Popper **05** ← Bubble

1. Find the center of circle $\boxed{x^2 + (y-2)^2 = 3}$

A. $(0, 2)$ B. $(2, 0)$ C. $(0, 0)$ D. $(\sqrt{3}, 0)$

2. Given parabola $\boxed{y^2 = 4x - 8}$, find the focus point coordinates.

A. $(1, 0)$ B. $(3, 0)$ C) $(1, 2)$ D. $(0, 1)$

3. Find the orientation of the ellipse $\boxed{\frac{x^2}{36} + \frac{y^2}{11} = 1}$

A. Vertical B. Horizontal

4. For the ellipse $\boxed{\frac{x^2}{36} + \frac{y^2}{11} = 1}$, find foci points

A. $(9, 0)$, $(-9, 0)$ B. $(\sqrt{11}, 0)$, $(-\sqrt{11}, 0)$ C. $(5, 0)$, $(-5, 0)$ D. none.

5. Bubble A.

6. Bubble B.

Ellipse – roughly speaking – elongated version of a circle.

Math 1330 – Section 8.2
Ellipses

Follow the coloured info:

Definition: An *ellipse* is the set of all points, the sum of whose distances from two fixed points is constant. Each fixed point is called a *focus* (plural = *foci*).

Basic ellipses (centered at origin): Vertices & foci on y-axis

Basic "vertical" ellipse:

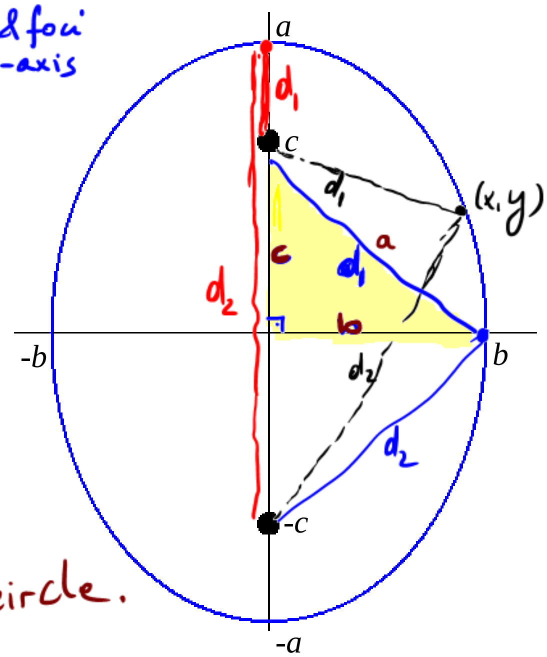
Equation: $\dfrac{x^2}{b^2}+\boxed{\dfrac{y^2}{a^2}}=1,\ a>b$

Foci: $(0,\pm c)$, where $c^2=a^2-b^2$

$\boxed{\text{Vertices: } (0,\pm a)}$ major axis

Eccentricity: $e=\dfrac{c}{a}$

shows how much the ellipse deviates from a circle.



$d_1+d_2=$ fixed ?

$d_1=a-c$

$d_2=a+c$

$d_1+d_2=2a$

"a" will give vertices.

$d_1=d_2$

$d_1+d_2=2a$

$d_1=d_2=a$

$c^2+b^2=a^2$

$\Rightarrow c^2=a^2-b^2$

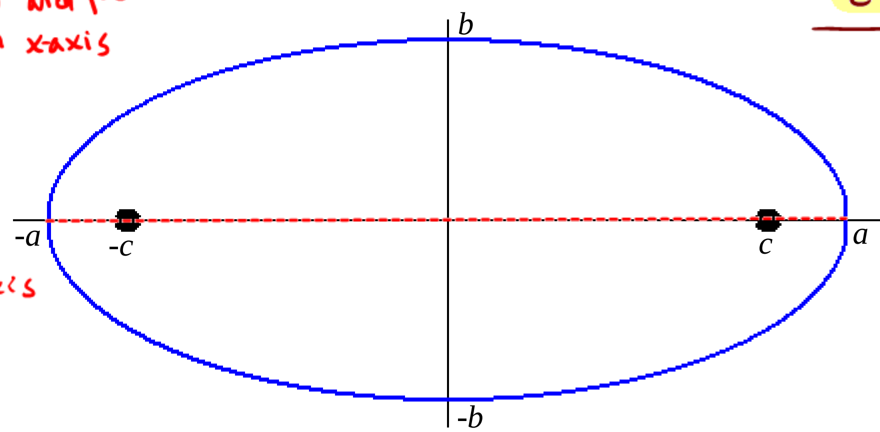
Basic "horizontal" ellipse: Vertices and foci on x-axis

Equation: $\boxed{\dfrac{x^2}{a^2}}+\dfrac{y^2}{b^2}=1,\ a>b$

Foci: $(\pm c,0)$, where $c^2=a^2-b^2$

$\boxed{\text{Vertices: } (\pm a,0)}$ major axis

Eccentricity: $e=\dfrac{c}{a}$

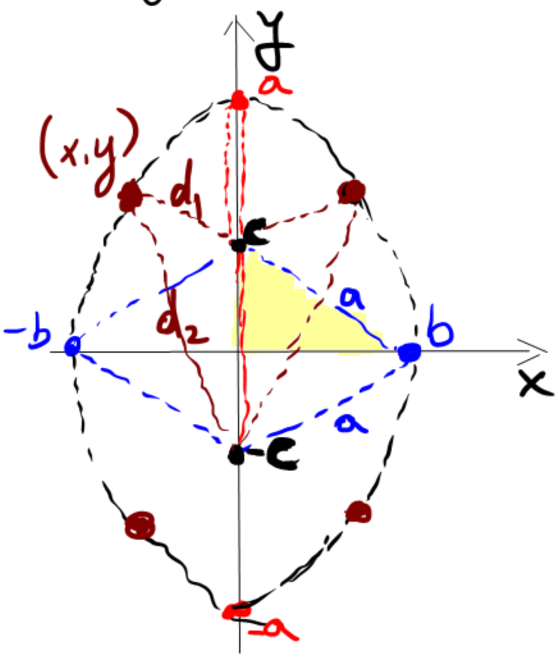


The eccentricity provides a measure on how much the ellipse deviates from being a circle. The *eccentricity e* is a number between 0 and 1.

- small *e*: graph resembles a circle (foci close together)
- large *e*: flatter, more elongated (foci far apart)
- if the foci are the same, it's a circle!

(The next two slides shows how the ellipse gets its formula $\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1$. – In case you wonder how.)

Getting an ellipse:



$$\boxed{c^2+b^2=a^2}.$$

Fix two points on the y-axis, say $(0,c)$ and $(0,-c)$.

Then plot all points, the sum of whose distances from the given points is fixed, we call the sum "$2a$".

Call the coordinates on x-axis to be $(b,0)$, $(-b,0)$.

Need to find a formula that describes ellipse: Let (x,y) be any point, then

$$\boxed{d_1 + d_2 = 2a = \text{fixed}}$$

$(x,y) \Leftrightarrow (0,c)$ distance

$(x,y) \Leftrightarrow (0,-c)$ distance

$$\sqrt{x^2+(y-c)^2} + \sqrt{x^2+(y+c)^2} = 2a$$

→ Square both sides:

$$x^2+(y-c)^2+x^2+(y+c)^2+2\sqrt{(x^2+(y-c)^2)(x^2+(y+c)^2)}=4a^2$$

→ Simplify, we get

$$x^2+y^2+c^2-2a^2=-\sqrt{(x^2+(y-c)^2)(x^2+(y+c)^2)}$$

→ Square both sides again:

$$(x^2+y^2+c^2-2a^2)^2=(x^2+y^2+c^2-2yc)(x^2+y^2+c^2+2yc)$$

→ Perform calculations and simplify:

$$\cancel{(x^2+y^2+c^2)^2}+4a^4-4a^2(x^2+y^2+c^2)=\cancel{(x^2+y^2+c^2)^2}-4y^2c^2$$

$$4a^4-4a^2x^2-4a^2y^2-4a^2\boxed{c^2}=-4y^2\boxed{c^2}$$

→ Substitute $\boxed{c^2=a^2-b^2}$ from construction

$$4a^4-4a^2x^2-4a^2y^2-4a^2\underbrace{(a^2-b^2)}_{}=-4y^2\underbrace{(a^2-b^2)}_{}$$

$$\cancel{4a^4}-4a^2x^2-\cancel{4a^2y^2}-\cancel{4a^4}+4a^2b^2=-\cancel{4a^2y^2}+4y^2b^2$$

$$\Longrightarrow\quad 4a^2x^2+4b^2y^2=4a^2b^2$$

→ Divide both sides by $4a^2b^2$ $\Longrightarrow$ $\boxed{\dfrac{x^2}{b^2}+\dfrac{y^2}{a^2}=1}$

Graphing ellipses: → Bring it in standard form

$\boxed{\frac{x^2}{a^2}} + \frac{y^2}{b^2} = 1$ or $\frac{x^2}{b^2} + \boxed{\frac{y^2}{a^2}} = 1$

bigger = horizontal

bigger = vertical

To graph an ellipse with center at the origin:

- Rearrange into the form $\frac{x^2}{number} + \frac{y^2}{number} = 1$.

- Decide if it's a "horizontal" or "vertical" ellipse.
    - o if the bigger number is under x^2, it's horizontal (longer in *x*-direction).
    - o if the bigger number is under y^2, it's vertical (longer in *y*-direction).

- Use the square root of the number under x^2 to determine how far to measure in *x*-direction.

- Use the square root of the number under y^2 to determine how far to measure in *y*-direction.

- Draw the ellipse with these measurements. Be sure it is smooth with no sharp corners

- $\boxed{c^2 = a^2 - b^2}$ where a^2 and b^2 are the denominators. So $c = \sqrt{big\ denom - small\ denom}$

- The foci are located *c* units from the center on the long axis.

To graph an ellipse with center not at the origin: Shifted Ellipse

- Rearrange (complete the square if necessary) to look like $\frac{(x-h)^2}{number} + \frac{(y-k)^2}{number} = 1$.

- Start at the center (h,k) and then graph it as before.

When graphing, you will need to find the orientation, center, values for a, b and c, vertices, foci, lengths of the major and minor axes and eccentricity.

Example 1: Find all relevant information and graph $\frac{x^2}{16}+\frac{y^2}{9}=1$. $\leftrightarrow$ horizontal

$\nearrow$ bigger

$a^2=16 \Rightarrow a=4$

$b^2=9 \Rightarrow b=3$

Orientation: horizontal

Center: $(0,0)$

Vertices: $(4,0)$, $(-4,0)$

Foci: $c^2=a^2-b^2=16-9=7 \Rightarrow c=\pm\sqrt{7}$

$(\sqrt{7},0)$, $(-\sqrt{7},0)$

Length of major axis:

$2\cdot a=2\cdot 4=8$

Length of minor axis:

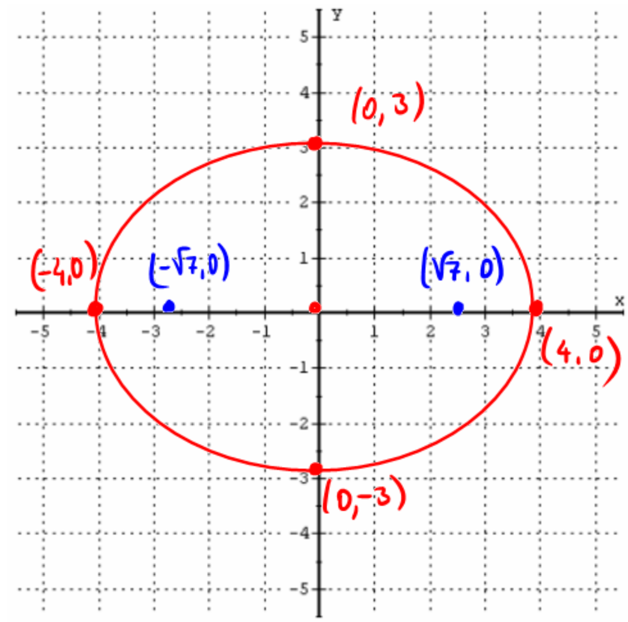
$2\cdot b=2\cdot 3=6$

Coordinates of the major axis:

$(4,0)$, $(-4,0)$

Coordinates of the minor axis: $(0,3)$, $(0,-3)$

Eccentricity: $e=\frac{c}{a}=\frac{\sqrt{7}}{4}\cong 0.66$



Example 2: Find all relevant information and graph $\frac{(x-1)^2}{9}+\frac{(y+2)^2}{25}=1$. Vertical

$\leftarrow$ bigger

$a^2=25$
$a=5$

$b^2=9$
$b=3$

$c^2=a^2-b^2$
$=16$
$c=4$

$\frac{x^2}{9}+\frac{y^2}{25}=1 \Rightarrow \frac{(x-1)^2}{9}+\frac{(y+2)^2}{25}=1$

shifted

shift 1 right, 2 down

Vertical Orientation: Vertical

$(0,0)$ Center: $(1,-2)$

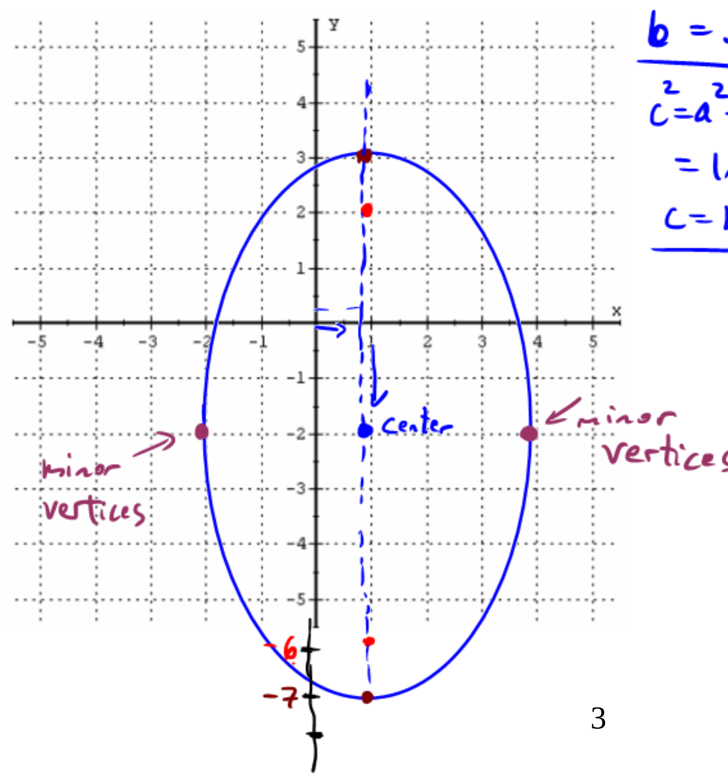
$(0,5)$, $(0,-5)$ Vertices: $(1,3)$, $(1,-7)$

$(0,4)$, $(0,-4)$ Foci: $(1,2)$, $(1,-6)$

$2\cdot 5=10$ Length of major axis: $2\cdot 5=10$

$2\cdot 3=6$ Length of minor axis: $2\cdot 3=6$

$e=\frac{c}{a}=\frac{3}{5}$ Eccentricity: $e=\frac{4}{5}=0.8$



Example 3: Write the equation in standard form. Find all relevant information and graph:
$4x^2 - 8x + 9y^2 - 54y = -49$.

⟹ group x terms together, y terms together

$(4x^2 - 8x) + (9y^2 - 54y) = -49$

⟹ Factor coefficients in front of squares

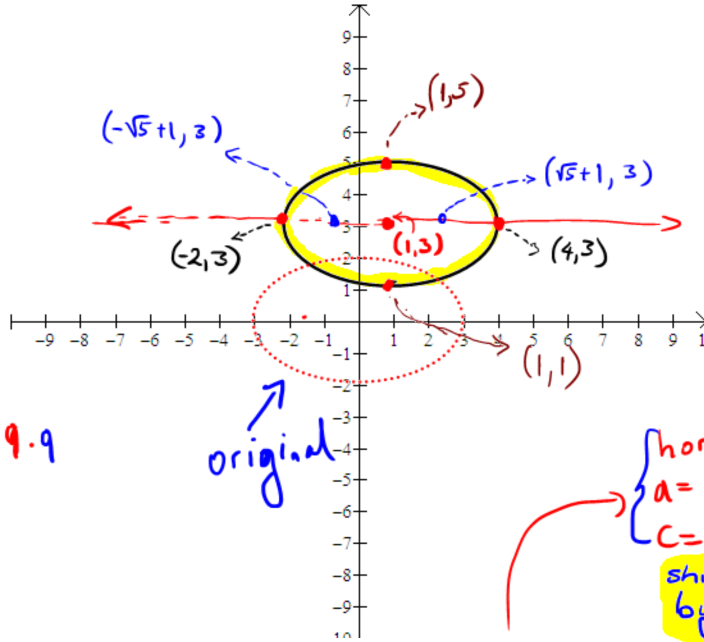
$4(x^2 - 2x + 1^2) + 9(y^2 - 6y + 9) = -49 + 4\cdot 1 + 9\cdot 9$

$\left(\frac{2}{2}=1\right)^2 \qquad \left(\frac{6}{2}=3\right)^2$

⟹ Complete the square:

$4(x-1)^2 + 9(y-3)^2 = 36$

⟹ Divide both sides by 36 ⟹ $\boxed{\dfrac{(x-1)^2}{9} + \dfrac{(y-3)^2}{4} = 1}$

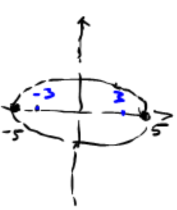


horizontal: $a = 3$, $b = 2$, $c = \sqrt{5}$

shifted by (1,3)

graph is easy now!

Example 4: Find the equation for the ellipse satisfying the given conditions.



Foci $(\pm 3, 0)$, vertices $(\pm 5, 0)$ → horizontal

$c = 3$ over x-axis $a = 5$

$c^2 = a^2 - b^2$

$3^2 = 5^2 - b^2 \Rightarrow b^2 = 16$

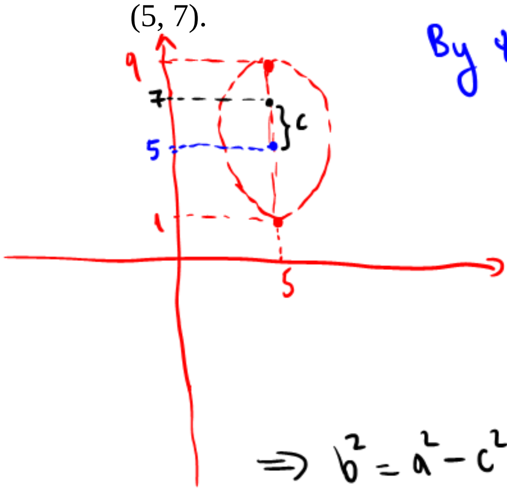
$\Rightarrow a^2 = 25$

$b^2 = 16$

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$\Rightarrow \boxed{\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1}$

Example 5: Write an equation of the ellipse with vertices (5, 9) and (5, 1) if one of the foci is (5, 7).



By the graph, it is vertical, and shifted.

Center = midpoint of major axis = (5,5)
shiftments

Foci (5,7) ⟹ $c = 2$,

length of major axis $= 2a = 8 \Rightarrow a = 4$

$\Rightarrow b^2 = a^2 - c^2 = 4^2 - 2^2 = 12$

$\Rightarrow \boxed{\dfrac{(x-5)^2}{12} + \dfrac{(y-5)^2}{16} = 1}$