Section 5.3b - Graphs of Tangent and Cotangent Functions

Tangent function: $f(x) = \tan(x) = \dfrac{\sin(x)}{\cos(x)}$;

Vertical asymptotes: when $\cos(x) = 0$, that is $x = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}, \ldots$

Domain: $x \neq \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}, \ldots$ Range: $(-\infty, \infty)$

x-intercepts: when $\sin(x) = 0$, that is $x = 0, \pm\pi, \pm 2\pi, \ldots$

$f(x) = \tan x = \dfrac{\sin x}{\cos x}$

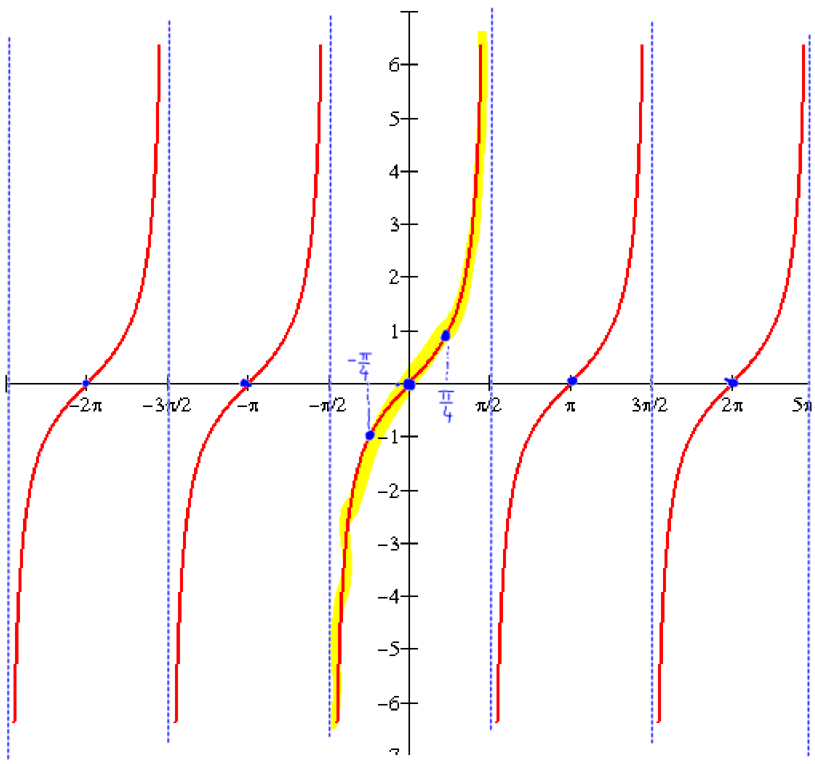
Period: π – Do not forget

x-intercepts:

$\tan x = 0$

$\Rightarrow \sin x = 0$

$x = 0, \pm\pi, \pm 2\pi \ldots$



V.A. $\cos x = 0$

$x = \pm\dfrac{\pi}{2}$

$\pm\dfrac{3\pi}{2}$

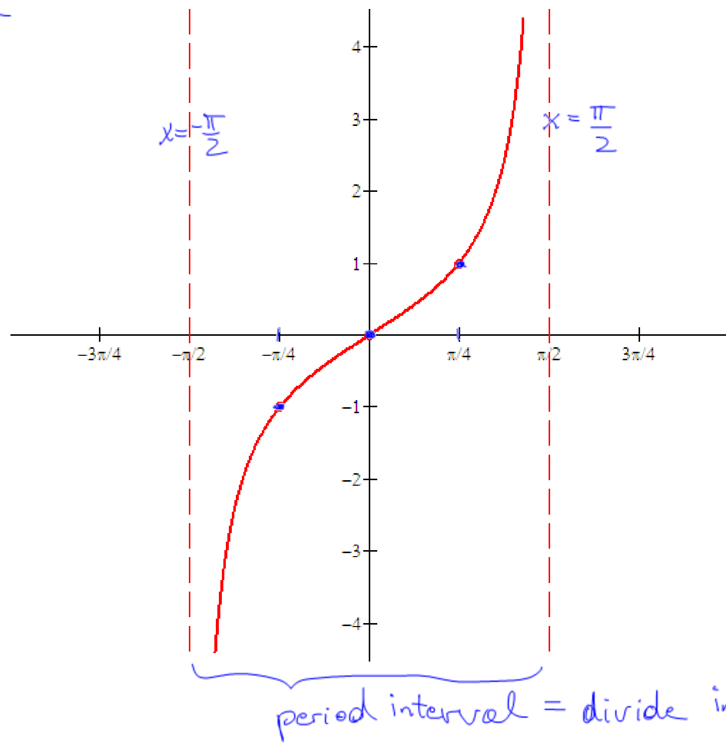
$\pm\dfrac{5\pi}{2}$

$\vdots$

$\boxed{\text{V.A. } x = \dfrac{k\pi}{2},\ k \text{ odd}}$

Need to plot these three points.

Often you will need to graph the function over just one period. In this case, you'll use the interval $\underbrace{\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)}_{\text{one-period interval}}$. Here's the graph of $f(x) = \tan(x)$ over this interval, with pertinent points marked.



Graph is always between two vertical asymptotes!

$x = -\pi/2$ $x = \frac{\pi}{2}$

Initial and final values of period interval

To graph $f(x) = A\tan(Bx - C) + D$;

Vertical Asymptotes for one period.

- The period is: $\boxed{\frac{\pi}{B}}$
- Find two consecutive asymptotes by solving: $\boxed{Bx - C = \frac{\pi}{2}}$ and $\boxed{Bx - C = -\frac{\pi}{2}}$.

  initial and final points of period

- Find an x-intercept by taking the average of the consecutive asymptotes.

  midpoint of period interval

- Find the x coordinates of the points halfway between the asymptotes and and the x-intercept. Evaluate the function at these values to find two more points on the graph of the function.

  Divide period interval in four equal pieces.

Note: If B > 1, it's a horizontal shrink. If 0 < B < 1, it's a horizontal stretch.

Example 1: Sketch $f(x) = \underbrace{2}_{A=2}\tan\left(\underbrace{\frac{x}{4}}_{B=\frac{1}{4}}\right)$ (No Shifting)

- $B = \frac{1}{4} \Rightarrow \text{period} = \frac{\pi}{1/4} = 4\pi$
- Vertical Asymptotes:

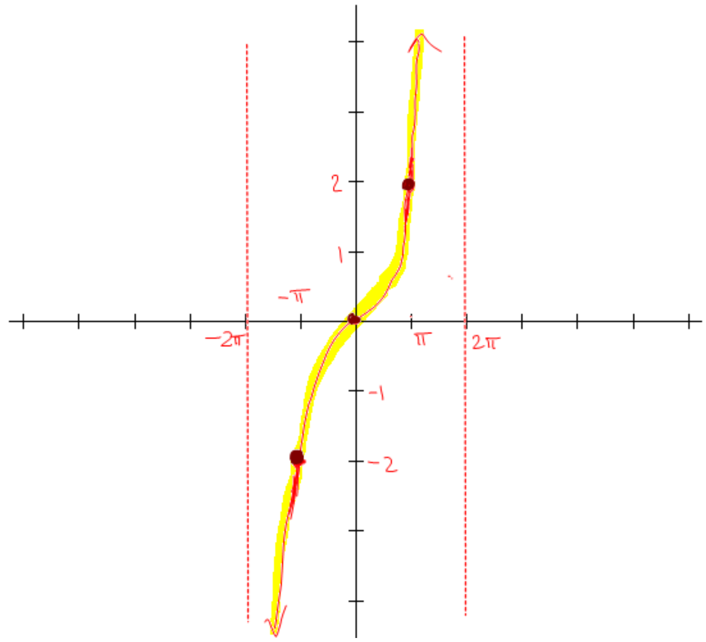
  $\cos\left(\frac{x}{4}\right) = 0$

  $\frac{x}{4} = -\frac{\pi}{2}$ or $\frac{x}{4} = \frac{\pi}{2}$

  $x = \frac{-4\pi}{2}$ or $x = \frac{4\pi}{2}$

  $\boxed{x = -2\pi \text{ or } x = 2\pi}$

  graph in between
- $A = 2 \rightarrow$ Vertical Stretch



Check $x = \pi$, $f(\pi) = 2\tan\left(\frac{\pi}{4}\right) = 2 \cdot 1 = 2$

$x = -\pi$, $f(-\pi) = 2\tan\left(\frac{-\pi}{4}\right) = 2(-1) = -2$

Example 2: Sketch $f(x) = 2\tan\left(\pi x - \frac{\pi}{4}\right)$

$A=2$ $B=\pi$ $C=\frac{\pi}{4}$

(Horizontal Shift $\frac{\pi/4}{\pi} = \frac{1}{4}$ to right)

- $B=\pi \Rightarrow$ period $= \frac{\pi}{\pi} = \boxed{1}$

- Vertical Asymptotes:

$\cos\left(\pi x - \frac{\pi}{4}\right) = 0$

graph in between

$\pi x - \frac{\pi}{4} = \frac{-\pi}{2} \Rightarrow \pi x = -\frac{\pi}{4}$

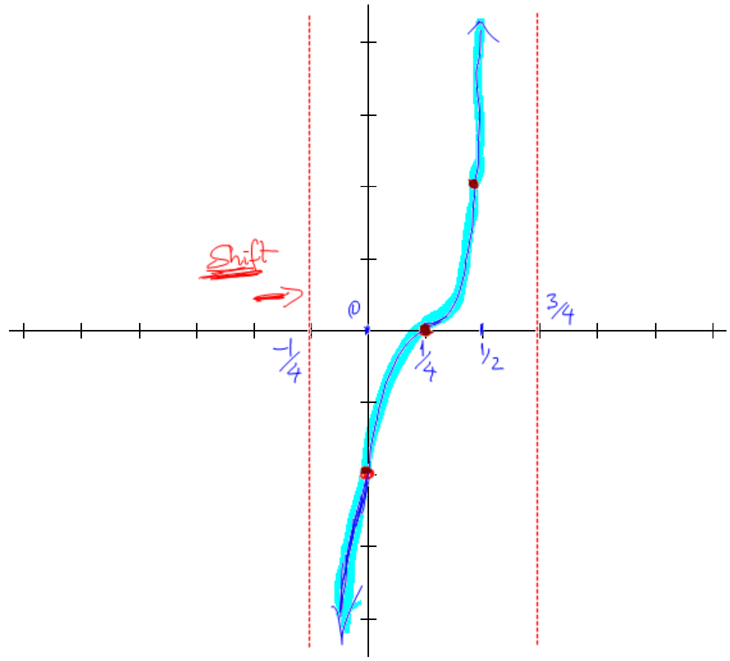
$\boxed{x = \frac{-1}{4}}$

or

$\pi x - \frac{\pi}{4} = \frac{\pi}{2} \Rightarrow \pi x = \frac{3\pi}{4}$

$\boxed{x = \frac{3}{4}}$

- $A=2 \rightarrow$ Vertical Stretch



divide into four intervals

Cotangent Function: $f(x) = \cot(x) = \dfrac{\cos(x)}{\sin(x)}$;

$f(x) = \cot(x) = \dfrac{\cos(x)}{\sin(x)}$

Vertical asymptotes: when $\sin(x) = 0$, that is $x = 0, \pm\pi, \pm 2\pi, \ldots$

Domain: $x \neq 0, \pm\pi, \pm 2\pi, \ldots$ Range: $(-\infty, \infty)$

x-intercepts: when $\cos(x) = 0$, that is $x = \pm\dfrac{\pi}{2}, \pm\dfrac{3\pi}{2}, \pm\dfrac{5\pi}{2}, \ldots$

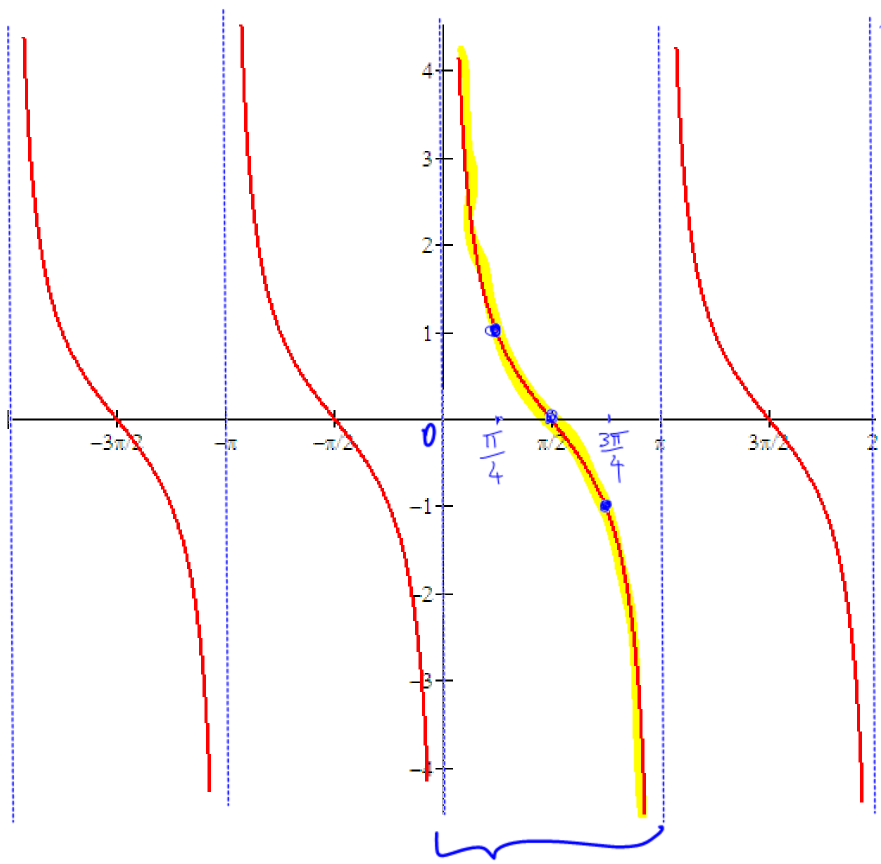
Period: π

x-intercepts:

$\cot x = 0$

$\Rightarrow \cos x = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \ldots$



V.A. $\Rightarrow \sin x = 0$

$x = 0, \pm\pi, \pm 2\pi \ldots$

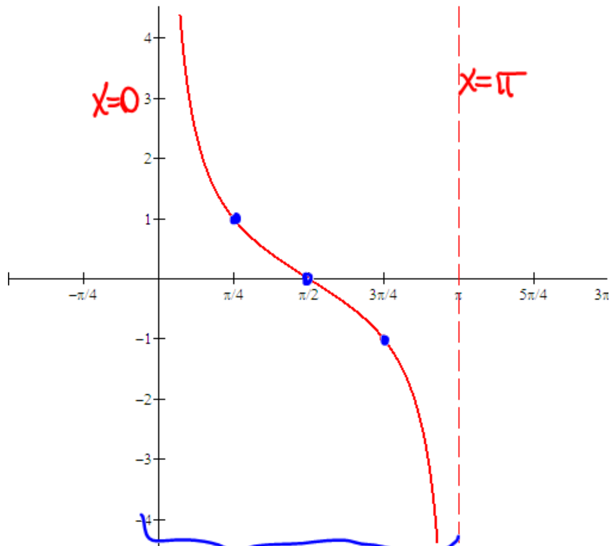
$\boxed{x = k\pi,\ k \text{ integer}}$

π = period

Plot three points.

Often you will need to graph the function over just one period. In this case, you'll use the interval $(0, \pi)$. Here's the graph of $f(x) = \cot(x)$ over this interval.

one-period interval



Graph is always between vertical asymptotes, which are
x=0 X=π
Initial and final values of period interval.

period interval = divide into four pieces.

You can take the graph of either of these basic functions and draw the graph of a more complicated function by making adjustments to the key elements of the basic function.

The key elements will be the location(s) of the asymptote(s), x intercepts, and the translations of the points at $\left(\frac{\pi}{4}, 1\right)$ and either $\left(\frac{-\pi}{4}, -1\right)$ or $\left(\frac{3\pi}{4}, -1\right)$.

To graph $g(x) = A\cot(Bx - C) + D$**;**

Vertical Asymptotes

- The period is: $\boxed{\frac{\pi}{B}}$
- Find two consecutive asymptotes by solving: $\boxed{Bx - C = 0}$ and $\boxed{Bx - C = \pi}$.
  initial and final endpoints of period
- Find an x-intercept by taking the average of the consecutive asymptotes.
  midpoint of period interval
- Find the x coordinates of the points halfway between the asymptotes and and the x-intercept. Evaluate the function at these values to find two more points on the graph of the function.
  Divide period into four equal pieces.

Note: If B > 1, it's a horizontal shrink. If 0 < B < 1, it's a horizontal stretch

Example 3: $f(x) = -4\cot\left(\pi x - \frac{\pi}{2}\right) + 6$ → Never Forget : Graph is between two vertical asymptotes.

A, B, C, D

Period: $\frac{\pi}{B} = \frac{\pi}{\pi} = 1$

Describe the transformations needed:
- $A=-4 \Rightarrow$ Vertical stretch and reflection wrt. x-axis
- $B=\pi \Rightarrow$ period $=1 \Rightarrow$ Horizontal shrinking
- $C=\frac{\pi}{2} \Rightarrow$ shift $= \frac{C}{B} = \frac{\pi/2}{\pi} = \frac{1}{2}$ to the right
- $D=6 \Rightarrow$ shift 6 units up.

Asymptotes:

$\pi x - \frac{\pi}{2} = 0$ ← first

$\Rightarrow \pi x = \frac{\pi}{2}$

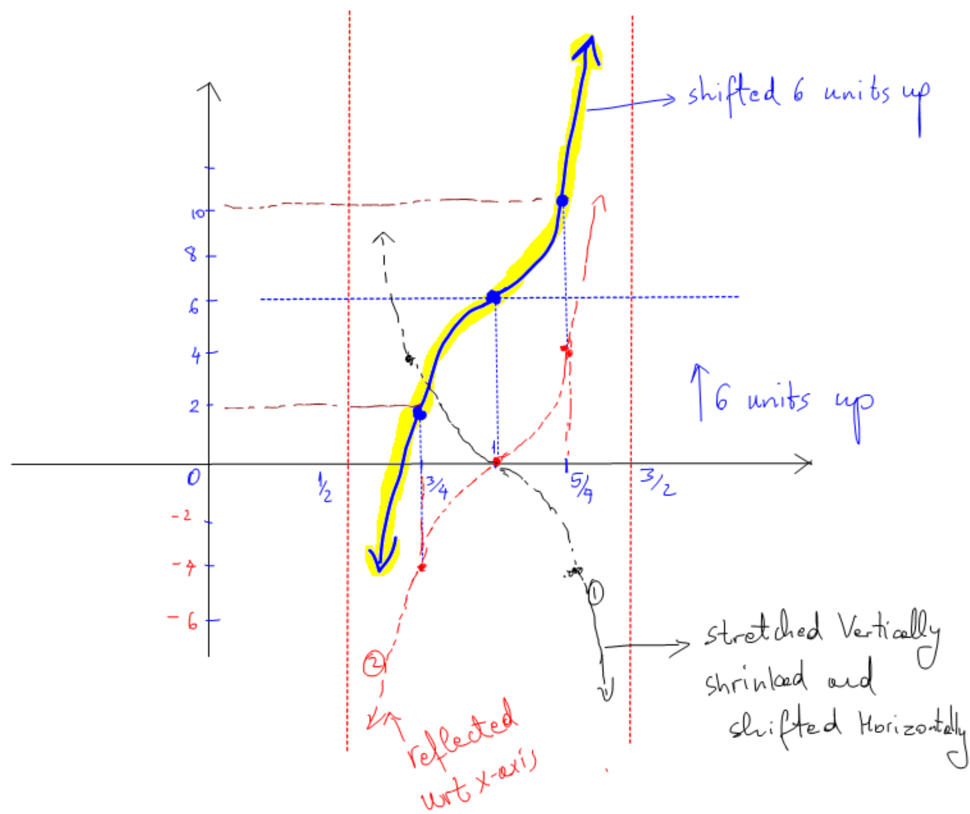
$\Rightarrow \boxed{x = \frac{1}{2}}$

or

$\pi x - \frac{\pi}{2} = \pi$ ← second

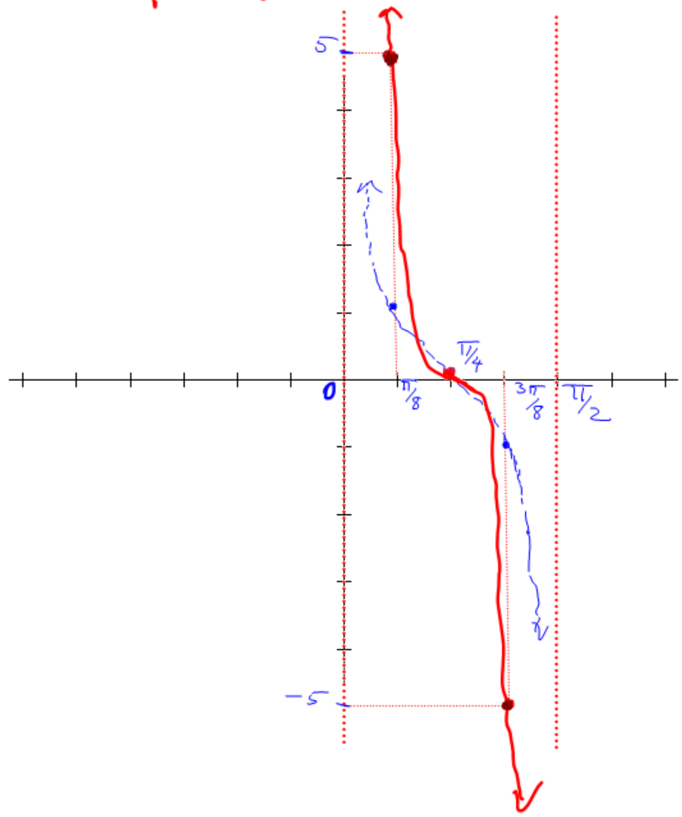
$\Rightarrow \pi x = \frac{3\pi}{2}$

$\Rightarrow \boxed{x = \frac{3}{2}}$



$A=5$ B

Example 4: Sketch $f(x)=5\cot(2x)$ $\Rightarrow$ No shiftment



- $A=5 \Rightarrow$ Vertical Stretching
- $B=2 \Rightarrow$ period $=\frac{\pi}{B}=\frac{\pi}{2}$

  Horizontal Shrinking

- Vertical Asymptotes

  $y=\cot x \Rightarrow x=0, \pi$

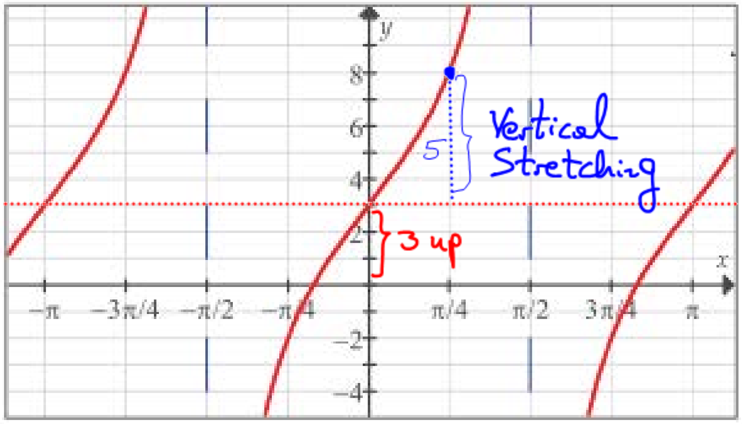
$\Rightarrow 2x=0$ or $2x=\pi$

$\boxed{x=0}$ or $\boxed{x=\frac{\pi}{2}}$

graph is between V.A.

To be continued on Monday, 3/28.

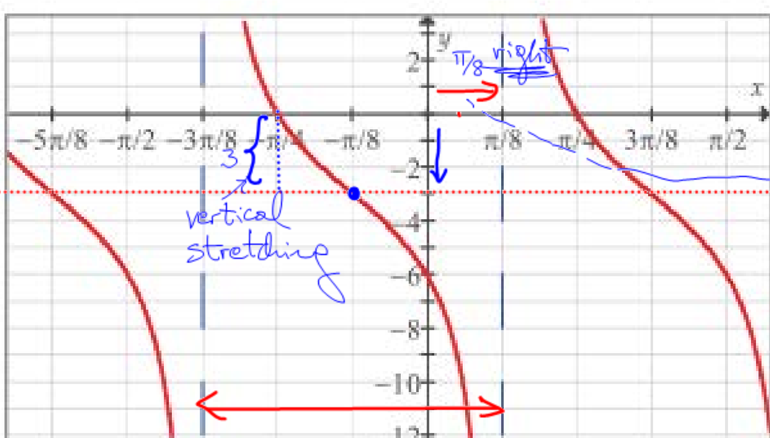
Example 5: Give an equation of the form f (x) = Atan(Bx - C)+ D and f (x) = Acot(Bx - C)+ D that could represent the following graph.



- It is a tangent function.
- Period $=\pi \Rightarrow \boxed{B=1}$
- From graph, it is shifted 3 units up $\Rightarrow \boxed{D=3}$
- No horizontal shifts $\boxed{C=0}$
- From graph, there is a vertical stretching $\boxed{A=5}$

$$\Rightarrow \boxed{y = 5\tan(x) + 3}$$

Exercise: Give an equation of the form f (x) = Atan(Bx - C)+ D and f (x) = Acot(Bx - C)+ D that could represent the following graph.



$$\text{period} = \frac{\pi}{8} - \left(-\frac{3\pi}{8}\right) = \frac{4\pi}{8} = \frac{\pi}{2}$$

- It is a cotangent fn.
- The period $= \frac{\pi}{2} = \frac{\pi}{B} \Rightarrow \boxed{B=2}$
- Horizontal shift $= \frac{\pi}{8} = \frac{C}{B}$

  $\frac{\pi}{8} = \frac{C}{2} \Rightarrow \boxed{C = \frac{2\pi}{8} = \frac{\pi}{4}}$
- There is a vertical shift of 3 units down $\Rightarrow \boxed{D=-3}$

There is a vertical stretching $\Rightarrow \boxed{A=3}$.

$$\Rightarrow \boxed{y = 3\cot\left(2x - \frac{\pi}{4}\right) - 3}$$