Popper # 17

$\frac{13\pi}{3} = \cancel{4\pi} + \frac{\pi}{3}$

1. $\sin\left(\frac{13\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

A. $-\frac{\sqrt{3}}{2}$ B. $\frac{1}{2}$ C. $\frac{\sqrt{3}}{2}$ D. $-\frac{1}{2}$

$\frac{23\pi}{6} = 3\pi + \frac{5\pi}{6}$

2. $\tan\left(-\frac{23\pi}{6}\right) = -\tan\left(\frac{23\pi}{6}\right) = -\tan\frac{5\pi}{6} = -\left(-\frac{\sqrt{3}}{3}\right) = \frac{\sqrt{3}}{3}$

A. $\sqrt{3}$ B. $-\sqrt{3}$ C. $\frac{\sqrt{3}}{3}$ D. $-\frac{\sqrt{3}}{3}$

$\frac{16\pi}{3} = 5\pi + \frac{\pi}{3}$

$= \cancel{4\pi} + \underbrace{\frac{\pi}{1} + \frac{\pi}{3}}_{\frac{4\pi}{3}}$

3. $\frac{1}{\cos\left(\frac{16\pi}{3}\right)} = \sec\left(\frac{16\pi}{3}\right) = \frac{1}{\cos\left(\frac{4\pi}{3}\right)} = \boxed{-2}$

A. $\frac{1}{2}$ B. $-\frac{1}{2}$ C. 2 D. -2

4. A 5. B 6. C

Section 5.1 Trigonometric Functions of Real Numbers

Here are some identities you need to know:

definition

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

$$\cot(t) = \frac{\cos(t)}{\sin(t)}$$

Reciprocal Identities

reciprocal

$$\csc(t) = \frac{1}{\sin(t)},\ \sin(t) \neq 0$$

$$\sec(t) = \frac{1}{\cos(t)},\ \cos(t) \neq 0$$

$$\cot(t) = \frac{1}{\tan(t)},\ \tan(t) \neq 0$$

Opposite Angle Identities

opposite

$$\sin(-t) = -\sin(t)$$
$$\cos(-t) = \cos(t)$$
$$\tan(-t) = -\tan(t)$$
$$\csc(-t) = -\csc(t)$$
$$\sec(-t) = \sec(t)$$
$$\cot(-t) = -\cot(t)$$

Pythagorean Identities

identities

$$\sin^2(t) + \cos^2(t) = 1$$
$$1 + \tan^2(t) = \sec^2(t)$$
$$1 + \cot^2(t) = \csc^2(t)$$

Periodicity

period

$$\sin(t + 2k\pi) = \sin(t) \qquad \tan(t + k\pi) = \tan(t)$$
$$\cos(t + 2k\pi) = \cos(t) \qquad \cot(t + k\pi) = \cot(t)$$
$$\sec(t + 2\pi k) = \sec(t)$$
$$\csc(t + 2\pi k) = \csc(t)$$

(for all real numbers t and all integers k.)

Sine, cosine $\Leftrightarrow$ period $= 2\pi$ $\Leftrightarrow$ multiple of 2π.

tangent, cotangent $\Leftrightarrow$ period $= \pi$ $\Leftrightarrow$ multiple of π.

Example 1 : Simplify: $\dfrac{\cot(-t)}{\cos(-t)} = \dfrac{-\cot(t)}{\cos(t)} = \dfrac{-\frac{\cos t}{\sin t}}{\frac{\cos t}{1}}$

$$= -\frac{\cancel{\cos t}}{\sin t} \cdot \frac{1}{\cancel{\cos t}}$$

$$= -\frac{1}{\sin t} = -\csc t = \csc(-t)$$

Example 2 : Simplify: $\dfrac{\sin(t \cancel{+ 6\pi})\csc(t \cancel{+ 2\pi})}{\cot(t \cancel{+ \pi})\tan(t \cancel{+ 2\pi})}$

$$= \frac{\sin t \cdot \csc t}{\cot t \cdot \tan t}$$

$$= \frac{\frac{\cancel{\sin t}}{1} \cdot \frac{1}{\cancel{\sin t}}}{\frac{\cancel{\cos t}}{\cancel{\sin t}} \cdot \frac{\cancel{\sin t}}{\cancel{\cos t}}}$$

$$= \boxed{1}$$

period for sine, cosine, secant, cosecant $= 2\pi$

period for tangent, cotangent $= \pi$

Example 3: Simplify: $\cos(-t)+\cos(-t)\tan^2(-t)$

$$= \cos t + \cos t \cdot (-\tan t)^2$$

$$= \cos t + \cos t \cdot \tan^2 t$$

$$= \cos t + \cancel{\cos t} \cdot \frac{\sin^2 t}{\cos^{\cancel{2}} t} \quad = \cancel{\cos t} \cdot \cos t$$

$$= \frac{\cos t}{\cos t} \cdot \frac{\cos t}{1} + \frac{\sin^2 t}{\cos t}$$

$$= \frac{\cos^2 t + \sin^2 t}{\cos t} = \frac{1}{\cos t} = \boxed{\sec t.}$$

Example 4: Simplify: $\dfrac{\sec(t+\cancel{4\pi})+\csc(t+\cancel{6\pi})}{1+\tan(t+\cancel{5\pi})}$ $= \dfrac{\sec t + \csc t}{1+\tan t}$

$$= \frac{\frac{\sin t}{\sin t} \cdot \frac{1}{\cos t} + \frac{1}{\sin t} \cdot \frac{\cos t}{\cos t}}{\frac{\cos t}{\cos t} \cdot 1 + \frac{\sin t}{\cos t}} = \frac{\frac{\sin t + \cos t}{\sin t \cdot \cos t}}{\frac{\cos t + \sin t}{\cos t}}$$

$$= \frac{\cancel{\sin t + \cos t}}{\sin t \cdot \cancel{\cos t}} \cdot \frac{\cancel{\cos t}}{\cancel{\cos t + \sin t}} = \frac{1}{\sin t} = \boxed{\csc t}$$