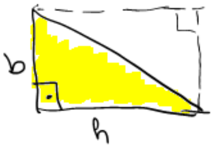
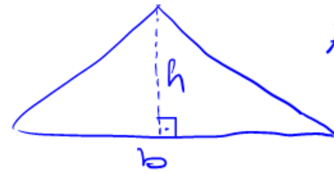


$\text{Area} = \frac{1}{2} * b * h$



$\text{Area} = \frac{1}{2} * b * h$

Section 7.2 - Area of a Triangle

In this section, we'll use a familiar formula and a new formula to find the area of a triangle.

What if we do not know the height, but only 2 sides and angle in between?

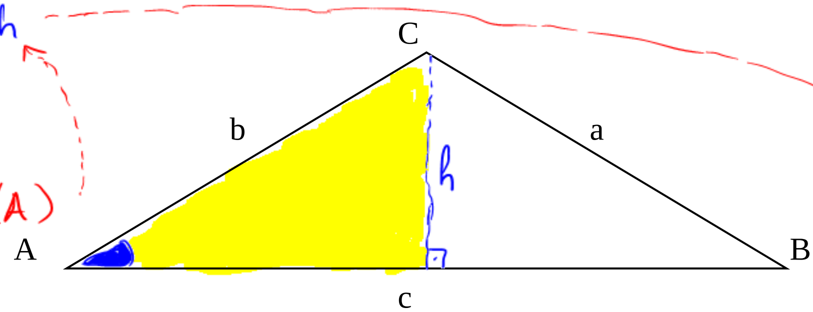
You have probably used the formula $K = \frac{1}{2}bh$ to find the area of a triangle, where b is the length of the base of the triangle and h is the height of the triangle. We'll use this formula in some of the examples here, but we may have to find either the base or the height using trig functions before proceeding.

Here's another approach to finding area of a triangle. Consider this triangle:

$\text{Area} = \frac{1}{2} c * h$

$\sin A = \frac{h}{b}$

$\Rightarrow h = b * \sin(A)$

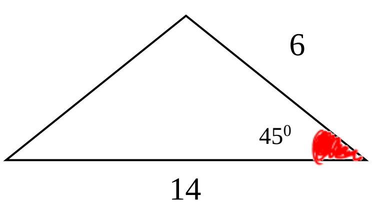


$\Rightarrow \boxed{A = \frac{1}{2} * c * b * \sin(A)}$

The area of the triangle ABC is: $\boxed{K = \frac{1}{2}bc\sin(A)}$

It is helpful to think of this as ***Area = ½*side*side*sine of the included angle.***

Example 1: Find the area of the triangle.



Check: side, side, angle in between ✓

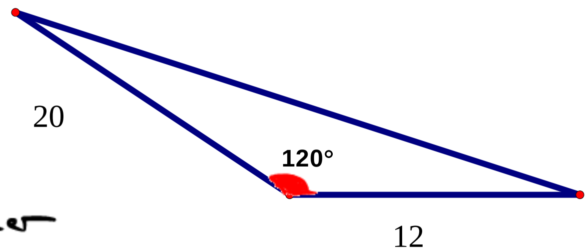
$A = \frac{1}{\cancel{2}} * \overset{3}{\cancel{6}} * 14 * \sin(45^\circ)$

$A = \overset{21}{\cancel{42}} * \frac{\sqrt{2}}{\cancel{2}} = 21\sqrt{2} \text{ unit}^2$ exact

$\boxed{A \approx 29.7 \text{ unit}^2}$ calculator

Example 2: Find the area of the triangle.

Check : side angle side ✓



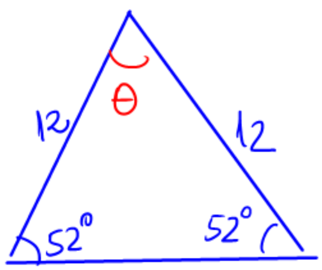
$A = \frac{1}{\cancel{2}} * \cancel{12}^{6} * 20 * \sin(120°)$

$A = 120 \cdot \frac{\sqrt{3}}{2} = 60\sqrt{3}$ ← exact

$A \cong 103.92$ ← calculator

Note: Consider the triangle with sides 12, 20 and angle 60°.
Area = 103.92 same as above triangle

Example 3: Find the area of an isosceles triangle with legs measuring 12 inches and base angles measuring 52 degrees each. Round to the nearest hundredth.



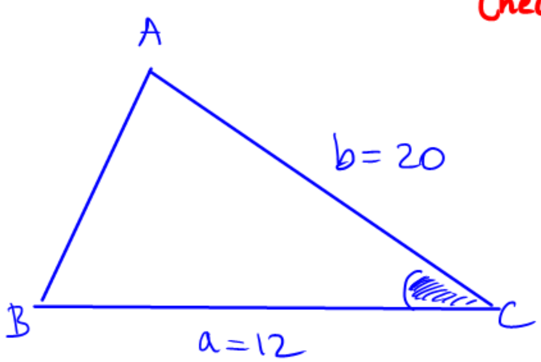
$\theta = 180° - 52° - 52° = 76°$ ⇒ Check : side angle side ✓

$A = \frac{1}{\cancel{2}} * \cancel{12}^{6} * 12 * \sin(76°) = 72\sin(76°)$

$\boxed{A \cong 69.86 \text{ in}^2}$

Example 4: In triangle ABC; $a = 12, b = 20$ and $\sin(C) = 0.42$. Find the area of the triangle.

Check : side angle side ✓



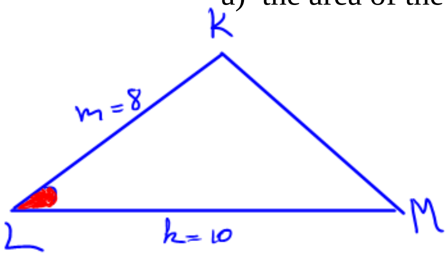
$A = \frac{1}{\cancel{2}} * \cancel{12}^{6} * 20 * \sin(C)$

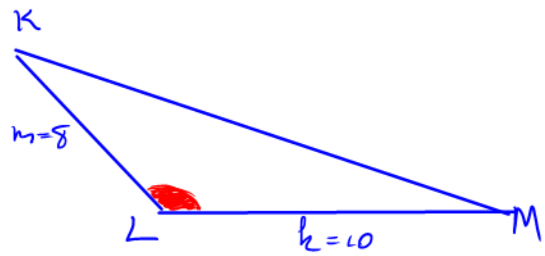
$A = 120 * 0.42$

$\boxed{A = 50.4 \text{ unit}^2}$

Example 5: In triangle KLM, k = 10 and m = 8. Find all possible measures of the angle L if

a) the area of the triangle is 20 unit squares.





$$A = \frac{1}{2} * k * m * \sin(L)$$

$$20 = \frac{1}{\cancel{2}} * \overset{5}{\cancel{10}} * 8 * \sin(L)$$

$$\Rightarrow \sin(L) = \frac{\cancel{20}}{\cancel{40}} = \frac{1}{2}$$

$$\Rightarrow \boxed{L = 30^\circ \text{ or } L = 150^\circ}$$

b) the area of the triangle is 25 unit squares.

$$A = \frac{1}{2} * k * m * \sin(L)$$

$$25 = \frac{1}{\cancel{2}} * \overset{5}{\cancel{10}} * 8 * \sin(L) \Leftrightarrow \sin(L) = \frac{\cancel{25}}{\cancel{40}} = \frac{5}{8} \Rightarrow \boxed{L = \sin^{-1}\left(\frac{5}{8}\right) \approx 39^\circ \text{ or } L = 180^\circ - 39^\circ = 141^\circ}$$

never forget

c) the area of the triangle is 80 unit squares.

$$A = \frac{1}{2} * k * m * \sin(L)$$

$$80 = \frac{1}{\cancel{2}} * \overset{5}{\cancel{10}} * 8 * \sin(L) \Rightarrow \sin(L) = \frac{80}{40} = 2 \leftarrow \underline{\text{No Solution}}$$

$$\boxed{\text{IMPOSSIBLE}}$$ no triangle

d) the area of triangle is 40 unit squares

$$A = \frac{1}{2} \cdot k \cdot m \cdot \sin(L)$$

$$40 = \frac{1}{2} \cdot \overset{5}{\cancel{10}} \cdot 8 \cdot \sin(L) \Rightarrow \sin(L) = \frac{40}{40} = 1$$

$$\Rightarrow \boxed{L = 90^\circ}$$

just one triangle

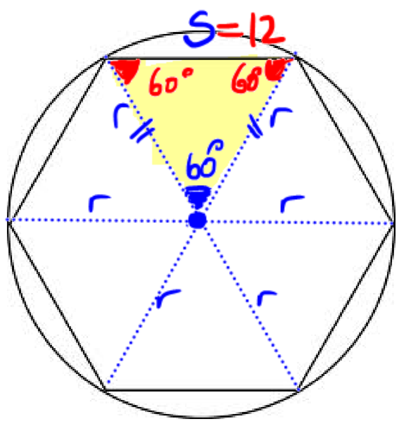
Formula for Area of a Regular Polygon Given a Side Length

$$\boxed{A = \frac{S^2 N}{4\tan\left(\frac{\pi}{N}\right)}}$$, where S = length of a side, N = number of sides.

→ Look at next page for the proof of formula.

Example 6: A regular hexagon is inscribed in a circle of radius 12. Find the area of the hexagon.

$N=6$ $r=12$



- Divide into 6 equal triangles.

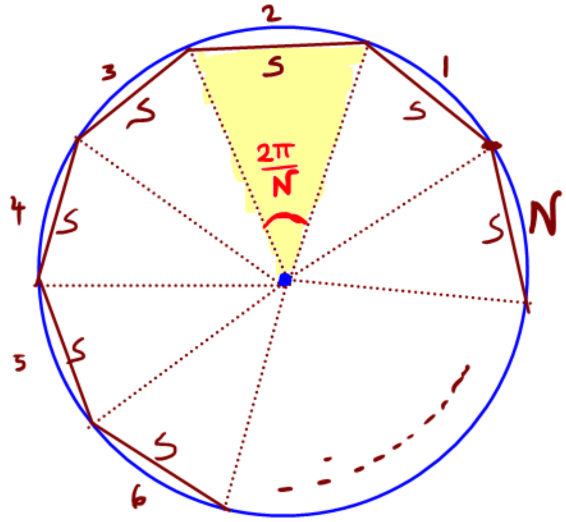
  Each central angle $= \frac{360°}{6} = 60°$

- Look at one triangle

  It is equilateral $\Rightarrow S=12$

$$\Rightarrow \text{Area} = \frac{S^2 \cdot N}{4\tan\left(\frac{\pi}{N}\right)} = \frac{12^2 \cdot 6}{4 \cdot \tan(30°)} = \frac{216}{\frac{1}{\sqrt{3}}} = \boxed{216\sqrt{3}}$$

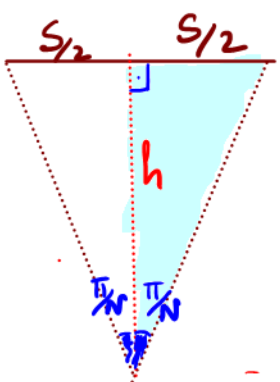
For reference, a pentagon has 5 sides, a hexagon has 6 sides, a heptagon has 7 sides, an octagon has 8 sides, a nonagon has 9 sides and a decagon has 10 sides.



- Think of a regular polygon with N sides, each side s long.
- Connect the vertices with the center of polygon.

$$\text{Area}_{\text{polygon}} = N \cdot \text{Area}_{\text{triangle}}$$

- You divided the polygon into N <u>equal</u> triangles.
- Each central angle is $\frac{2\pi}{N}$.



$$\text{Area}_{\text{triangle}} = \frac{1}{2} \cdot s \cdot h = \frac{1}{2} \cdot s \cdot \frac{s}{2\tan\left(\frac{\pi}{N}\right)} = \frac{s^2}{4\tan\left(\frac{\pi}{N}\right)}$$

$$\tan\left(\frac{\pi}{N}\right) = \frac{s/2}{h} = \frac{s}{2h}$$

$$\Rightarrow h = \frac{s}{2\tan\left(\frac{\pi}{N}\right)}$$

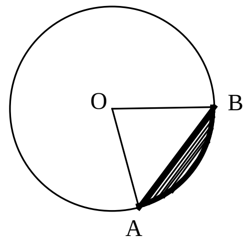
$$\Rightarrow \text{Area}_{\text{polygon}} = N \cdot \text{Area}_{\text{triangle}}$$

$$\boxed{A = \frac{N s^2}{4\tan\left(\frac{\pi}{N}\right)}}$$

exercise

Area of a segment of a circle

You can also find the area of a segment of a circle. The shaded area of the picture is an example of a segment of a circle.



To find the area of a segment, find the area of the sector with central angle θ and radius *OA*. Then find the area of ΔOAB. Then subtract the area of the triangle from the area of the sector.

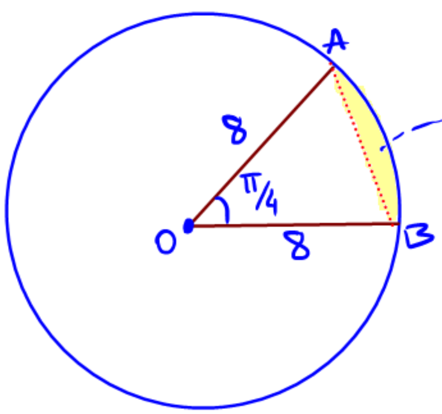
Area of segment = Area of sector AOB - Area of ΔAOB

$$= \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin(\theta)$$

Example 7: Find the area of the segment of the circle with radius 8 inches and central angle measuring $\frac{\pi}{4}$.

Solution:

$$A_{segment} = A_{sector} - A_{\Delta AOB}$$



$$\boxed{\text{Area} = 8\pi - 16\sqrt{2}}$$

$$\text{Area}_{sector} = \frac{1}{2}r^2\cdot\theta = \frac{1}{2}\cdot 8^2\cdot\frac{\pi}{4} = 8\pi$$

$$\text{Area}_{\Delta AOB} = \frac{1}{2}\cdot 8\cdot 8\cdot\sin\frac{\pi}{4} = \frac{32}{1}\cdot\frac{\sqrt{2}}{2} = 16\sqrt{2}$$