## MATH 1330 Review for Test -4

Time: 50 minutes Friday, April 29 - Monday, May 2
Number of questions: 11 Multiple Choice Questions (total: 100 pts )
What is covered: 5.1, 4.4, Chapters 6 and 7.
Do not forget to reserve a seat for Test -4!
Take practice Test - 4! $10 \%$ of your best score will be added to your test grade.
Remember the make-up policy: No make-ups!
The following formulas will be provided. It is your responsibility to locate the formula sheet before you start your test. If you can't find it, ask proctors for help.

Handy Formulas $\rightarrow$ Only these formulas will be provided to you. $\sin (\mathrm{s}+\mathrm{t})=\sin \mathrm{s} \cos \mathrm{t}+\cos \mathrm{s} \sin \mathrm{t}$
$\sin (\mathrm{s}-\mathrm{t})=\sin \mathrm{s} \cos \mathrm{t}-\cos \mathrm{s} \sin \mathrm{t}$
$\cos (\mathrm{s}+\mathrm{t})=\cos \mathrm{s} \cos \mathrm{t}-\sin \mathrm{s} \sin \mathrm{t}$
$\cos (\mathrm{s}-\mathrm{t})=\cos \mathrm{s} \cos \mathrm{t}+\sin \mathrm{s} \sin \mathrm{t}$
$\tan (\mathrm{s}+\mathrm{t})=\frac{\tan \mathrm{s}+\tan \mathrm{t}}{1-\tan \mathrm{s} \tan \mathrm{t}}$
$\tan (s-t)=\frac{\tan s-\tan t}{1+\tan s \tan t}$
$\sin (2 t)=2 \sin t \cos t$
$\cos (2 \mathrm{t})=\cos ^{2} \mathrm{t}-\sin ^{2} \mathrm{t}$
$\sin \frac{\mathrm{s}}{2}= \pm \sqrt{\frac{1-\cos \mathrm{s}}{2}} \quad \cos \frac{\mathrm{~s}}{2}= \pm \sqrt{\frac{1+\cos \mathrm{s}}{2}} \quad \tan \frac{\mathrm{~s}}{2}=\frac{\sin \mathrm{s}}{1+\cos \mathrm{s}}$

1) Simplify: $-\frac{\cos (-x)}{\cot (-x)}-\frac{\cos x}{-\cot x}=\frac{\cos x}{\frac{\cos x}{\sin x}}=\frac{\cos x}{1} \cdot \frac{\sin x}{\cos x}=\sin x$

Question \#1

$$
\begin{aligned}
=3 \cos ^{2} x+\left(\frac{\left(\sin ^{2} x-\cos ^{2} x\right.}{\sin ^{2} x}\right) \cdot \frac{\sin ^{2} x}{1} & =3 \cos ^{2} x+2 \sin ^{2} x-\cos ^{2} x=2 \cos ^{2} x+2 \sin ^{2} x \\
& =2(\underbrace{\cos ^{2} x+\sin ^{2} x}_{=1})=2
\end{aligned}
$$

Question \#2

$$
\text { 3) Simplify: } \begin{aligned}
\tan (x)-\frac{\cos (x)}{1-\sin (x)}-\frac{\sin x}{\cos x(1-\sin x)} \cdot(1-\sin x) \\
=\frac{\sin x-\sin ^{2} x-\cos ^{2} x}{1-\sin x} \cdot \frac{\cos x}{\cos x} \\
\cos x(1-\sin x)
\end{aligned}=\frac{\sin x-\left(\sin ^{2} x+\cos ^{2} x\right)}{\cos x(1-\sin x)}=\frac{\sin x-1}{\cos x(1-\sin x)}
$$

Another version: $\frac{1}{1-\cos (x)}+\frac{1}{1+\cos (x)}$ common deneminat or

$$
\begin{aligned}
=\frac{(1+\cos x)+(1-\cos x)}{(1-\cos x)(1+\cos x)} & =\frac{2}{1-\cos ^{2} x}=\frac{2}{\sin ^{2} x} \\
& =2 \csc ^{2} x
\end{aligned}
$$

Question \#3
4) Given $\sin (x)=\frac{1}{4}, 90^{\circ}<x<180^{\circ}$, and $\sin (y)=-\frac{2}{5}, 180^{\circ}<y<270^{\circ}$, find:


$$
\begin{aligned}
& \sin x=\frac{1}{4} \\
& \cos x=-\frac{\sqrt{15}}{4}
\end{aligned}
$$



$$
\begin{aligned}
& \sin (y)=-\frac{2}{5} \\
& \cos (y)=\frac{-\sqrt{21}}{5}
\end{aligned}
$$

a) $\sin (x-y)=\sin x \cos y-6 \sin \sin y=\frac{1}{4}\left(\frac{-\sqrt{21}}{5}\right)-\left(\frac{-\sqrt{15}}{4}\right) \cdot\left(\frac{-2}{5}\right)=\frac{-\sqrt{21}-2 \sqrt{15}}{20}$
b) $\sin (x+y)=\sin x \cos y+\cos x \sin y=\frac{1}{4}\left(\frac{-\sqrt{21}}{5}\right)+\left(\frac{-\sqrt{15}}{4}\right)\left(\frac{-2}{5}\right)=\frac{-\sqrt{21}+2 \sqrt{15}}{20}$
c) $\cos (x-y)=\cos x \cos y+\sin x \sin y=\left(\frac{-\sqrt{15}}{4}\right)\left(\frac{-\sqrt{21}}{5}\right)+\left(\frac{1}{4}\right)\left(\frac{-2}{5}\right)=\frac{3 \sqrt{35}-2}{20}$
d) $\cos (x+y)=\cos x \cos y-\sin x \sin y=\left(\frac{-\sqrt{15}}{4}\right)\left(\frac{-\sqrt{21}}{5}\right)-\left(\frac{1}{4}\right) \cdot\left(\frac{-2}{5}\right)=\frac{3 \sqrt{35}+2}{20}$

Question \#4
5) Given $\tan (x)=-\frac{1}{5}, 90^{\circ}<x<180^{\circ}$,
a) $\sin (2 x)=2 \sin x \cos x$

$$
\begin{aligned}
& =2 \frac{1}{\sqrt{26}} \cdot \frac{-5}{\sqrt{26}} \\
& =\frac{-10}{26}=\frac{-5}{13}
\end{aligned}
$$



$$
\begin{aligned}
& \sin x=\frac{1}{\sqrt{26}} \\
& \cos x=\frac{-5}{\sqrt{26}}
\end{aligned}
$$

b) $\cos (2 x)$

$$
\begin{aligned}
& =\cos ^{2} x-\sin ^{2} x \\
& =\left(\frac{-5}{\sqrt{26}}\right)^{2}-\left(\frac{1}{\sqrt{26}}\right)^{2} \\
& =\frac{25}{26}-\frac{1}{26}=\frac{24}{26}=\frac{12}{13}
\end{aligned}
$$

6) Given $\sin (x)=\frac{3}{5}$, where $x$ is an acute angle, find $\sin \left(\frac{x}{2}\right) \cdot=+\sqrt{\frac{1-\cos x}{2}}=\sqrt{\frac{1-\frac{4}{5}}{2}}=\sqrt{\frac{\frac{1}{5}}{2}}$


$$
\cos (x)=\frac{4}{5}=\sqrt{\frac{1}{10}}=\sqrt{\frac{\sqrt{10}}{10}}
$$

7) Find the following using the sum or difference formulas:
a) $\cos \left(75^{\circ}\right)=\cos \left(30^{\circ}+45^{\circ}\right)=\ldots$
b) $\sin \left(105^{\circ}\right) \quad=\sin \left(60^{\circ}+45^{\circ}\right)=\cdots$.

Question \#5 period $=\frac{2 \pi}{3} \longrightarrow 3$ times $\Rightarrow 6$ solutions
8) Find all solutions of the equation $\cos (\underline{3} x)=\frac{1}{2} \quad$ on the interval $[0,2 \pi)$.

$$
\begin{aligned}
& \text { Unit } \cos 3 x=\frac{1}{2} \\
& \begin{array}{ll}
3 x & =\frac{\pi}{3} \\
-3 & \text { or } \frac{3 x}{3} \\
-\frac{5 \pi}{3} & \frac{5 \pi}{3} \\
\hline
\end{array} \\
& x=\frac{\pi}{9} \text { or } \frac{5 \pi}{9} \\
& \xrightarrow{\text { dst }} x=\frac{\pi}{9} \text { or } \frac{5 \pi}{9} \\
& { }_{\text {ard }}^{\text {end }} x=\frac{\pi}{9}+\frac{2 \pi}{3} \cdot 1=\frac{7 \pi}{9} \text { or } \frac{5 \pi}{9}+\frac{2 \pi}{3} \cdot 1=\frac{11 \pi}{9} \\
& \xrightarrow{3 r d} \\
& \rightarrow x=\frac{\pi}{9}+\frac{2 \pi}{3} \cdot 2=\frac{13 \pi}{9} \text { or } \frac{5 \pi}{9}+\frac{2 \pi}{3} \cdot 2=\frac{17 \pi}{9} \\
& \frac{\pi}{9}, \frac{5 \pi}{9}, \frac{7 \pi}{9}, \frac{11 \pi}{9}, \frac{13 \pi}{9}, \frac{17 \pi}{9}
\end{aligned}
$$

Alternate version: Solve $\sin (2 x)=\frac{\sqrt{3}}{2}$ on the interval $[0,2 \pi)$.

$$
\begin{equation*}
\text { period }=\frac{2 \pi}{2}=\pi \tag{times}
\end{equation*}
$$

Unit $\operatorname{cin}(2 x)=\frac{\sqrt{3}}{2}$
1st

$$
\begin{aligned}
& \rightarrow 2 x=\frac{\pi}{3} \text { or } \quad 2 x=\frac{2 \pi}{3} \\
& \rightarrow x=\frac{\pi}{6} \text { or } \frac{\pi}{3}
\end{aligned}
$$

and

$$
\frac{\pi}{6}, \frac{\pi}{3}, \frac{7 \pi}{6}, \frac{4 \pi}{3}
$$

Question \#6
9) Solve the following equation on the interval $[0,2 \pi)$ :

Factor

$$
\zeta(4 \sin x+5)(\sin x+1)=0
$$



$$
4 \sin x+5=0 \text { or } \sin x+1=0
$$

$$
4 \sin x=-5
$$

Question \#7
12) The angle of elevation from a point that is 120 ft away from a building to the top of the building is $25^{0}$. Find the height of the building.


$$
\tan \left(25^{\circ}\right)=\frac{x}{120} \Rightarrow x=120 \tan \left(25^{\circ}\right)
$$

Note: If the angle is $30,45,60^{\circ}$, make sure you evaluate!

Question \#8
10) Find the area of triangle ABC if $\angle \mathrm{B}=30^{\circ}, \mathrm{c}=10$ and $\mathrm{a}=12$.


Check: S.A.S

$$
\begin{aligned}
\text { Area } & =\frac{1}{2} * \text { side } \operatorname{sid} e-\sin \text { (angle) } \\
& =\frac{1}{2} * 10 * 12 * \sin \left(30^{\circ}\right)
\end{aligned}
$$

$$
=30
$$

Alurays Graph!
Question \#9
15- a) Given a triangle ABC with $\mathrm{AB}=6 \mathrm{~cm}$ and $\stackrel{O P P}{\mathrm{OCP}} 4$, angle $\left(\mathrm{A}\right.$ measures $120^{\circ}$.
How many choices are there for the measure of angle C ?

$\hat{\theta}=120^{\circ}$
adj $>$ opp
$\Rightarrow N_{0}$ triangle
ie. $\theta$ choices
b) Given a triangle ABC with $\stackrel{\text { ad }}{\mathrm{AB}}=6 \mathrm{~cm}$ and $\stackrel{\mathrm{BPP}}{\mathrm{BC}}=4$, angle A measures $30^{\circ}$. How many choices are there for the measure of angle $C$ ?


$$
B C=4>\text { height }=3
$$

Two triangles
$\Rightarrow 2$ choices
c) In triangle ABC with $\mathrm{AB}=6, \mathrm{BC} \equiv 7$, the measure of angle A is $30^{\circ}$. How many choices are there for the measure of angle C ?


$$
\begin{aligned}
& \text { opp }>\text { adj } \\
& \Rightarrow \text { Always } 1 \text { triangle } \\
& \quad \text { I choice }
\end{aligned}
$$

Question \#10 - Law of Cosine
11) ABC is a triangle with $\mathrm{AB}=10, \mathrm{BC}=13$, and $\mathrm{AC}=7$. Find $\cos (\mathrm{A})$.


$$
\begin{aligned}
& \text { Pythagorean in "disguise" } \\
& 13^{2}=7^{2}+10^{2}-2 \cdot 10 \cdot 7 \cdot \cos A \\
& 169=\underbrace{49+100}_{149}-140 \cdot \cos A \\
& \frac{20}{-140}=\frac{-140}{-1450} \cdot \cos A \Rightarrow \cos A=\frac{-200}{140}=\frac{-1}{7} \\
& 6
\end{aligned}
$$

Question \#10 - Law of Cosine
13) Two boats leave the dock at the same time and they travel with an angle of $150^{\circ}$ between them. What is the distance between them after they each travel 50 meters and 70 meters respectively?


$$
\begin{aligned}
& d^{2}=50^{2}+70^{2}-2 \cdot 50 \cdot 70 \cdot \cos \left(150^{\circ}\right) \\
& d^{2}=2500+4900-700 \cdot\left(-\frac{\sqrt{3}}{2}\right) \\
& d^{2}=7400+3500 \sqrt{3} \\
& d^{2}=100(74+35 \sqrt{3}) \Rightarrow d=\sqrt{100(74+35 \sqrt{3})} \\
& d=10 \sqrt{74+35 \sqrt{3}}
\end{aligned}
$$

Question \#11 - Law of Sines
14) Given a triangle $\mathrm{ABC}, \mathrm{A}=45^{\circ}, \mathrm{B}=30^{\circ}, \mathrm{BC}=60 \mathrm{~cm}$, find AC .


$$
\begin{aligned}
& \frac{\sin \left(30^{\circ}\right)}{x}=\frac{\sin \left(45^{\circ}\right)}{60} \\
& x \cdot \sin \left(45^{\circ}\right)=60 * \sin \left(30^{\circ}\right) \\
& \sin \left(45^{\circ}\right)
\end{aligned}
$$

$$
\Rightarrow x=\frac{60 \cdot \sin \left(30^{\circ}\right)}{\sin \left(45^{\circ}\right)}=\frac{60 \cdot \frac{1}{2}}{\frac{\sqrt{2}}{2}}=30 \cdot \frac{2}{\sqrt{2}}
$$

$$
x=\frac{60}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}=\frac{60 \sqrt{2}}{2}=30 \sqrt{2}
$$

$$
\Rightarrow x=30 \sqrt{2}
$$

extra examples

1. given a triangle $A B C, \quad \hat{A}=120^{\circ}, \quad A B=6, \quad$ adj: $\quad B C=10$, how many choices are there for/angle $C$ ?


Always 1 triangle
$\Rightarrow \quad l$ choice
2. Given a triangle $A B C, \hat{A}=30^{\circ}, A B=10,>B C=5 \sqrt{\text { opp }}, \overrightarrow{=}$, how many choices are there for angle $C$ ?

Do the trick (Pull the height)


$$
\begin{aligned}
& h=10 \cdot \sin 30^{\circ}=10 \cdot \frac{1}{2}=5 \\
& h<\text { opp }=B C=5 \sqrt{3}
\end{aligned}
$$

2 triangles $\Rightarrow$ 2choices

