MATH 1330 Review for Test #3

When: **April 1st – April 4th** Friday – Monday.
Where: **CASA** Testing Center (222 Garrison Gym)
Time: **50 minutes**
Number of questions: **17**
14 Multiple Choice Questions (total of 70 points) – each 5 points
3 Free Response Questions (total of 30 points) – each 10 points

What is covered: **All sections covered since Test 2 (4.1 – 4.3, 5.1 – 5.4)**

Do not forget to reserve a seat for Test #3.

Do not be late for your test. Plan to be at the testing center 10-15 minutes before your scheduled time. If you miss your reserved seat, log in to your CASA account and try to reschedule; you can do this if there are any available seats.

Remember the make-up policy: NO MAKE-UPS! If you miss your test, you will get a zero. When you take the final, it will replace ONE missed test.

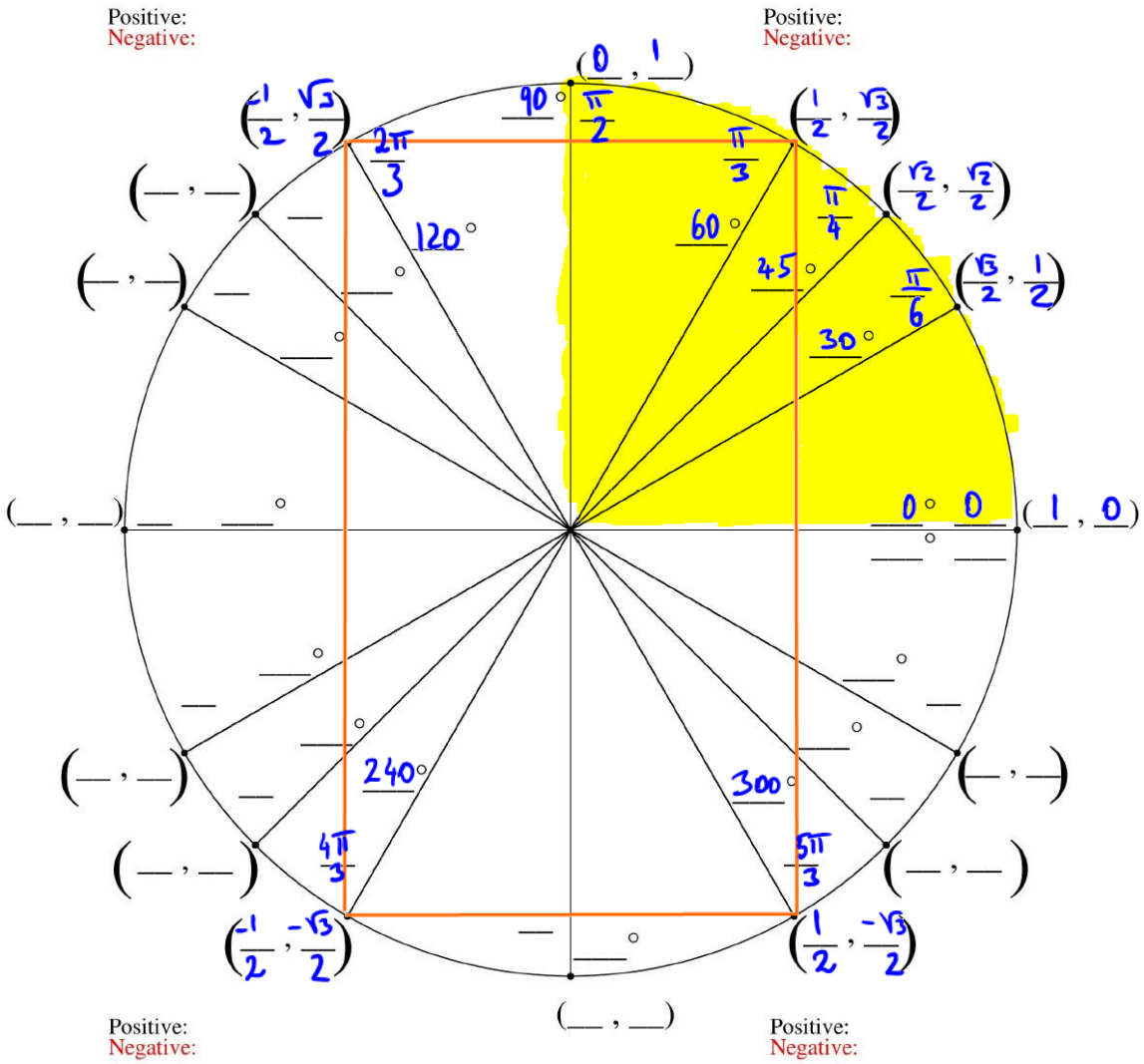
Take Practice Test – 3! 10% of your best score will be added to your test grade. Make 100.

For the free response part, please show your work neatly. Do not skip steps.

When you take the test, you will see a score in your CASA grade sheet right away. That score is for the multiple choice part only. So, **it is out of 70 points.** The grade for the Free Response Part will be posted later, after the papers are graded.

Fill in the unit circle! PRACTICE UNTIL YOU KNOW ALL! $(x, y) = (\cos\theta, \sin\theta)$

1st quadrant is enough.



$x = \cos\theta$

$y = \sin\theta$

How to evaluate: $\sin\left(\frac{7\pi}{6}\right) = -\sin(30^\circ) = -\frac{1}{2}$ (III)

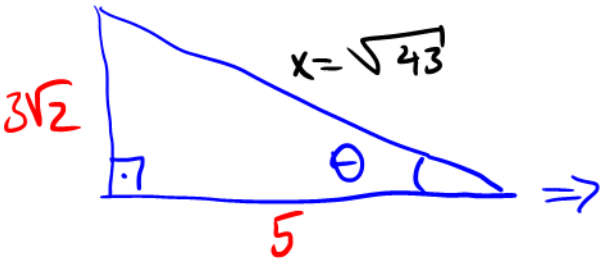
1. Locate the angle → $\frac{7\pi}{6} = 210^\circ$ QIII
2. Determine the sign → sine is negative in QIII
3. Find the corresponding reference angle and evaluate → $210^\circ - 180^\circ = 30^\circ$

EXAM-LIKE PRACTICE PROBLEMS

Question #1.

Suppose that θ is an acute angle of a right triangle and that $\tan(\theta) = \frac{3\sqrt{2}}{5}$ = $\frac{Opp}{Adj}$

Find $\sin(\theta)$ and $\cos(\theta)$. .



$x^2 = (3\sqrt{2})^2 + 5^2 = 43 \Rightarrow x = \sqrt{43}$

$\Rightarrow \sin\theta = \frac{3\sqrt{2}}{\sqrt{43}} = \frac{3\sqrt{86}}{43}$, $\cos\theta = \frac{5}{\sqrt{43}} = \frac{5\sqrt{43}}{43}$

Question #2.

A motorcycle has wheels with a 12 inch radius. If each wheel's rate of turn is 4 revolutions per second, how fast is the car moving in units of inches/sec?

$r = 12$ in

$w = 4$ rps

$= 4 \times 2\pi = 8\pi$ rad/s

→ linear velocity

$v = r \cdot w$

$v = 12 \cdot 8\pi = 96\pi$ in/sec

Question #3.

Find the area of the sector if Formula for the area will be given.

a) $r = 4, \theta = \frac{\pi}{6}$

$A = \pi r^2 \cdot \frac{\theta}{2\pi} = \frac{1}{2} r^2 \cdot \theta = \frac{1}{2} \cdot 4^2 \cdot \frac{\pi}{6} = \frac{4\pi}{3}$

b) $r = 8, \theta = 60^0$

$A = \pi r^2 \cdot \frac{\theta}{360} = \pi \cdot 8^2 \cdot \frac{60}{360} = \frac{32\pi}{3}$

Popper # 23

Question #4.

Substitute 180° for π.

(1) a) Convert $\theta = \frac{5\pi}{8}$ radians to degrees. (multiply by $\frac{180}{\pi}$)

$\theta = \frac{5\pi}{8} \cdot \frac{180}{\pi} = \frac{5 \cdot 180}{8} = \frac{225^\circ}{2}$

A. 100

B. 100/8

C. 225/2

b) Convert $\theta = 275^0$ to radians. (multiply by $\frac{\pi}{180}$)

(2) $\theta = 275^\circ \cdot \frac{\pi}{180} = \frac{55\pi}{36}$

A. $\frac{55}{36}$ B. $\frac{55\pi}{36}$ C. $\frac{275}{\pi}$

Question #5

Find three angles, two positive and one negative, that are coterminal with the given angle

a) $\theta = 140^0$

$\theta_1^+ = 140^\circ + 360^\circ = 500^\circ$

$\theta_2^+ = 500^\circ + 360^\circ = 860^\circ$

$(140^\circ + 2 \times 360^\circ = 860^\circ)$

$\theta_3^- = 140^\circ - 360^\circ = -220^\circ$

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b) $\theta = -70^0$

$\theta_1^+ = -70^\circ + 360^\circ = 290^\circ$

$\theta_2^+ = 290^\circ + 360^\circ = 650^\circ$

$\theta_3^- = -70^\circ - 360^\circ = -430^\circ$

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c) $\theta = \frac{11\pi}{18}$

$\theta_1^+ = \frac{11\pi}{18} + 2\pi \cdot \frac{18}{18} = \frac{47\pi}{18}$

$\theta_2^+ = \frac{47\pi}{18} + 2\pi \cdot \frac{18}{18} = \frac{83\pi}{18}$

$\theta_3^- = \frac{11\pi}{18} - 2\pi \cdot \frac{18}{18} = \frac{-25\pi}{18}$

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d) $\theta = -\frac{5\pi}{9}$

(3) $\theta_1^+ = -\frac{5\pi}{9} + 2\pi \cdot \frac{9}{9} = \frac{13\pi}{9}$

$\theta_2^+ = \frac{13\pi}{9} + 2\pi \cdot \frac{9}{9} = \frac{31\pi}{9}$

$\theta_3^- = -\frac{5\pi}{9} - 2\pi \cdot \frac{9}{9} = \frac{-23\pi}{9}$

A. $\frac{-13\pi}{9}$ B. $\frac{-23\pi}{9}$ C. $\frac{-7\pi}{9}$

1. Find the quadrant where θ belongs
2. Determine the sign.
3. Evaluate at reference angle.

Questions #6, #7, #8, #9 --- EVALUATIONS

Evaluate the followings. A completed unit circle might be useful.

a) $\tan\left(\frac{5\pi}{3}\right) = \tan(300^\circ) = -\tan(60^\circ)$ (QIV) $= \boxed{-\sqrt{3}}$

b) $\cot\left(\frac{5\pi}{6}\right) = \cot(150^\circ) = -\cot(30^\circ)$ (QII) $= \boxed{-\sqrt{3}}$

c) $\cos\left(\frac{2\pi}{3}\right) = \cos(120^\circ) = -\cos(60^\circ)$ (QII) $= \boxed{-1/2}$

d) $\sin\left(-\frac{5\pi}{3}\right) = \sin(-300^\circ) = \sin(60^\circ)$ (QI) $= \boxed{\sqrt{3}/2}$

e) $\csc\left(\frac{4\pi}{3}\right) = \frac{1}{\sin(240^\circ)} = -\frac{1}{\sin(60^\circ)} = \boxed{\frac{-2\sqrt{3}}{3}}$ (QIII)

f) $\sec\left(\frac{7\pi}{6}\right) = \frac{1}{\cos(210^\circ)} = -\frac{1}{\cos(30^\circ)} = \boxed{\frac{-2\sqrt{3}}{3}}$ (QIII)

g) $\sin\left(\frac{11\pi}{6}\right) = \sin(330^\circ) = -\sin(30^\circ)$ (QIV) $= \boxed{-1/2}$

h) $\cot\left(\frac{3\pi}{4}\right) = \cot(135^\circ) = -\cot(45^\circ)$ (QII) $= \boxed{-1}$

i) $\tan\left(\frac{5\pi}{4}\right) = \tan(225^\circ) = \tan(45^\circ)$ (QIII) $= \boxed{1}$

j) $\cos\left(-\frac{7\pi}{4}\right) = \cos(-315^\circ) = \cos(45^\circ)$ (QI) $= \boxed{\sqrt{2}/2}$

k) $\tan(135^0) = -\tan(45^\circ)$ (QII) $= \boxed{-1}$

l) $\cos(360^0) = \cos 0^\circ = \boxed{1}$

m) $\tan(-45^0) = -\tan(45^\circ)$ (QIV) $= \boxed{-1}$

n) $\cot(-225^0) = -\cot(45^\circ) = \boxed{-1}$ (QII)

o) $\tan(-315^0) = \tan(45^\circ) = \boxed{1}$ (QI)

p) $\sin(-45^0) = -\sin(45^\circ) = \boxed{-\frac{\sqrt{2}}{2}}$ (QIV)

q) $\cos(-60^0) = \cos(60^\circ) = \boxed{\frac{1}{2}}$ (QIV)

r) $\cos(300^0) = \cos(60^\circ) = \boxed{\frac{1}{2}}$ (QIV)

s) $\sin(240^0) = -\sin(60^\circ) = \boxed{\frac{-\sqrt{3}}{2}}$ (QIII)

t) $\sin(210^0) = -\sin(30^\circ) = \boxed{\frac{-1}{2}}$ (QIII)

Question #10

Find the exact value of the following expression:

a) $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$ in QIV

$\sin\theta = -\frac{1}{2} \Rightarrow \theta = -\frac{\pi}{6}$

b) $\cos^{-1}\left(-\frac{1}{2}\sqrt{3}\right) = \theta$ in QII

$\cos\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{5\pi}{6}$

c) $\arcsin\left(\frac{\sqrt{2}}{2}\right) = \theta$ in QI

$\sin\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{\pi}{4}$

d) $\arccos\left(-\frac{\sqrt{2}}{2}\right) = \theta$ in QII

$\cos\theta = -\frac{\sqrt{2}}{2} \Rightarrow \theta = \frac{3\pi}{4}$

e) $\arctan(\sqrt{3}) = \theta$ in QI

$\tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$

(4) f) $\arctan(-1) = \theta$ in QIV

$\tan\theta = -1 \Rightarrow \theta = -\frac{\pi}{4}$

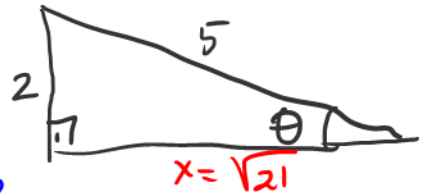
A. $\frac{7\pi}{4}$ B. $-\frac{\pi}{4}$ C. $\frac{\pi}{4}$

Question #11

Find the exact value of the following expression:

a) $\sec\left(\underbrace{\sin^{-1}\left(\frac{2}{5}\right)}_{\theta}\right) = \sec\theta$

$\sin^{-1}\left(\frac{2}{5}\right) = \theta \Leftrightarrow \sin\theta = \frac{2}{5}$



$x^2 + 2^2 = 5^2$

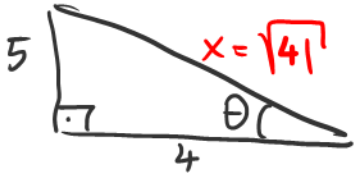
$x^2 = 21 \Rightarrow x = \sqrt{21}$

$\Rightarrow \sec\theta = \frac{5}{\sqrt{21}} = \frac{5\sqrt{21}}{21}$

b) $\cos\left(\underbrace{\tan^{-1}\left(\frac{5}{4}\right)}_{\theta}\right) = \cos\theta$

(5)

$\tan^{-1}\left(\frac{5}{4}\right) = \theta \Leftrightarrow \tan\theta = \frac{5}{4}$



$x^2 = 5^2 + 4^2 = 1$

$x = \sqrt{41}$

$\Rightarrow \cos\theta = \frac{4}{\sqrt{41}} = \frac{4\sqrt{41}}{41}$

A. $\frac{4}{5}$ B. $\frac{\sqrt{41}}{4}$ C. $\frac{4\sqrt{41}}{41}$

Question #12

Find the phase shift (horizontal shift) of the following functions. Phase Shift $= \frac{C}{B}$

a) $f(t) = 12\cos\left(\frac{1}{5}t + \frac{\pi}{4}\right)$ $\Rightarrow$ $A = 12$, $B = \frac{1}{5}$, $C = -\frac{\pi}{4}$

phase shift $= \frac{C}{B} = \frac{-\pi/4}{1/5} = \boxed{-\frac{5\pi}{4} \text{ i.e. } \frac{5\pi}{4} \text{ to the left}}$

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b) $f(t) = 12\sin\left(6t - \frac{\pi}{5}\right)$ $\Rightarrow$ $A = 12$, $B = 6$, $C = \pi/5$

phase shift $= \frac{C}{B} = \frac{\pi/5}{6} = \boxed{\frac{\pi}{30} \text{ i.e. } \frac{\pi}{30} \text{ to the right}}$

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c) $f(x) = 10\tan\left(5x - \frac{\pi}{20}\right)$ $\Rightarrow$ $A = 10$, $B = 5$, $C = \pi/20$

phase shift $= \frac{C}{B} = \frac{\pi/20}{5} = \boxed{\frac{\pi}{100} \text{ i.e. } \frac{\pi}{100} \text{ to the right}}$

Question #13

Write a sine function with amplitude 10, horizontal shift 5 to the left, vertical shift 2 down, and period 5.

$f(x) = 10 \cdot \sin\left(\frac{2\pi}{5}(x+5)\right) - 2$

$\Rightarrow$ $\boxed{f(x) = 10\sin\left(\frac{2\pi}{5}x + 2\pi\right) - 2}$

period $= 5$

$\frac{2\pi}{B} = 5$

$\Rightarrow B = \frac{2\pi}{5}$

Question #14

The voltage V produced by an alternating current generator is

$V(t) = 284\sin(140\pi t)$. $\Rightarrow$ $A = 284$, $B = 140\pi$

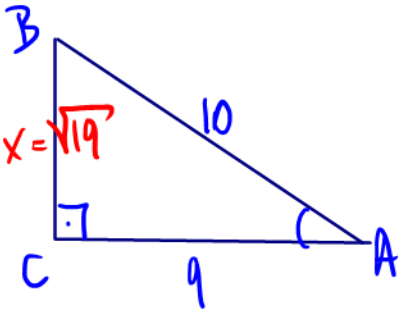
What are the amplitude and period of $V(t)$?

$\boxed{\text{amplitude} = 284, \quad \text{period} = \frac{2\pi}{140\pi} = \frac{1}{70}}$

Question #15 – Free Response #1

In the right triangle ABC with right angle C, $AB = 10$ and $AC = 9$.

Find the six trigonometric functions of angle *A*.



$x^2 + 9^2 = 10^2$

$x^2 + 81 = 100$

$x^2 = 19 \Rightarrow x = \sqrt{19}$

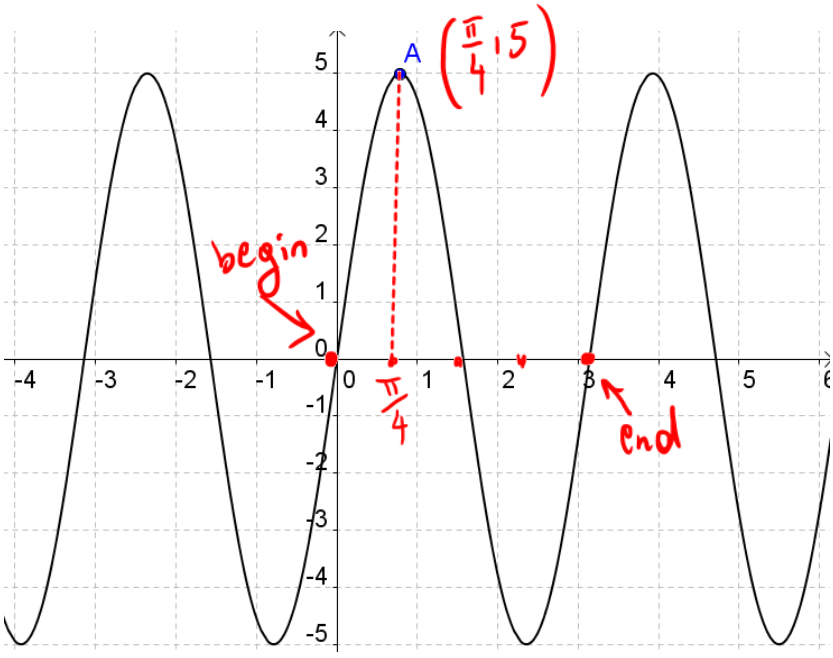
$\sin(A) = \frac{\sqrt{19}}{10}$ $\quad \csc(A) = \frac{10}{\sqrt{19}} = \frac{10\sqrt{19}}{19}$

$\cos(A) = \frac{9}{10}$ $\quad \sec(A) = \frac{10}{9}$

$\tan(A) = \frac{\sqrt{19}}{9}$ $\quad \cot(A) = \frac{9}{\sqrt{19}} = \frac{9\sqrt{19}}{19}$

Question #16 – Free Response #2

Find an equation for the sine function passing through (0,0) given that the first maximum point on the right of the origin is $\left(\frac{\pi}{4}, 5\right)$.



- amplitude = 5 $\Rightarrow$ $A = 5$
- period $= 4 \cdot \frac{\pi}{4} = \pi$

$\frac{2\pi}{B} = \pi \Rightarrow B = 2$

- $f(x) = 5\sin(2x)$

Question #17 – Free Response #3

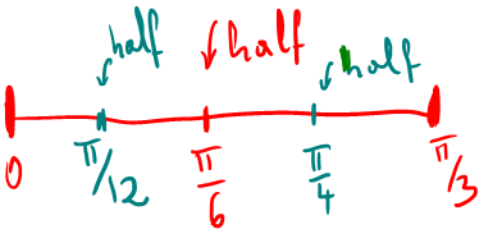
Given $f(x) = 5\sin(6x)$:

What is the domain of this function? $(-\infty, \infty)$

What is the range of this function? $[-5, 5]$

Graph the function over one period.
Label the intercepts, minimum/ maximum value(s) with an ordered pair.

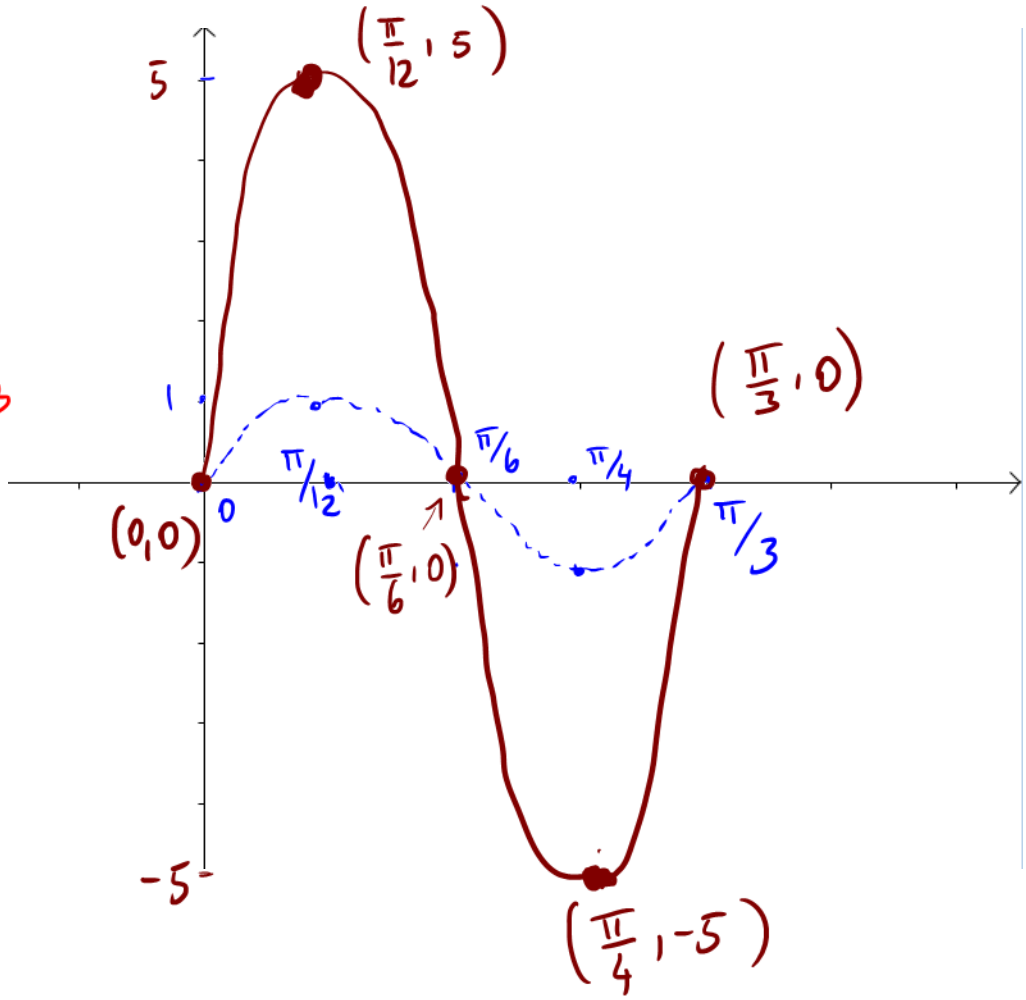
- period $= \frac{2\pi}{6} = \frac{\pi}{3}$



$\frac{\frac{\pi}{6} + \frac{\pi}{3}}{2} = \frac{\pi}{4}$

- amplitude $= 5$

Vertical stretch



Exercise: $f(x) = 6\cos(2x)$

What is the domain of this function? $(-\infty, \infty)$

What is the range of this function? $[-6, 6]$

Graph the function over one period.
Label the intercepts, minimum/ maximum value(s) with an ordered pair.

- period $= \frac{2\pi}{2} = \pi$

- amplitude $= 6$

vertical stretch