Math 1330 - Section 1.2
Functions and Graphs

The graph of a function is the set of ordered pairs (x, y) where x is in the domain of $f(x)$ and $y = f(x)$.

input output

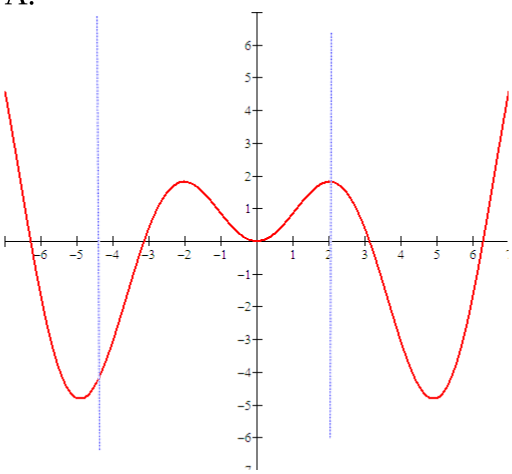
Vertical Line Test (VLT)

A curve is the graph of a function if and only if each vertical line intersects it in at most one point.

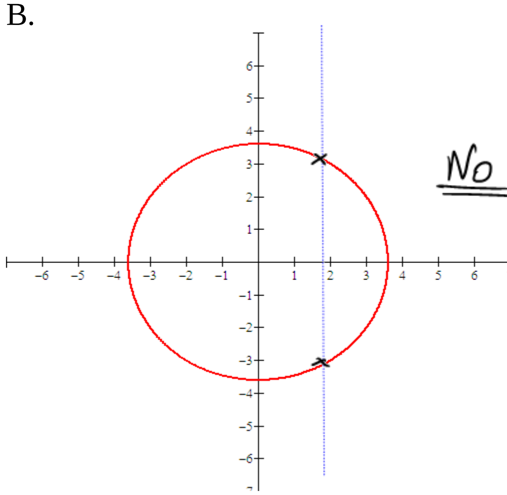
Example 1: Determine if these graphs are graphs of functions.

A.

Yes, it passes V.L.T



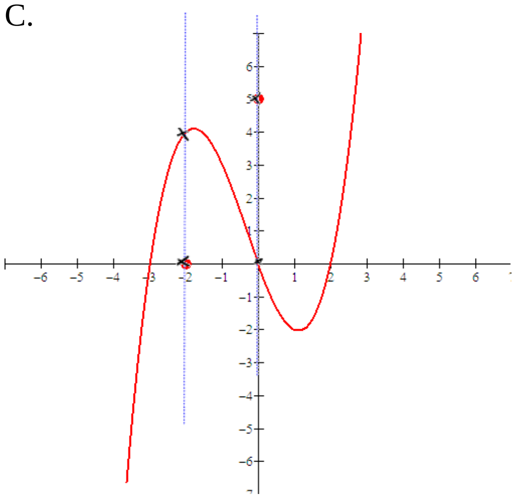
B.



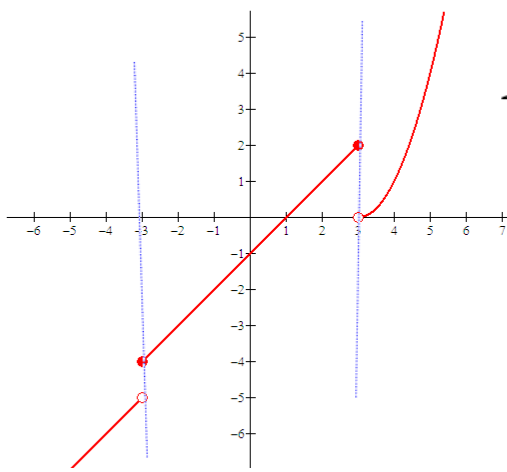
No it fails VLT

C.

No, it fails V.L.T



D.



Yes, it passes V.L.T.

think of $f(x)=x^2$

The next topic may be new to you. We can identify certain functions as **even functions** or **odd functions**. These functions have certain symmetries and knowing that a function is even or odd is another piece of information to help you graph it efficiently.

think of $g(x)=x^3$

A function is **even** if $f(-x)=f(x)$ for all x in the domain of the function. Even functions are symmetric with respect to the y axis. A very common even function is $f(x)=x^2$ whose graph is shown here.

- Note that $f(x)=x^2$ is symmetric wrt y-axis.
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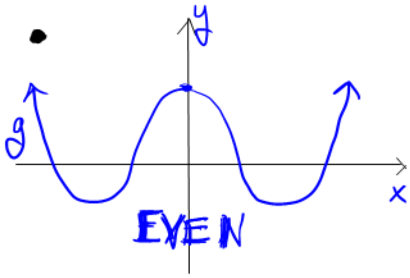
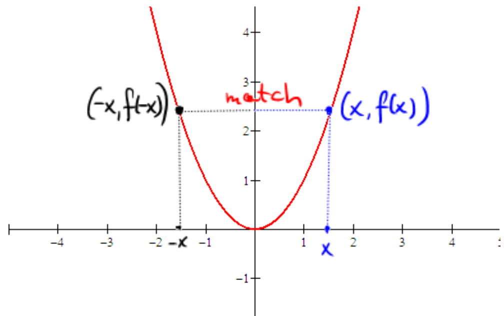




By symmetry, we have $\boxed{f(-x)=f(x)}$

To determine whether f is even or not, we check $f(-x)=f(x)$.

Apply: $f(x)=x^2+x^4$

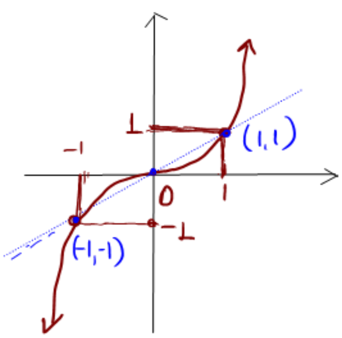
$\Rightarrow f(-x)=(-x)^2+(-x)^4$

$=x^2+x^4=f(x)$

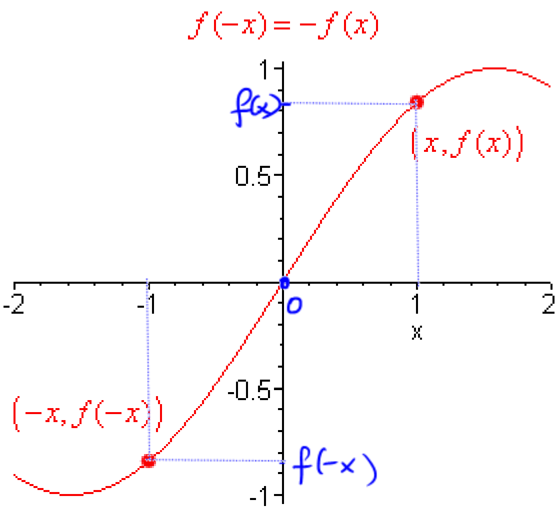
$\Rightarrow f$ even

A function is **odd** if $f(-x)=-f(x)$ for all x in the domain of the function. Odd functions have symmetry with respect to the origin. Here are a couple of examples of odd functions.

Think of $f(x)=x^3$



Such graphs are symmetric wrt. origin.



By symetry wrt. origin $f(-x)$ and $f(x)$ are opposite of each other $\boxed{f(-x)=-f(x)}$

To determine whether f is odd or not check $f(-x)=-f(x)$

Apply: $f(x)=3x^3-x$

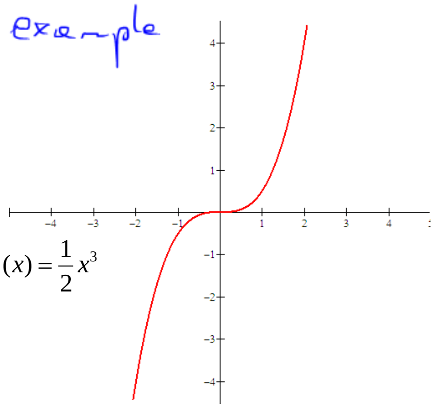
$f(-x)=3(-x)^3-(-x)$

$=-3x^3+x$ ← all signs changed

$=-(3x^3-x)=-f(x)$

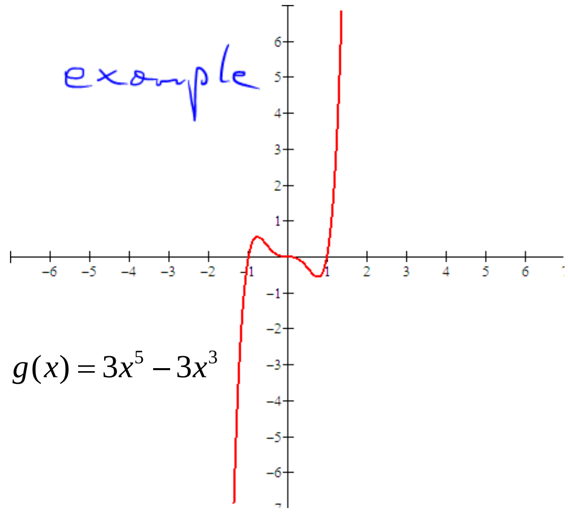
$\Rightarrow f$ odd.

example



$f(x) = \frac{1}{2}x^3$

example



$g(x) = 3x^5 - 3x^3$

Example 2: Determine if $f(x) = 5x^{4} - 3x^{2} + 2x^{0}$ is odd, even or neither. All powers involved are even.

even even even

Check $f(-x) = 5(-x)^4 - 3(-x)^2 + 2$
$= 5x^4 - 3x^2 + 2 = f(x) \implies$ EVEN

Example 3: Determine if $f(x) = x^{3} - x^{1}$ is odd, even or neither. All powers involved are odd.

odd odd

Check $f(-x) = (-x)^3 - (-x) = -x^3 + x = -f(x) \implies$ ODD

Example 4: Determine if $f(x) = x^{3} - x + 1x^{0}$ is odd, even or neither. – Some powers odd and some even.

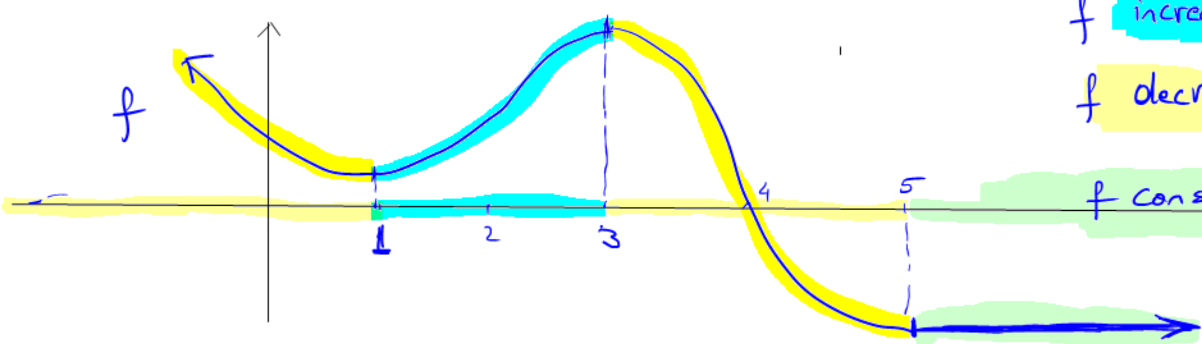
odd odd even

Check $f(-x) = (-x)^3 - (-x) + 1 = -x^3 + x \underline{+} 1 \implies$ NEITHER

A function is **increasing** on an interval (a,b) if $f(x_1) < f(x_2)$ for each $x_1 < x_2$ in (a,b). You can think: a function is increasing if the y values are getting bigger as we look from left to right.

A function is **decreasing** on an interval (a,b) if $f(x_1) > f(x_2)$ for each $x_1 < x_2$ in (a,b). You can think: a function is decreasing if the y values are getting smaller as we look from left to right.

A **turning point** is a point where the graph of a function changes from increasing to decreasing or where it changes from decreasing to increasing.



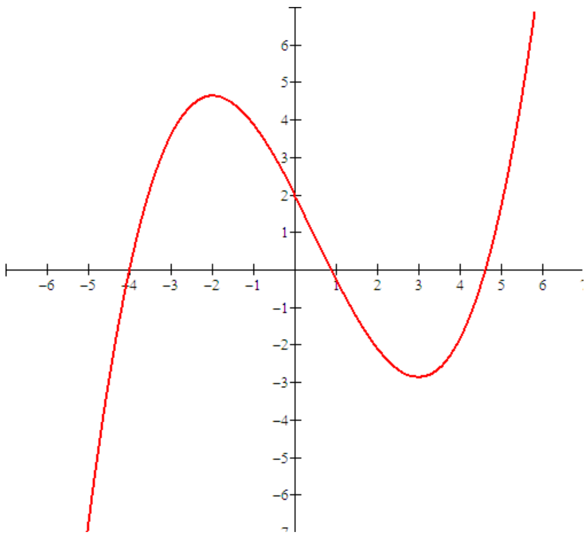
f increasing $(1,3)$

f decreasing $(3,5); (-\infty, 1)$

f constant $(5, \infty)$

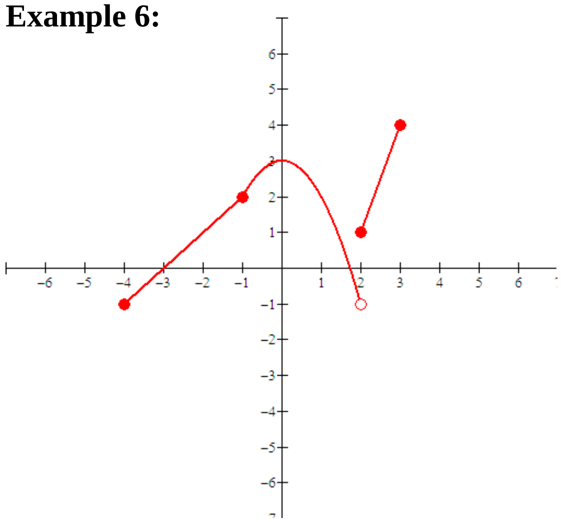
To be finished on Tuesday; 01/26

Example 5: State the intervals on which the function graphed is increasing and the intervals on which it is decreasing. Identify any turning points on the graph of this function.



You should be able to state the domain and range of a function, given its graph.

Example 6:



Find the domain and range of the given function.

When is the function decreasing?

NOTE: You should be familiar with some vocabulary from College Algebra. You should know what we mean by a maximum value, a minimum value, a turning point, and an increasing function and a decreasing function.

A **maximum value** is the biggest *y* value a function takes on. A function may or may not have a maximum value.
A **minimum value** is the smallest *y* value a function takes on. A function may or may not have a minimum value.

Example 7: State the maximum and minimum values of the function that is graphed.