

MATH 1330 - Review for Final Exam

When: Monday, May 9 – Thursday, May 12.

Where: CASA – Schedule the final exam!!!

Begins on Friday!

What is covered: All chapters.

What to bring: **Picture ID**, pencil and eraser.

Number of questions: **31 multiple choice questions** (100 points).

Time: 110 minutes

Take practice Final: 10% of your grade will be added to your final score.

Do the teacher evaluation before Thursday, May 5, midnight.
(5 bonus points will be added to your final score!)

Handy Formulas

$$\sin (s+t)=\sin s \cos t+\cos s \sin t$$

$$\sin (s-t)=\sin s \cos t-\cos s \sin t$$

$$\cos (s+t)=\cos s \cos t-\sin s \sin t$$

$$\cos (s-t)=\cos s \cos t+\sin s \sin t$$

$$\tan (s+t)=\frac{\tan s+\tan t}{1-\tan s \tan t}$$

$$\tan (s-t)=\frac{\tan s-\tan t}{1+\tan s \tan t}$$

$$\sin (2t)=2 \sin t \cos t$$

$$\cos (2t)=\cos ^2 t-\sin ^2 t$$

$$\sin \frac{s}{2}=\pm \sqrt{\frac{1-\cos s}{2}}$$

$$\cos \frac{s}{2}=\pm \sqrt{\frac{1+\cos s}{2}}$$

$$\tan \frac{s}{2}=\frac{\sin s}{1+\cos s}$$

Given Formulas:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin(2\theta) = 2 \sin \theta \cos \theta$$

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta$$

$$\tan(2\theta) = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \frac{\sin \theta}{1 + \cos \theta}$$

1. What is the y intercept(s) for $f(x) = -2(x-2)^2(x+1)$?

- A. (0, 8)
- B. (0, 0)
- C. (0, -8)
- D. (2, 0) and (-1, 0)
- E. None of the above

$$\begin{cases} \text{y-intercept} \Rightarrow y = f(0) \\ x = 0 \end{cases} = -2(0-2)^2(0+1) = -2 \cdot 4 \cdot 1 = -8$$

$$\Rightarrow \boxed{(0, -8)}$$

Vertical Asymptotes / Holes

2. Find the vertical asymptote(s) and/or hole(s), if any, for $f(x) = \frac{x^2 + 3x - 18}{x^2 + 7x + 6}$. \Rightarrow factor

- A. There are no vertical asymptotes or holes.
- B. Vertical asymptotes at $x = -6$ and $x = -1$; no holes
- C. Vertical asymptote at $x = -6$ and hole at $x = -1$
- D. Vertical asymptote at $x = -1$ and hole at $x = -6$
- E. Holes at $x = -6$ and $x = -1$; no vertical asymptotes

$$= \frac{\cancel{(x+6)}(x-3)}{\underbrace{\cancel{(x+6)}}_{\text{hole}} \underbrace{(x+1)}_{\text{V.A.}}}$$

$$\boxed{\begin{array}{ll} x+6=0 & x+1=0 \\ x=-6 \text{ hole} & x=-1 \text{ V.A.} \end{array}}$$

3. Suppose that $f(x) = \frac{2x+3}{x-1}$. Find $f^{-1}(x)$.

A. $f^{-1}(x) = \frac{x-1}{2x+3}$

B. $f^{-1}(x) = \frac{x-3}{2x-3}$

C. $f^{-1}(x) = \frac{2x+1}{x+3}$

D. $f^{-1}(x) = \frac{x+3}{x-2}$

E. None of the above

1. Rewrite $y = \frac{2x+3}{x-1}$

2. Exchange $\frac{x}{1} = \frac{2y+3}{y-1}$

3. Solve for y : Cross-Product

$$\begin{aligned} x(y-1) &= 2y+3 \\ xy-2y &= x+3 \\ \frac{y(x-2)}{\cancel{x-2}} &= \frac{x+3}{x-2} \end{aligned}$$

$$\Rightarrow \boxed{f^{-1}(x) = \frac{x+3}{x-2}}$$

4. Determine the end behavior of the polynomial function $f(x) = -3x^2(x+1)(x-2)^3$.

- A. Both ends up ↗ ↗
- B. Both ends down ↘ ↘
- C. Rising left to right ↘ ↗
- D. Falling left to right ↗ ↘

Find the leading term:

• $-3 \cdot x^2 \cdot x^1 \cdot x^3 = -3x^6$

even-degree, negative ↘ ↘

Polynomial Equation

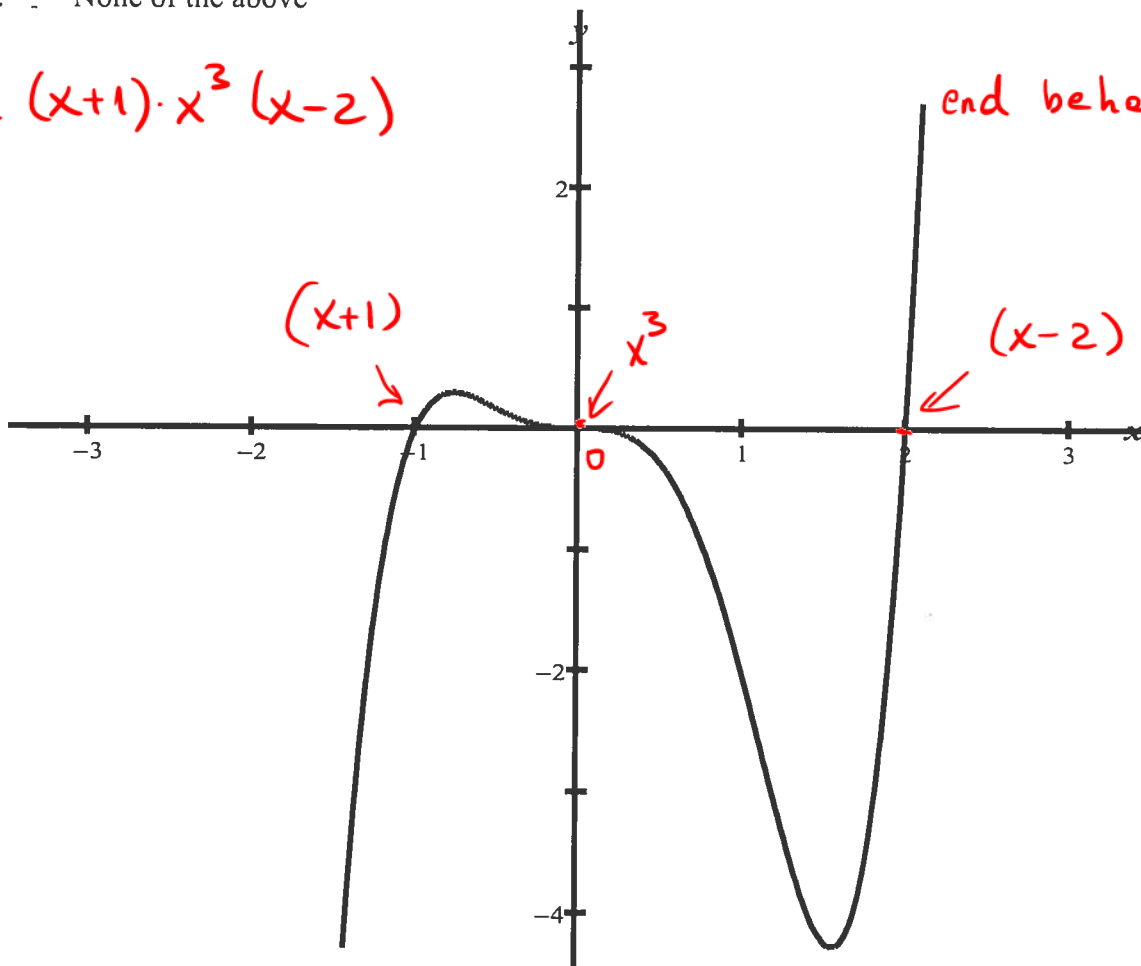
5. Which of the following functions gives this graph?

- A. $f(x) = x(x-1)^3(x+2)$
- B. $f(x) = -x^3(x+1)(x-2)$
- C. $f(x) = x^3(x+1)(x-2)$
- D. $f(x) = x^3(x-1)(x+2)$
- E. None of the above

• Find zeros and their multiplicities

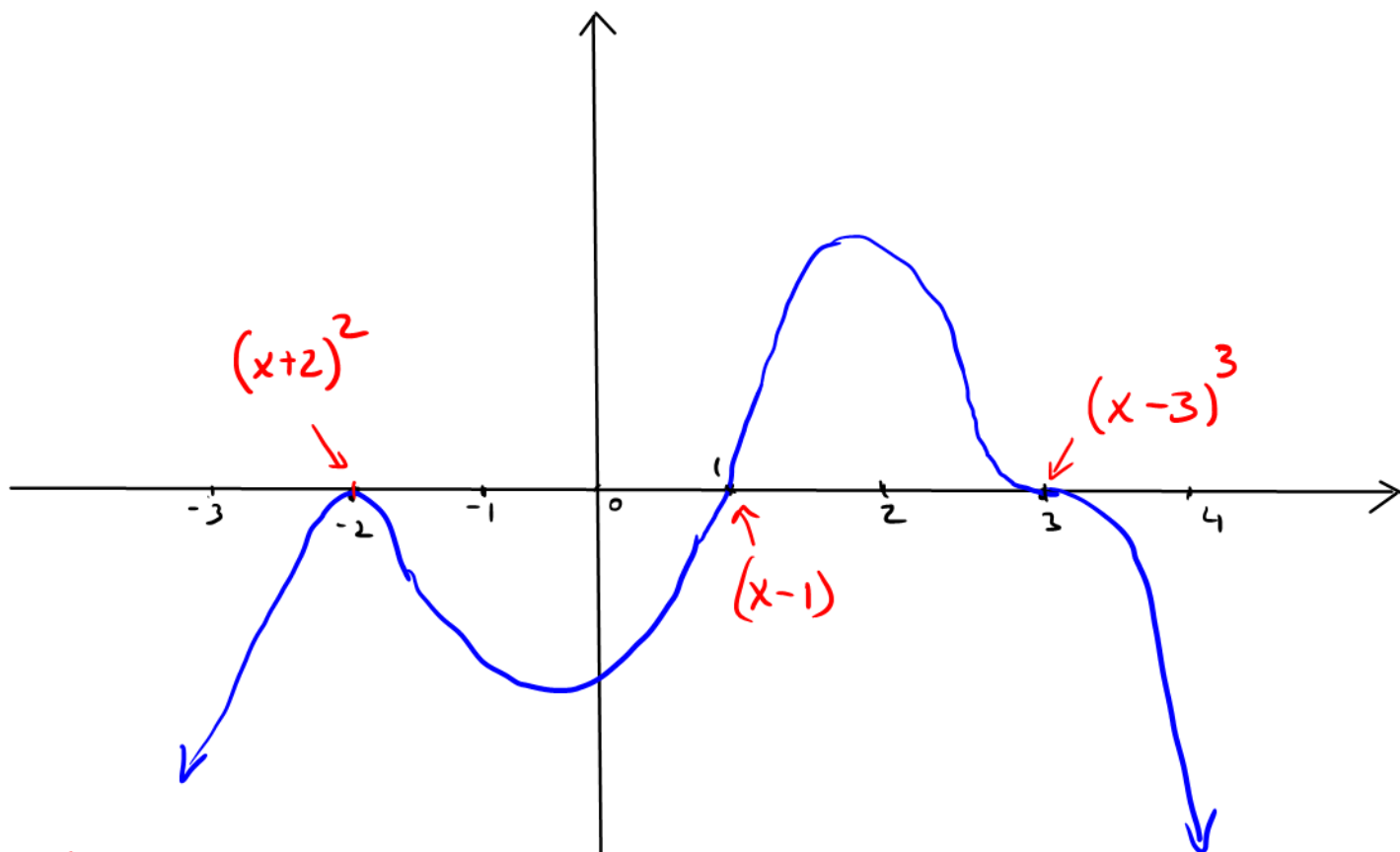
• Find coefficient from end behaviour

$f(x) = (x+1) \cdot x^3 (x-2)$



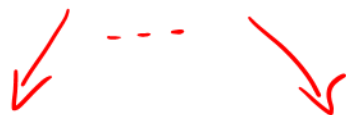
end behavior ↗ ⊕

extra:



end behaviour :

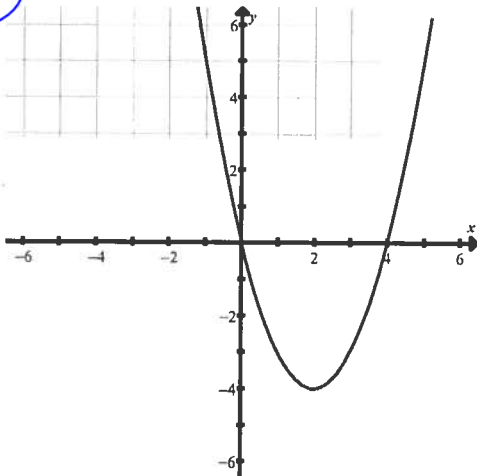
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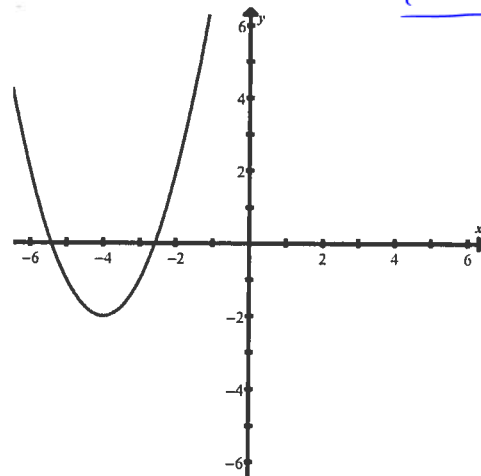
$$f(x) = -2(x+2)^2(x-1)(x-3)^3$$

6. Which of the following is the graph of $f(x) = (x-2)^2 - 4$ → shift x^2 2 to right

(A.)

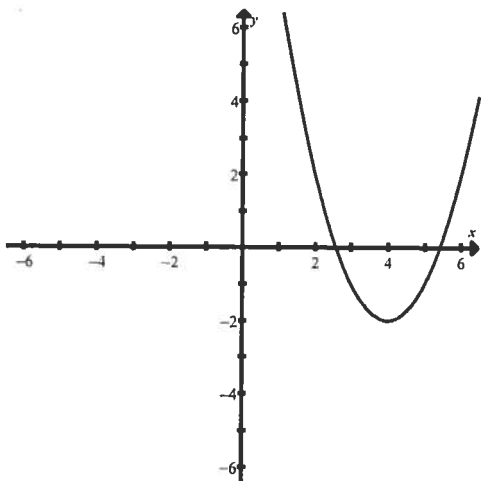


B.

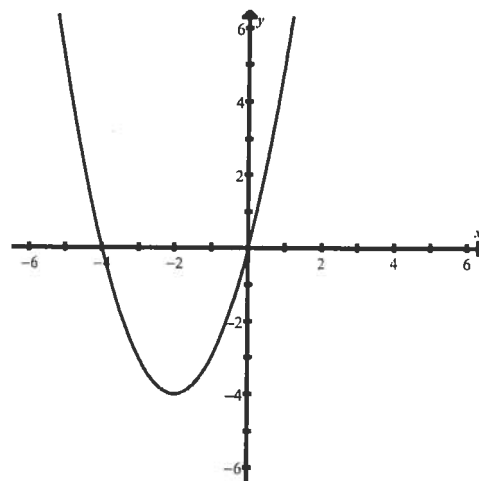


4 down

C.



D.



Circle Equation

7.

Write the equation of the circle with center $(-1, 2)$ and radius 9.

A. $(x-1)^2 + (y+2)^2 = 9$

B. $(x+1)^2 + (y-2)^2 = 9$

(C.) $(x+1)^2 + (y-2)^2 = 81$

D. $(x-1)^2 + (y+2)^2 = 81$

$$(x-h)^2 + (y-k)^2 = r^2$$

↓ substitute

$$(x-(-1))^2 + (y-2)^2 = 9^2$$

$$(x+1)^2 + (y-2)^2 = 81$$

Ellipse Vertices

8. What are the vertices for $\frac{x^2}{9} + \frac{y^2}{64} = 1$?

- A. (0, 8) and (0, -8)
- B. (8, 0) and (-8, 0)
- C. (0, -3) and (0, 3)
- D. (-3, 0) and (3, 0)
- E. None of the above

- Compare the coefficients
- The bigger one determines whether horizontal/vertical

vertical ellipse

$$b^2 = 64 \Rightarrow b = \pm 8$$

$$\Rightarrow \text{Vertices } (0, 8), (0, -8)$$

Intersection Points

9. Find the x-coordinates of the points of intersection for the functions shown below:

$$f(x) = x^2 + 2x - 1$$

$$g(x) = 3x + 5$$

- A. $\left\{2, \frac{3}{2}\right\}$
- B. $\{-5, 0\}$
- C. $\{-3, 2\}$
- D. $\{-4, 6\}$
- E. $\{-2, 3\}$

In such questions, make

$f(x) = g(x)$ and solve:

$$x^2 + 2x - 1 = 3x + 5$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

\Rightarrow

$$x = 3, -2$$

Vertex of Parabola

10. State the coordinates of the vertex for the given parabola.

- A. (2, -5)
- B. (4, -5)
- C. (5, -4)
- D. (-5, 4)
- E. (-4, 5)

$$x^2 - 8x + 31 + 3y = 0$$

$$\hookrightarrow x^2 - \underbrace{8x} + 4^2 = -3y - 31 + 4^2$$
$$\frac{8}{2} = 4$$

$$\Rightarrow (x-4)^2 = -3y - 15$$

$$(x-4)^2 = -3(y+5) \Rightarrow V = (4, -5)$$

11. Simplify the following expression:

$$\frac{\cos(\theta)}{1 - \sin(\theta)} - \frac{\cos(\theta)}{1 + \sin(\theta)} \quad \leftarrow \text{Common Denominator}$$

A. $-\csc(\theta)$

B. $-\cos(\theta)$

C. $2 \tan(\theta)$

D. $2 \sec(\theta)$

E. $-\cot^2(\theta)$

$$= \frac{\cos\theta(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)} - \frac{\cos\theta \cdot (1 - \sin\theta)}{(1 + \sin\theta) \cdot (1 - \sin\theta)}$$

$$= \frac{\cancel{\cos\theta} + \cos\theta \sin\theta - \cancel{\cos\theta} + \cos\theta \sin\theta}{1 - \sin^2\theta = \cos^2\theta}$$

$$= \frac{2 \cos\theta \cdot \sin\theta}{\cos^2\theta} = \frac{2 \sin\theta}{\cos\theta} = \boxed{2 \tan\theta}$$

12. Which of the following equals $1 - (\sin x + \cos x)^2$?

A. 0

B. $-\sin(2x)$

C. $2 \cos^2(x)$

D. $-\cos(2x)$

E. None of the above.

FOIL →

$$= 1 - (\sin x + \cos x)(\sin x + \cos x)$$

$$= 1 - [\sin^2 x + \sin x \cos x + \cos x \sin x + \cos^2 x]$$

$$= 1 - [1 + 2 \sin x \cos x]$$

$$= \cancel{1} - \cancel{1} - 2 \sin x \cos x = \boxed{-\sin(2x)}$$

Unit Circle

13. Evaluate: $\cos\left(\frac{2\pi}{3}\right) = -\cos\left(\frac{\pi}{3}\right) = -\frac{1}{2}$

II →
reference

A. $-\frac{\sqrt{3}}{3}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. $\frac{\sqrt{3}}{2}$

E. $-\frac{\sqrt{3}}{2}$

Unit Circle

III

14. Evaluate: $\sin\left(-\frac{5\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$
reference

- A. $-\frac{\sqrt{3}}{2}$ B. $\frac{\sqrt{3}}{2}$ C. $\frac{\sqrt{3}}{3}$ D. $\frac{1}{2}$ E. $-\frac{1}{2}$

15. Find the exact value of $\sqrt{1 + \cot^2\left(\frac{2\pi}{3}\right)}$.

- A. $\sqrt{2}$
B. $\frac{4}{3}$
C. 2

$$\cot\left(\frac{2\pi}{3}\right) = \frac{\cos\left(\frac{2\pi}{3}\right)}{\sin\left(\frac{2\pi}{3}\right)} = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}}$$

D. $\frac{2\sqrt{3}}{3}$

$$\sqrt{1 + \cot^2\left(\frac{2\pi}{3}\right)} = \sqrt{1 + \left(-\frac{1}{\sqrt{3}}\right)^2} = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

Inverse Sine / Cosine

16. Evaluate: $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \theta$ $\Leftrightarrow \cos \theta = -\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{5\pi}{6}$

- A. $-\frac{\pi}{6}$ B. $\frac{2\pi}{3}$ C. $\frac{5\pi}{6}$ D. $-\frac{\pi}{3}$ E. undefined

17. Find the exact value of the following expression. If undefined, state, *undefined*.

$$\sec\left(\underbrace{\sin^{-1}\left(\frac{3}{5}\right)}_{\theta}\right) = \sec\theta$$

A. $\frac{5}{4}$

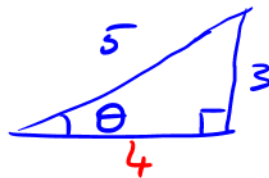
B. $\frac{3}{4}$

C. $\frac{4}{5}$

D. $\frac{4}{3}$

E. undefined

$$\Rightarrow \sin^{-1}\left(\frac{3}{5}\right) = \theta \iff \sin\theta = \frac{3}{5}$$



$$\Rightarrow \boxed{\sec\theta = \frac{5}{4}}$$

18. Suppose that $\pi < \theta < \frac{3\pi}{2}$ and that $\tan\theta = \frac{1}{4}$. Compute $\sin\theta$. \rightarrow negative

III

$$1^2 + 4^2 = 17$$

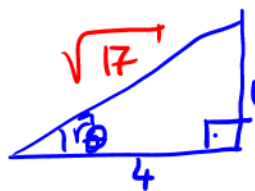
A. $\frac{\sqrt{17}}{17}$

B. $\frac{\sqrt{15}}{4}$

C. $-\frac{\sqrt{15}}{4}$

D. $-\frac{\sqrt{17}}{17}$

E. None of the above



\Rightarrow

$$\sin\theta = -\frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{\sqrt{17}} = -\frac{\sqrt{17}}{17}$$

19. Determine the equation of the sine function which has amplitude 2 and period 4. *No shiftments*

A. $y = 2\sin\left(\frac{\pi}{2}x\right)$

B. $y = 2\sin(4x)$

C. $y = 4\sin(2x)$

D. $y = 4\sin\left(\frac{\pi}{4}x\right)$

$$A = 2, \text{ period} = 4$$

$$\text{period} = \frac{2\pi}{B} = \frac{4}{1}$$

$$\boxed{f(x) = 2\sin\left(\frac{\pi}{2}x\right)}$$

$$4B = 2\pi$$

$$\Rightarrow B = \frac{\pi}{2}$$

Vertical Asymptotes for Secant/Cosecant

20. Which of these is an equation of one of the asymptotes of the following function?

$$f(x) = 7 \sec\left(\frac{1}{5}x\right) + 6 = 7 \cdot \frac{1}{\underbrace{\cos\left(\frac{1}{5}x\right)}_{=0}} + 6$$

A. $x = \frac{5}{4}\pi$

B. $x = \frac{5}{8}\pi$

C. $x = 6$

D. $x = 5\pi$

E. $x = \frac{5}{2}\pi$

$$\cos\left(\frac{1}{5}x\right) = 0$$

$$\Rightarrow \frac{1}{5}x = \frac{\pi}{2} \Rightarrow x = \frac{5\pi}{2}$$

Phase Shift = $\frac{C}{B}$

21. Find the horizontal shift for the following function:

$$f(x) = 5 \cos\left(\frac{1}{4}\pi x - \pi\right) - 2$$

A. 4 left

B. 4 right

C. π right

D. 2 left

E. π left

$$\text{Shift} = \frac{\pi}{\frac{1}{4}\pi} = 4 \text{ right}$$

Amplitude Definition = $\frac{1}{2} \cdot \text{distance from max. to min.}$

22. A sine graph with amplitude of 8 has a maximum value of 3. What is its minimum value?

$$A = 8$$

$$M = 3$$

$$\rightarrow m = ?$$

A. -5

B. -11

C. -13

D. 19

$$\text{let max} = M, \text{ min} = m \Rightarrow \underbrace{M - m}_{\text{distance}} = 2 \cdot A$$

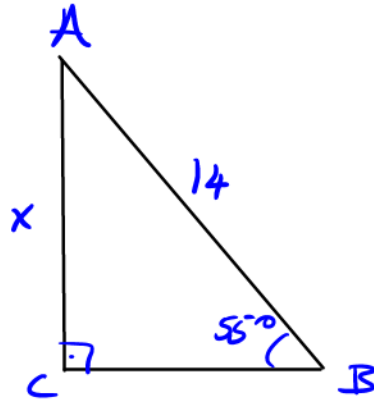
$$3 - m = 2 \cdot 8$$

$$-m = 16 - 3 = 13$$

$$\Rightarrow m = -13$$

23. In right triangle ABC with $m\angle C = 90^\circ$, $m\angle B = 55^\circ$ and AB measures 14 units. Find the length of AC.

- A. $14 \tan(55^\circ)$
- B. $14 \sin(55^\circ)$**
- C. $14 \cos(55^\circ)$
- D. $\sin(55^\circ)$
- E. 7

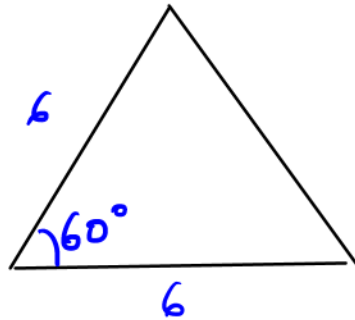


$$\Rightarrow \frac{\sin(55^\circ)}{1} = \frac{x}{14}$$

$$x = 14 \cdot \sin(55^\circ)$$

24. Find the area of an equilateral triangle with side length 6 feet.

- A. $12\sqrt{3} \text{ ft}^2$
- B. $9\sqrt{3} \text{ ft}^2$**
- C. 6 ft^2
- D. $18\sqrt{3} \text{ ft}^2$
- E. $6\sqrt{3} \text{ ft}^2$

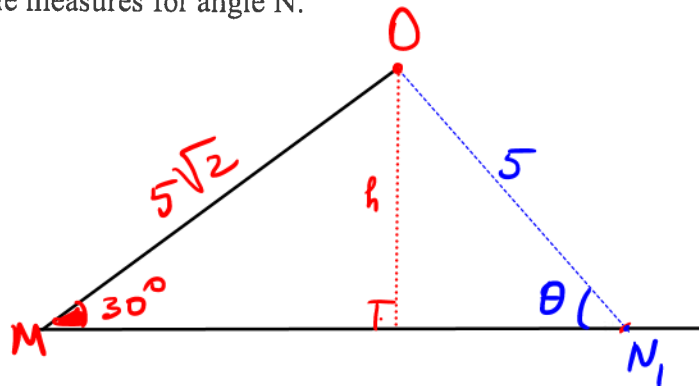


$$\begin{aligned} \text{Area} &= \frac{1}{2} \cdot 6 \cdot 6 \cdot \sin(60^\circ) \\ &= \frac{1}{2} \cdot \frac{36}{1} \cdot \frac{\sqrt{3}}{2} \\ &= 9\sqrt{3} \end{aligned}$$

Law of Sines

25. In triangle MNO, the measure of angle M is 30° , the length of NO is 5, and the length of MO is $5\sqrt{2}$. Find all possible measures for angle N.

- A. 60°
- B. 30° or 60°
- C. 135°
- D. 45°
- E. 45° or 135°**



- Check
- $MO > NO$
 $5\sqrt{2} > 5$
 - draw h
- $$h = 5\sqrt{2} \cdot \sin(30^\circ) = \frac{5\sqrt{2}}{2} < \underline{NO}$$

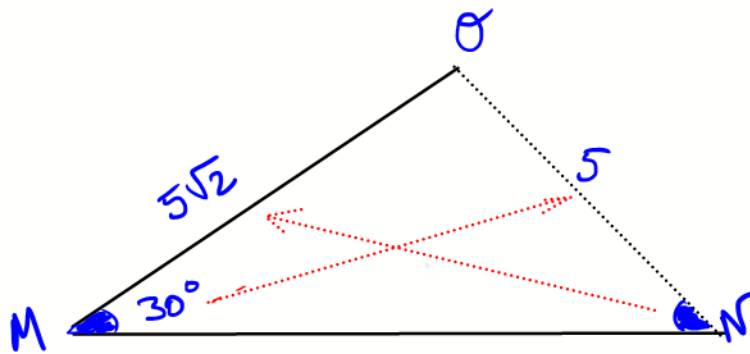
\Rightarrow two triangles

$$\frac{\sin(30^\circ)}{5} = \frac{\sin(N)}{5\sqrt{2}}$$

$$\Rightarrow \sin(N) = \frac{\sqrt{2}}{2} \rightarrow N = 45^\circ \text{ or } 135^\circ$$

25. In triangle MNO, the measure of angle M is 30° , the length of NO is 5, and the length of MO is $5\sqrt{2}$. Find all possible measures for angle N.

- A. 60°
- B. 30° or 60°
- C. 135°
- D. 45°
- E. 45° or 135°



Just apply
The
Law of Sines.

If it is possible, then Law of Sines gives

$$\frac{\sin(30^\circ)}{5} = \frac{\sin(N)}{5\sqrt{2}}$$

- if this equation has 1 solution, then just 1 triangle can be built.
- If this equation has 2 solutions then 2 triangles \Rightarrow 2 angles.
- If it has no solution \Rightarrow 0 triangle

$$\frac{\sin(30^\circ)}{5} = \frac{\sin(N)}{5\sqrt{2}} \Rightarrow \cancel{5} \sin(N) = \frac{5\sqrt{2}}{5} \cdot \underbrace{\sin(30^\circ)}_{=1/2}$$

$$\Rightarrow \sin(N) = \frac{\sqrt{2}}{2} \Rightarrow \boxed{N = 45^\circ \text{ or } 135^\circ}$$

Apply Formula

26. If $\sin \theta = \frac{1}{3}$ and θ lies in Quadrant II, find the exact value of $\sin\left(\theta + \frac{\pi}{6}\right)$.

- A. $\frac{5}{6}$
- B. $\frac{\sqrt{3}-1}{2}$
- C. $\frac{\sqrt{3}+2\sqrt{2}}{6}$

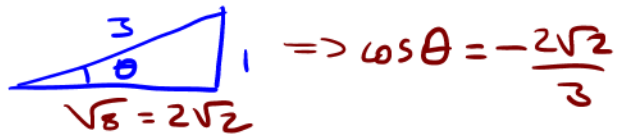
D. $\frac{\sqrt{3}-2\sqrt{2}}{6}$

$$\rightarrow \sin\left(\theta + \frac{\pi}{6}\right) = \sin \theta \cos \frac{\pi}{6} + \cos \theta \sin \frac{\pi}{6}$$

✓
✓
↓ find
✓

$$= \frac{1}{3} \cdot \frac{\sqrt{3}}{2} + \left(-\frac{2\sqrt{2}}{3}\right) \cdot \frac{1}{2}$$

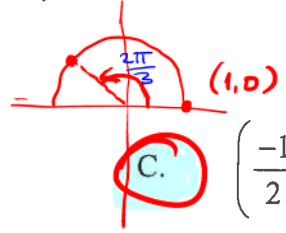
$$= \frac{\sqrt{3} - 2\sqrt{2}}{6}$$



27. Suppose an ant is sitting on the perimeter of the unit circle at the point (1, 0). If the ant travels a distance of $2\pi/3$ in the counter-clockwise direction, then the coordinates of the point where the ant stops will be

A. $\left(\frac{-\sqrt{3}}{2}, \frac{1}{2}\right)$

B. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



C. $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$

D. $\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$

$$\Rightarrow \left(\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}, \sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}\right)$$

Angle of Elevation

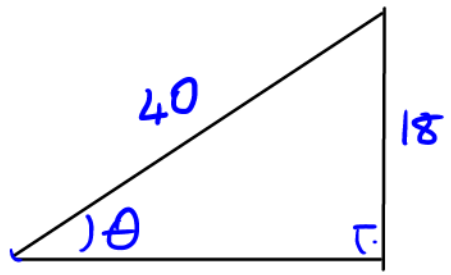
28. A skateboard ramp is 40 feet long and rises 18 feet. What is the angle of elevation of the ramp?

A. $\tan^{-1}\left(\frac{9}{20}\right)$

B. $\sin^{-1}\left(\frac{9}{20}\right)$

C. $\cos^{-1}\left(\frac{9}{20}\right)$

D. $\sin^{-1}\left(\frac{20}{9}\right)$



$$\Rightarrow \sin \theta = \frac{18}{40}$$

$$\sin \theta = \frac{9}{20}$$

$$\Rightarrow \theta = \sin^{-1}\left(\frac{9}{20}\right)$$

32. Solve $\sin(2x) = -\cos(x)$ on the interval $[0, 2\pi]$.

- A. $\left\{x = \frac{\pi}{2}, x = \frac{3\pi}{2}\right\}$
 B. $\left\{x = \frac{7\pi}{6}, x = \frac{11\pi}{6}\right\}$
 C. $\left\{x = \frac{\pi}{6}, x = \frac{5\pi}{6}, x = \frac{\pi}{2}, x = \frac{3\pi}{2}\right\}$

D. $\left\{x = \frac{7\pi}{6}, x = \frac{11\pi}{6}, x = \frac{\pi}{2}, x = \frac{3\pi}{2}\right\}$

E. No solution.

$$\sin(2x) = -\cos x$$

$$2 \sin x \cdot \cos x = -\cos x$$

$$2 \sin x \cdot \cos x + \cos x = 0$$

$$\cos x (2 \sin x + 1) = 0$$

$$\cos x = 0 \quad \text{or} \quad 2 \sin x + 1 = 0$$

$$\hookrightarrow \sin x = -\frac{1}{2}$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Law of Cosines

33. ABC is a triangle with $AB = 4$, $BC = 7$, and $AC = 9$. Find $\cos(A)$.

Note: You are asked to find $\cos(A)$ not the measure of angle A.

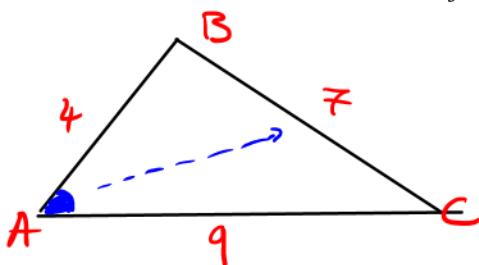
A. $\frac{19}{12}$

B. $\frac{2}{3}$

C. $\frac{9}{4}$

D. $\frac{7}{4}$

E. $\frac{4}{3}$



$$7^2 = 4^2 + 9^2 - 2 \cdot 4 \cdot 9 \cdot \cos A$$

$$49 = 16 + 81 - 72 \cdot \cos A$$

$$-48 = -72 \cos A$$

$$\Rightarrow \cos A = \frac{48}{72} = \frac{2}{3}$$

→ Evaluate the following expression: $\tan^{-1}(1) + \sin^{-1}(1/2)$

$$\frac{\pi}{4} + \frac{\pi}{6} = \frac{\pi \cdot 3}{4 \cdot 3} + \frac{\pi \cdot 2}{6 \cdot 2} = \frac{5\pi}{12}$$

→ If A and B are two acute angles such that $\sin(A) = 3/5$ and $\tan(B) = 12/5$, find $\cos(A+B)$.

$$\cos(A+B) = \cos A \cdot \cos B - \sin A \sin B$$

$$= \frac{4}{5} \cdot \frac{5}{13} - \frac{3}{5} \cdot \frac{12}{13} = \frac{-16}{65}$$

