Popper # 12 ← Bubble

$\sin\theta = -\frac{1}{2}$

I, IV

1. $\cos\left(\underbrace{\sin^{-1}\left(-\frac{1}{2}\right)}_{\theta}\right) = \cos\left(-\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

A. $\frac{\sqrt{3}}{2}$ B. $\frac{1}{2}$ C. $-\frac{1}{2}$ D. $-\frac{\sqrt{3}}{2}$

I, II

2. $\tan\left(\underbrace{\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)}_{\theta}\right) = \tan\left(\frac{3\pi}{4}\right) = -1$

A. $+1$ B. -1 C. $\sqrt{2}$ D. $-\sqrt{2}$

I, IV

3. $\sin\left(\underbrace{\tan^{-1}(-1)}_{\theta}\right) = \sin\left(-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

A. $\frac{\sqrt{2}}{2}$ B. $-\frac{\sqrt{2}}{2}$ C. 1 D. -1

4. A

5. B

Section 5.1 Trigonometric Functions of Real Numbers

In calculus and in the sciences many of the applications of the trigonometric functions require that the inputs be real numbers, rather than angles. By making this small but crucial change in our viewpoint, we can define the trigonometric functions in such a way that the inputs are real numbers. The definitions of the trig functions, and the identities that we have already met (and will meet later) will remain the same, and will be valid whether the inputs are angles or real numbers.

That's why we use radian measure. They are "the real numbers" where we define the trigonometric functions.

Here are some identities you need to know:

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

$$\cot(t) = \frac{\cos(t)}{\sin(t)} = \frac{1}{\tan t}$$

Reciprocal Identities

$$\sec(t) = \frac{1}{\cos(t)}, \quad \cos(t) \neq 0$$

$$\csc(t) = \frac{1}{\sin(t)}, \quad \sin(t) \neq 0$$

$$\cot(t) = \frac{1}{\tan(t)}, \quad \tan(t) \neq 0$$

When becomes "0" you get V.A. for each function.

Pythagorean Identities

$$\sin^2(t) + \cos^2(t) = 1$$

→ since $x = \cos t$, $y = \sin t$ and $x^2 + y^2 = 1$, unit circle.

$$1 + \tan^2(t) = \sec^2(t)$$

→ $1 + \tan^2 t = 1 + \frac{\sin^2 t}{\cos^2 t} = \frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t} = \sec^2 t$

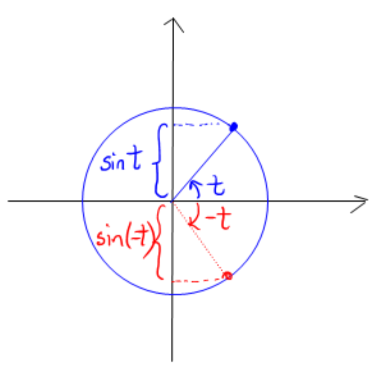
$$1 + \cot^2(t) = \csc^2(t)$$

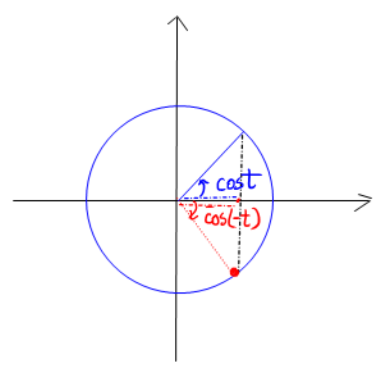
→ similar.

Opposite Angle Identities

We have been using these all the time! Now, we just state them!!!

$$\sin(-t) = -\sin(t)$$
$$\cos(-t) = \cos(t)$$
$$\tan(-t) = -\tan(t)$$
$$\cot(-t) = -\cot(t)$$
$$\sec(-t) = \sec(t)$$
$$\csc(-t) = -\csc(t)$$





$\overline{\text{III}}$

Example 1: Given that $\cos(t) = -\frac{1}{4}$ and $\pi < t < \frac{3\pi}{2}$; find $\sin(t)$ and $\tan(t)$.

$\cos^2 t + \sin^2 t = 1$

$\left(-\frac{1}{4}\right)^2 + \sin^2 t = 1$

$\sin^2 t = 1 - \frac{1}{16} = \frac{15}{16}$

$\Rightarrow \sin t = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$, $\underline{\text{III}}$

$\tan t = \frac{\sin t}{\cos t} = \frac{-\sqrt{15}/4}{-1/4} = \sqrt{15}$

Example 2: If $\sin(t) = \frac{1}{4}$ and $\tan(t) < 0$, find $\sec(t)$.

$\sin t = \frac{1}{4} > 0$

$\tan t = \frac{\sin t}{\cos t} < 0 \Rightarrow \cos t < 0$

$\sin^2 t + \cos^2 t = 1$

$\left(\frac{1}{4}\right)^2 + \cos^2 t = 1$

$\cos^2 t = \frac{15}{16}$

$\Rightarrow \cos t = -\sqrt{\frac{15}{16}} = -\frac{\sqrt{15}}{4}$

$\boxed{\sec t = \frac{1}{\cos t} = -\frac{4}{\sqrt{15}}}$

Example 3: Use the opposite-angle identities to find:

a. $\sin\left(-\frac{2\pi}{3}\right) = -\sin\frac{2\pi}{3} = -\frac{\sqrt{3}}{2}$

b. $\cos\left(-\frac{5\pi}{6}\right) = \cos\left(\frac{5\pi}{6}\right) = -\frac{\sqrt{3}}{2}$

c. $\tan\left(-\frac{\pi}{4}\right) + \cot\left(-\frac{3\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) - \cot\left(\frac{3\pi}{4}\right)$

$= -1 - (-1)$

$= 0$

Here's another set of identities:

Periodicity (the interval where the function gets repeated)

The sine and cosine functions are periodic functions. That means that there is some number p such that $f(x+p)=f(x)$. The number p is the **period** of the function. So

$\sin(t+2\pi)=\sin(t)$ more generally $\sin(t+2k\pi)=\sin(t)$

$\cos(t+2\pi)=\cos(t)$ $\cos(t+2k\pi)=\cos(t)$

After a full

for all real numbers t and all integers k.

revolution, sine and cosine repeat!

The tangent and cotangent functions are also periodic functions. However, these functions repeat themselves when $\mathbf{p=\pi}$. So

$\tan(t+\pi)=\tan(t)$ more generally $\tan(t+k\pi)=\tan(t)$

$\cot(t+\pi)=\cot(t)$ $\cot(t+k\pi)=\cot(t)$

for all real numbers t and all integers k.

Tangent and Cotangent repeat in every π-lengthened interval.

Note: the period for the sine and cosine functions is 2π while the period for the tangent and cotangent functions is π.

The secant and cosecant functions are the reciprocal functions, so they will follow the same periodicity rules as sine and cosine.

$\sec(t+2\pi k)=\sec(t)$

$\csc(t+2\pi k)=\csc(t)$

for all real numbers t and all integers k.

They depend on cosine and sine, respectively.

We will use the identities and periodicity to evaluate trig functions of real numbers.

So the periods coincide.

Example 4: Evaluate $\tan\left(\frac{15\pi}{4}\right) = \tan\left(\cancel{3\pi} + \frac{3\pi}{4}\right) = \tan\left(\frac{3\pi}{4}\right) = \boxed{-1}$

$$\frac{15\pi}{4} = 3\pi + \frac{3\pi}{4}$$

Example 5: Evaluate $\cos\left(\frac{25\pi}{6}\right) = \cos\left(\underbrace{\cancel{4\pi}}_{\text{2 rotations}} + \frac{\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{2}}$

$$\frac{25\pi}{6} = 4\pi + \frac{\pi}{6}$$

Example 6: Evaluate $\sin\left(\frac{19\pi}{3}\right) = \sin\left(\underbrace{\cancel{6\pi}}_{\text{3 rotations}} + \frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

$$\frac{19\pi}{3} = 6\pi + \frac{\pi}{3}$$

Example 7: Evaluate $\sin\left(\frac{-20\pi}{3}\right) = -\sin\left(\frac{20\pi}{3}\right) = -\sin\left(\cancel{6\pi} + \frac{2\pi}{3}\right)$

$$\frac{20\pi}{3} = 6\pi + \frac{2\pi}{3}$$

$$= -\sin\left(\frac{2\pi}{3}\right)$$

$$= \boxed{-\frac{\sqrt{3}}{2}}$$

extra example: not full rotation

- $\cos\left(\frac{16\pi}{3}\right) = \cos\left(5\pi + \frac{\pi}{3}\right) = \cos\left(4\pi + \pi + \frac{\pi}{3}\right)$

???

$$\frac{16\pi}{3} = 5\pi + \frac{\pi}{3}$$

$$= \cos\left(\pi + \frac{\pi}{3}\right) = \boxed{-\frac{1}{2}}$$

Example 8: Evaluate $\dfrac{\cos\left(\frac{19\pi}{3}\right)\tan\left(\frac{21\pi}{4}\right)}{\cos(8\pi)\sin\left(\frac{25\pi}{6}\right)} = \dfrac{\cos\left(\frac{\pi}{3}\right)\cdot\tan\left(\frac{\pi}{4}\right)}{\cos 0 \cdot \sin\left(\frac{\pi}{6}\right)}$

$\frac{19\pi}{3} = 6\pi + \frac{\pi}{3}$

$\frac{21\pi}{4} = 5\pi + \frac{\pi}{4}$

$8\pi = 8\pi + 0$

$\frac{25\pi}{6} = 4\pi + \frac{\pi}{6}$

$= \dfrac{\frac{1}{2}\cdot 1}{1\cdot\frac{1}{2}} = \boxed{1}$

Example 9: Evaluate $\cot\left(\frac{21\pi}{4}\right) + \dfrac{\tan\left(\frac{17\pi}{4}\right)}{\cos(11\pi)\sin\left(\frac{17\pi}{6}\right)} = \cot\left(\frac{\pi}{4}\right) + \dfrac{\tan\left(\frac{\pi}{4}\right)}{\cos(\pi)\sin\left(\frac{5\pi}{6}\right)}$

$\frac{21\pi}{4} = \cancel{5}\pi + \frac{\pi}{4}$

$\frac{17\pi}{4} = \cancel{4}\pi + \frac{\pi}{4}$

$11\pi = \cancel{10}\pi + \pi$

$\frac{17\pi}{6} = \cancel{2}\pi + \frac{5\pi}{6}$

$= 1 + \dfrac{1}{(-1)\cdot\frac{1}{2}}$

$= 1 - 2 = \boxed{-1}$

Example 10: Simplify: $\cot(-t)\sec(-t)$

$= -\cot(t)\cdot\sec(t)$

$= -\dfrac{\cancel{\cos t}}{\sin t}\cdot\dfrac{1}{\cancel{\cos t}}$

$= -\dfrac{1}{\sin t} = -\csc(t)$