x) = 4x-1$

Domain of $f \circ g$? dom $f \circ g = \left(-\infty, \frac{3}{8}\right) \cup \left(\frac{3}{8}, \infty\right)$

$$(f \circ g)(x) = f(g(x)) = \frac{g(x)+1}{2 \cdot g(x) - 1} = \frac{4x-1+1}{2(4x-1)-1} = \boxed{\frac{4x}{8x-3}}$$

$$(g \circ f)(x) = g(f(x)) = 4 \cdot f(x) - 1 = 4 \cdot \frac{(x+1)}{2x-1} - 1$$

$$= \frac{4x+4}{2x-1} - \frac{2x-1}{2x-1} = \boxed{\frac{2x+5}{2x-1}}$$

Example 5: $f(x) = -x^2 + 4x + 5$.

a) Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$

Step I: $f(x+h) = -(x+h)^2 + 4(x+h) + 5$

$$= -x^2 - 2xh - h^2 + 4x + 4h + 5$$

Step II: $f(x+h) - f(x) = -x^2 - 2xh - h^2 + 4x + 4h + 5 - (-x^2 + 4x + 5)$

$$= \cancel{-x^2} - 2xh - h^2 + \cancel{4x} + 4h + \cancel{5} + \cancel{x^2} - \cancel{4x} - \cancel{5}$$

$$= -2xh - h^2 + 4h = h(-2x - h + 4)$$

Step III: $\dfrac{f(x+h) - f(x)}{h} = \dfrac{\cancel{h}(-2x-h+4)}{\cancel{h}} = \boxed{-2x - h + 4}$

b) Simplify $\dfrac{f(x+h) - f(x)}{h}$ when $x = 5$.

$$= -2x - h + 4 \quad , \quad x = 5$$

$$= -2 \cdot 5 - h + 4 = \boxed{-6 - h}$$

Another example:

$$f(x) = \frac{3}{x} + 2 \implies \frac{f(x+h) - f(x)}{h}$$

Step I: $f(x+h) = \dfrac{3}{x+h} + 2$

Step II.

$$f(x+h) - f(x) = \left(\frac{3}{x+h} + 2\right) - \left(\frac{3}{x} + 2\right)$$

$$= \frac{3 \cdot x}{(x+h) \cdot x} - \frac{3 \cdot (x+h)}{x \cdot (x+h)}$$

$$= \frac{3x - 3(x+h)}{x(x+h)} = \frac{-3h}{x(x+h)}$$

Step III.

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{-3\cancel{h}}{x(x+h)}}{\cancel{h}} = \boxed{\frac{-3}{x(x+h)}}$$

Evaluate for $x = 5 \implies \dfrac{-3}{5(5+h)} = \boxed{\dfrac{-3}{25+5h}}$

Example 6: Find the inverse of the function, if possible.

a) $f(x)=\dfrac{4x+2}{x-1}$.

Rewrite

I. $y=\dfrac{4x+2}{x-1}$

Exchange

II. $x=\dfrac{4y+2}{y-1}$

Solve for y

III. $\dfrac{x}{1}=\dfrac{4y+2}{y-1}$ Cross-product

$x(y-1)=1\cdot(4y+2)$

$xy-x=4y+2$

$y(x-4)=x+2$

$y=\dfrac{x+2}{x-4} \Rightarrow \boxed{f^{-1}(x)=\dfrac{x+2}{x-4}}$

b) $f(x)=\sqrt{5-2x}$.

dom f : $5-2x\geq 0 \Rightarrow x\leq \frac{5}{2}$, range f : $y\geq 0$. range f becomes dom f^{-1}.

I. $y=\sqrt{5-2x}$

II. $x=\sqrt{5-2y}$

III. $x^2=(\sqrt{5-2y})^2$

$x^2=5-2y$

$y=\dfrac{-x^2+5}{2} \Rightarrow \boxed{f^{-1}(x)=\dfrac{-x^2+5}{2},\ x\geq 0}$

c) $f(x)=x^2+12$, where $x\geq 0$.

dom f → becomes range for f^{-1}.

I. $y=x^2+12$

II. $x=y^2+12$

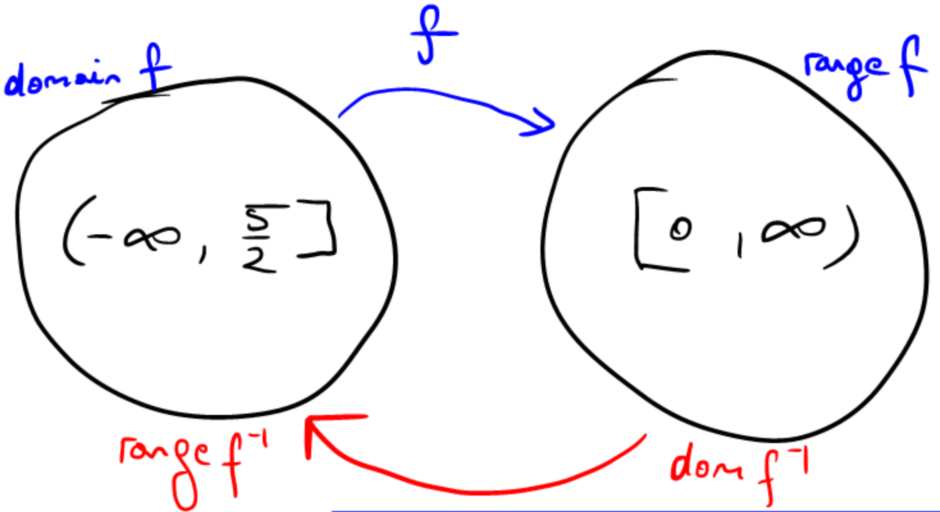
III. $x=y^2+12$

$y^2=x-12 \Rightarrow y=\sqrt{x-12}$

$\left(y=\begin{cases}\sqrt{x-12}\\ -\sqrt{x-12}\end{cases}\right) \Rightarrow \boxed{f^{-1}(x)=\sqrt{x-12}}$

b) $f(x) = \sqrt{5-2x}$

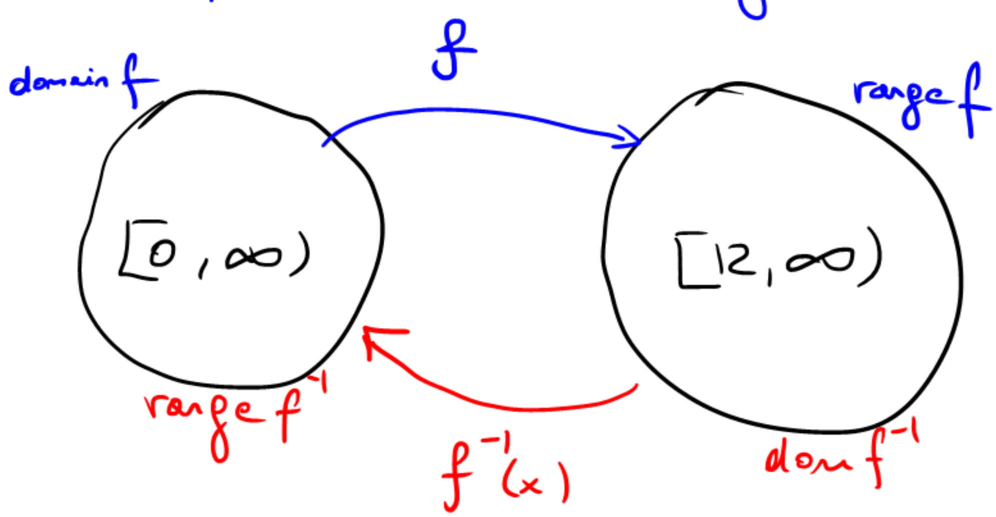
$\text{dom } f = \left(-\infty, \frac{5}{2}\right] \qquad \text{range } f = [0, \infty)$



$$\boxed{f^{-1}(x) = \frac{-x^2+5}{2}, \quad x \geq 0}$$

c) $f(x) = x^2 + 12, \quad x \geq 0$

$\text{dom } f = [0, \infty), \quad \text{range } f = [12, \infty)$



$$\boxed{f^{-1}(x) = \sqrt{x-12}}$$

Example 7: Find the linear function $f(x)$ given that $(1,4)$ is on the graph of f and $(2,5)$ is on the graph of f^{-1}.

For a function, need two points:

$(1,4)$ ✓ $\qquad (2,5)$ is on $f^{-1} \Leftrightarrow f^{-1}(2)=5$

$(5,2)$ ✓ $\qquad \Leftrightarrow f(5)=2$

$\Rightarrow$ slope: $m=\dfrac{4-2}{1-5}=\dfrac{2}{-4}=-\dfrac{1}{2} \Rightarrow \boxed{y-4=-\frac{1}{2}(x-1) \Rightarrow y=-\frac{1}{2}x+\frac{9}{2}}$

Example 8: Find the coordinates of the center and the radius for the given circle:

$x^2+6x+21+y^2-8y=0$

$(x^2+6x+3^2)+(y^2-8y+4^2)=-21+3^2+4^2$

$\frac{6}{2}=3 \qquad \frac{8}{2}=4$

$(x+3)^2+(y-4)^2=4 \Rightarrow \boxed{\text{Center } (-3,4),\ r=2}$

3 Radius A. 4 B. 2 C. 1 D. none

Example 9: Given $\dfrac{x^2}{81}-\dfrac{y^2}{16}=1$. $\Leftrightarrow$ horizontal $\Leftrightarrow y=\pm\dfrac{b}{a}x$

$a^2 \qquad b^2$

a) What are the asymptotes?

$a^2=81 \Rightarrow a=9$

$b^2=16 \Rightarrow b=4$

$\Rightarrow \boxed{y=\frac{4}{9}x \text{ and } y=-\frac{4}{9}x}$

b) What are the coordinates of the vertices and foci?

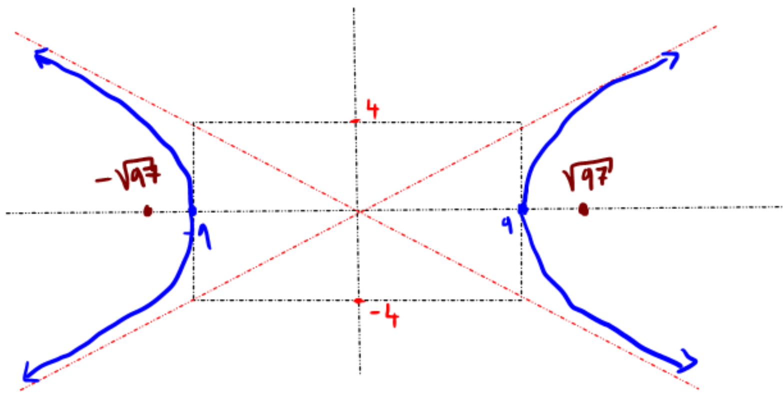
(next page)

Vertices on x-axis $(-9,0)$, $(9,0)$

Foci: $c^2 = a^2 + b^2$ $(-\sqrt{97}, 0)$, $(\sqrt{97}, 0)$

$c^2 = 81 + 16$

$c^2 = 97$



4. Vertices

A. $(9,0)$, $(-9,0)$

B. $(0,9)$, $(0,-9)$

C. none

5. A.

6. B.

Written Question #1: Find the point(s) of intersection for the following equations:

$$7\cdot\begin{cases}4x^2+7y^2=11\\3x^2-y^2=2\end{cases} \Longrightarrow +\begin{cases}4x^2+7y^2=11\\21x^2-7y^2=14\end{cases} \Longrightarrow 25x^2=25$$

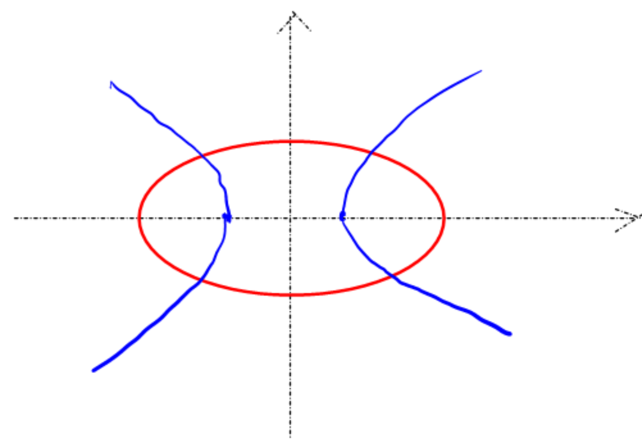
$$x^2=1 \Rightarrow x=\pm 1$$

- $x=1 \Rightarrow 3\cdot 1^2-y^2=2$

  $y^2=1 \Rightarrow y=\pm 1$

- $x=-1 \Rightarrow 3(-1)^2-y^2=2$

  $y^2=1 \Rightarrow y=\pm 1$

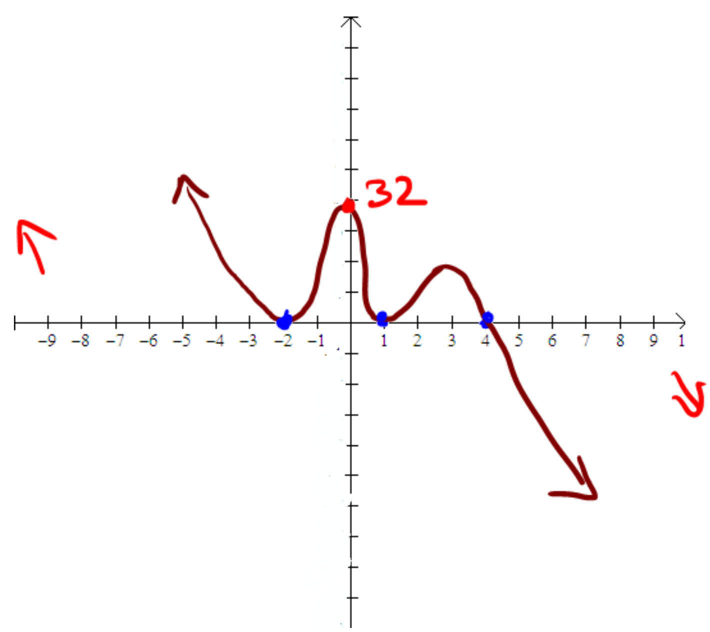


$$\Rightarrow \boxed{(1,1),\ (1,-1),\ (-1,1),\ (-1,-1)}$$

Written Question #2: Graph the polynomial $P(x)=-2(x-1)^2(x-4)^1(x+2)^2$

Leading term: $-2x^5$

End Behavior: $\nwarrow \ \cdots \ \searrow$



Zeros (x-intercepts):

$x-1=0 \Rightarrow x=1$ mult. 2 (parabola)

$x-4=0 \Rightarrow x=4$ mult. 1 (line)

$x+2=0 \Rightarrow x=-2$ mult. 2 (parabola

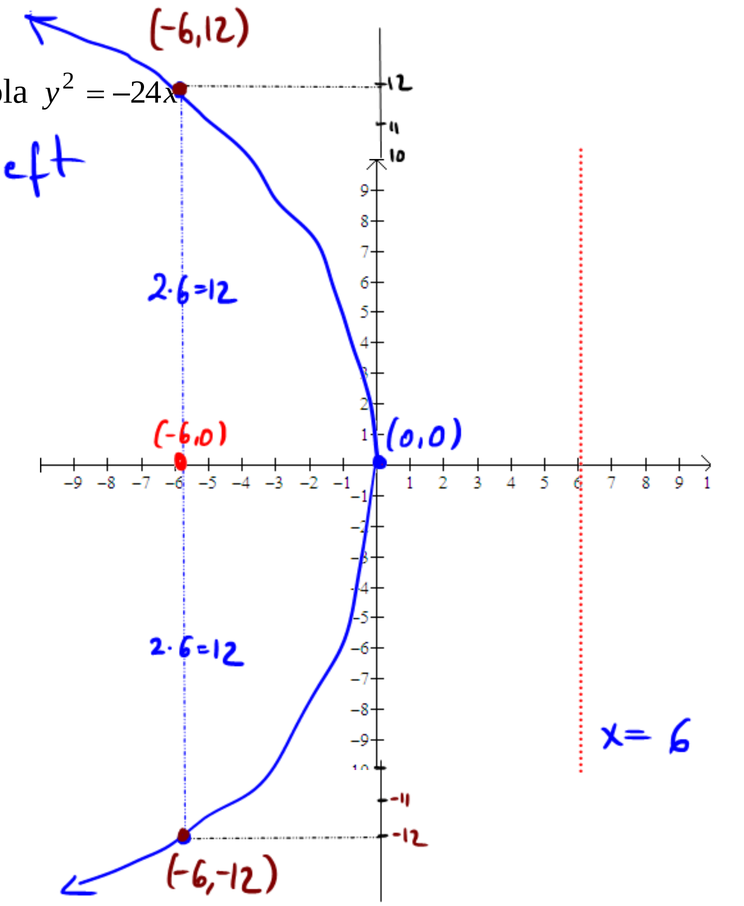
y-intercept:

$P(0)=-2(0-1)^2(0-4)(0+2)^2$

$y=32$

To be continued on Wednesday, 02/24.

Written Question #3: Graph the parabola $y^2 = -24x$



Orientation: Horizontal - open left

Vertex: $(0,0)$

$4p=-24$

$p = -6$

$p=-6$

directrix: $x = -(-6) = 6$

focus (focal point): $(-6,0)$

end points of the focal chord:

$(-6,12)$, $(-6,-12)$

(Exercise!) Graph the parabola $x^2 = -20y$.

Orientation: Vertical downward

$4p=-20$

Vertex: $(0,0)$

$p=-5$

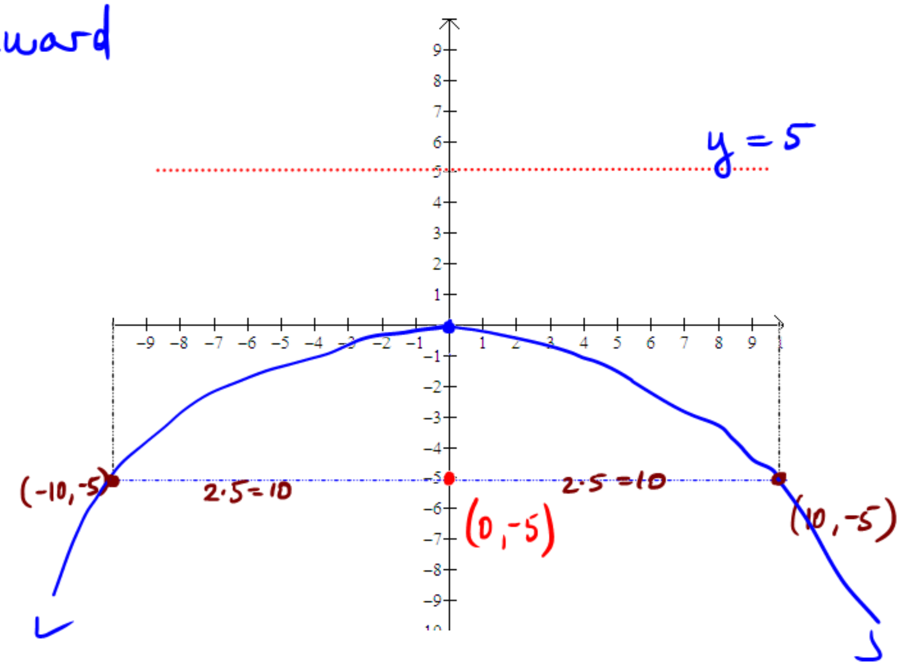
$p = -5$

directrix: $y = -(-5) = 5$

focus (focal point): $(0,-5)$

End points of the focal chord:

$(-10,-5)$, $(10,-5)$



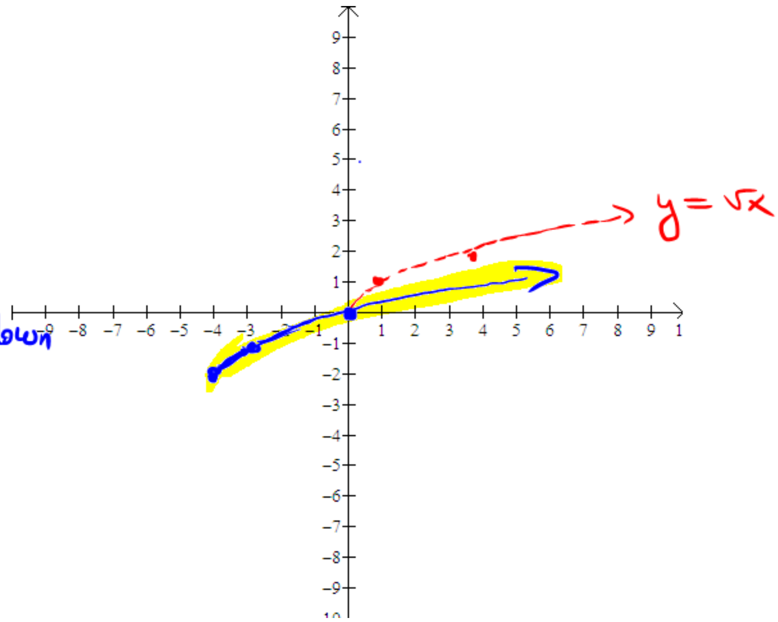
Written Question #4: Graph the function $f(x)=\sqrt{x+4}-2$ using transformations.

graph $y=\sqrt{x}$

Domain: $x+4 \geq 0 \Rightarrow [-4,\infty)$
$x \geq -4$

Range: $[-2,\infty)$

Transformations of the key point:



• shift 4 unit left, 2 units down

$(0,0) \longrightarrow (-4,-2)$

$(1,1) \longrightarrow (-3,-1)$

$(4,2) \longrightarrow (0,0)$

Is this function one-to-one?

Yes, it is 1-1.

(Exercise!) Graph the function $f(x)=|x-5|+2$ using transformations.

Domain: $(-\infty,\infty)$

Range: $[2,\infty)$

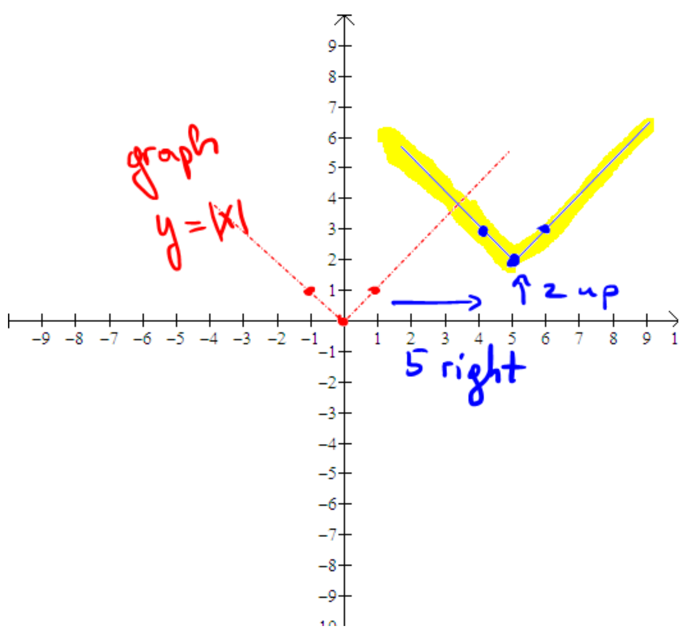
Transformations of the key point:

shift 5 right, 2 up

$(0,0) \longrightarrow (5,2)$

$(1,1) \longrightarrow (6,3)$

$(-1,1) \longrightarrow (4,3)$



is this function one-to-one?

No, it is not.