MATH 1330 Review for Test -4

Time: 50 minutes Friday, April 29 – Monday, May 2

Number of questions: 11 Multiple Choice Questions (total: 100 pts)

What is covered: **5.1, 4.4, Chapters 6 and 7.**

Do not forget to reserve a seat for Test – 4!

Very Important → **Take practice Test – 4!** 10% of your best score will be added to your test grade.

Remember the make-up policy: No make-ups!

The following formulas will be provided. It is your responsibility to locate the formula sheet before you start your test. If you can't find it, ask proctors for help.

Handy Formulas → Only these formulas will be provided to you.

$\sin(s + t) = \sin s \cos t + \cos s \sin t$

$\sin(s - t) = \sin s \cos t - \cos s \sin t$

$\cos(s + t) = \cos s \cos t - \sin s \sin t$

$\cos(s - t) = \cos s \cos t + \sin s \sin t$

$$\tan(s + t) = \frac{\tan s + \tan t}{1 - \tan s \tan t}$$

$$\tan(s - t) = \frac{\tan s - \tan t}{1 + \tan s \tan t}$$

$\sin(2t) = 2\sin t \cos t$ $\qquad$ $\cos(2t) = \cos^2 t - \sin^2 t$

$$\sin\frac{s}{2} = \pm\sqrt{\frac{1-\cos s}{2}} \qquad \cos\frac{s}{2} = \pm\sqrt{\frac{1+\cos s}{2}} \qquad \tan\frac{s}{2} = \frac{\sin s}{1+\cos s}$$

Tuesday, 04/26 : We did question types 8-11 (Skip toward the end)

Thursday, 04/28 : We'll do the rest

1) Simplify: $-\dfrac{\cos(-x)}{\cot(-x)} = -\dfrac{\cos x}{-\cot x} = \dfrac{\cos x}{\frac{\cos x}{\sin x}} = \dfrac{\cos x}{1}\cdot\dfrac{\sin x}{\cos x} = \boxed{\sin x}$

Question #1

2) Simplify: $3\cos^2(x)+\dfrac{2-\cot^2(x)}{1+\cot^2(x)}$ ($1+\cot^2(x) = \csc^2 x$)

$= 3\cos^2 x + \dfrac{\frac{2}{1}\cdot\frac{\sin^2 x}{\sin^2 x} - \frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}}$

$= 3\cos^2 x + \dfrac{(2\sin^2 x - \cos^2 x)}{\sin^2 x}\cdot\dfrac{\sin^2 x}{1} = 3\cos^2 x + 2\sin^2 x - \cos^2 x = 2\cos^2 x + 2\sin^2 x$

$= 2(\underbrace{\cos^2 x + \sin^2 x}_{=1}) = \boxed{2}$

Question #2

3) Simplify: $\tan(x)-\dfrac{\cos(x)}{1-\sin(x)} = \dfrac{\sin x}{\cos x}\cdot\dfrac{(1-\sin x)}{(1-\sin x)} - \dfrac{\cos x}{1-\sin x}\cdot\dfrac{\cos x}{\cos x}$

$= \dfrac{\sin x - \sin^2 x - \cos^2 x}{\cos x(1-\sin x)} = \dfrac{\sin x - (\sin^2 x + \cos^2 x)}{\cos x(1-\sin x)} = \dfrac{\sin x - 1}{\cos x(1-\sin x)}$ (-1)

$= -\dfrac{1}{\cos x} = \boxed{-\sec x}$

Another version: $\dfrac{1}{1-\cos(x)}+\dfrac{1}{1+\cos(x)}$ common denominator

$= \dfrac{(1+\cos x)+(1-\cos x)}{(1-\cos x)(1+\cos x)} = \dfrac{2}{1-\cos^2 x} = \dfrac{2}{\sin^2 x}$

$= \boxed{2\csc^2 x}$

Question #3

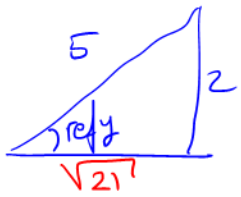
II

4) Given $\sin(x)=\frac{1}{4}$, $90^0 < x < 180^0$, and $\sin(y)=-\frac{2}{5}$, $180^0 < y < 270^0$, find:

III

$\sin x = \frac{1}{4}$

$\cos x = -\frac{\sqrt{15}}{4}$



$\sin(y) = -\frac{2}{5}$

$\cos(y) = -\frac{\sqrt{21}}{5}$

a) $\sin(x-y) = \sin x\cos y - \cos x \sin y = \frac{1}{4}\left(\frac{-\sqrt{21}}{5}\right) - \left(\frac{-\sqrt{15}}{4}\right)\cdot\left(\frac{-2}{5}\right) = \boxed{\frac{-\sqrt{21}-2\sqrt{15}}{20}}$

b) $\sin(x+y) = \sin x\cos y + \cos x \sin y = \frac{1}{4}\left(\frac{-\sqrt{21}}{5}\right) + \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{-2}{5}\right) = \boxed{\frac{-\sqrt{21}+2\sqrt{15}}{20}}$

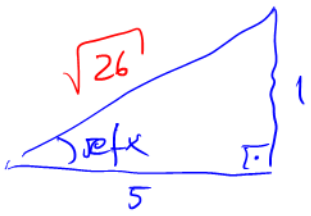
c) $\cos(x-y) = \cos x\cos y + \sin x \sin y = \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{-\sqrt{21}}{5}\right) + \left(\frac{1}{4}\right)\left(\frac{-2}{5}\right) = \boxed{\frac{3\sqrt{35}-2}{20}}$

d) $\cos(x+y) = \cos x\cos y - \sin x \sin y = \left(\frac{-\sqrt{15}}{4}\right)\left(\frac{-\sqrt{21}}{5}\right) - \left(\frac{1}{4}\right)\left(\frac{-2}{5}\right) = \boxed{\frac{3\sqrt{35}+2}{20}}$

Question #4

II

5) Given $\tan(x) = -\frac{1}{5}$, $90^0 < x < 180^0$,



$\sin x = \frac{1}{\sqrt{26}}$

$\cos x = \frac{-5}{\sqrt{26}}$

a) $\sin(2x) = 2\sin x\cos x$

$= 2\cdot\frac{1}{\sqrt{26}}\cdot\frac{-5}{\sqrt{26}}$

$= \frac{-10}{26} = \boxed{\frac{-5}{13}}$

b) $\cos(2x)$

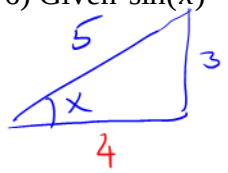
$= \cos^2 x - \sin^2 x$

$= \left(\frac{-5}{\sqrt{26}}\right)^2 - \left(\frac{1}{\sqrt{26}}\right)^2$

$= \frac{25}{26} - \frac{1}{26} = \frac{24}{26} = \boxed{\frac{12}{13}}$

6) Given $\sin(x)=\frac{3}{5}$, where x is an acute angle, find $\sin\left(\frac{x}{2}\right)$.

I



$\cos(x)=\frac{4}{5}$

quadrant I

$=+\sqrt{\frac{1-\cos x}{2}}=\sqrt{\frac{1-\frac{4}{5}}{2}}=\sqrt{\frac{\frac{1}{5}}{2}}$

$=\sqrt{\frac{1}{10}}=\boxed{\frac{\sqrt{10}}{10}}$

7) Find the following using the sum or difference formulas:

a) $\cos(75^0)$ $= \cos(30^\circ+45^\circ) = \ldots$

b) $\sin(105^0)$ $= \sin(60^\circ+45^\circ) = \ldots$

Question #5

8) Find all solutions of the equation $\cos(3x)=\frac{1}{2}$ on the interval $[0,2\pi)$.

period $=\frac{2\pi}{3}$ 3 times $\Rightarrow$ 6 solutions

Unit Circle $\rightarrow$ $\cos(3x)=\frac{1}{2}$

$3x=\frac{\pi}{3}$ or $3x=\frac{5\pi}{3}$

$x=\frac{\pi}{9}$ or $\frac{5\pi}{9}$

1st $\rightarrow$ $x=\frac{\pi}{9}$ or $\frac{5\pi}{9}$

2nd $\rightarrow$ $x=\frac{\pi}{9}+\frac{2\pi}{3}\cdot 1=\frac{7\pi}{9}$ or $\frac{5\pi}{9}+\frac{2\pi}{3}\cdot 1=\frac{11\pi}{9}$

3rd $\rightarrow$ $x=\frac{\pi}{9}+\frac{2\pi}{3}\cdot 2=\frac{13\pi}{9}$ or $\frac{5\pi}{9}+\frac{2\pi}{3}\cdot 2=\frac{17\pi}{9}$

$\boxed{\frac{\pi}{9},\frac{5\pi}{9},\frac{7\pi}{9},\frac{11\pi}{9},\frac{13\pi}{9},\frac{17\pi}{9}}$

Alternate version: Solve $\sin(2x)=\frac{\sqrt{3}}{2}$ on the interval $[0,2\pi)$.

period $=\frac{2\pi}{2}=\pi$ 2 times $\Rightarrow$ 4 solutions

Unit Circle $\rightarrow$ $\sin(2x)=\frac{\sqrt{3}}{2}$

$2x=\frac{\pi}{3}$ or $2x=\frac{2\pi}{3}$

$x=\frac{\pi}{6}$ or $\frac{\pi}{3}$

1st $\rightarrow$ $x=\frac{\pi}{6}$ or $\frac{\pi}{3}$

2nd $\rightarrow$ $x=\frac{\pi}{6}+\pi\cdot 1=\frac{7\pi}{6}$ or $\frac{\pi}{3}+\pi\cdot 1=\frac{4\pi}{3}$

$\boxed{\frac{\pi}{6},\frac{\pi}{3},\frac{7\pi}{6},\frac{4\pi}{3}}$

Question #6

9) Solve the following equation on the interval $[0,2\pi)$:

$$4\sin^2(x)+9\sin(x)+5=0$$

Factor

$$(4\sin x+5)(\sin x+1)=0$$

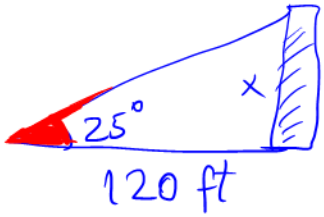
$4\sin x+5=0$ or $\sin x+1=0$

$4\sin x=-5$ $\qquad \sin x=-1 \Rightarrow$ $\boxed{x=\frac{3\pi}{2}}$

$\sin x=\frac{-5}{4}$ ✗

Question #7

12) The angle of elevation from a point that is 120 ft away from a building to the top of the building is 25^0. Find the height of the building.

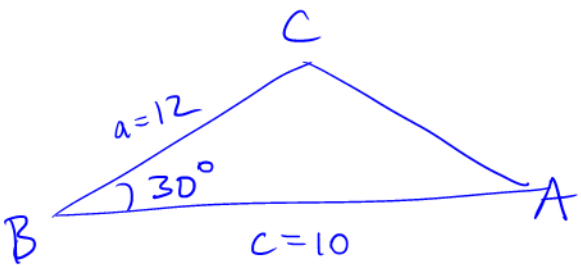


$$\tan(25^\circ)=\frac{x}{120} \Rightarrow \boxed{x=120\tan(25^\circ)}$$

Note: If the angle is 30, 45, 60, make sure you evaluate!

Question #8

10) Find the area of triangle ABC if $\angle B = 30°$, $c = 10$ and $a = 12$.



Check: S.A.S ✓

$$\text{Area}=\frac{1}{2}\times \text{side}\times\text{side}\times\sin(\text{angle})$$

$$=\frac{1}{2}\times 10\times 12\times\sin(30^\circ)$$

$$=\boxed{30}$$

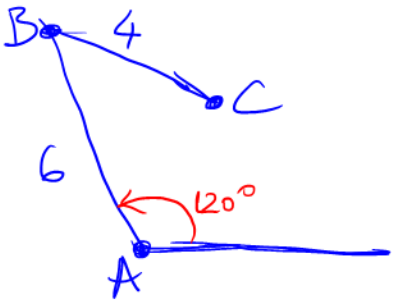
Note: Formula for area is not given!!!

Always Graph

Question #9

15- a) Given a triangle ABC with AB = 6 cm and BC = 4, angle A measures 120^0.
How many choices are there for the measure of angle C?

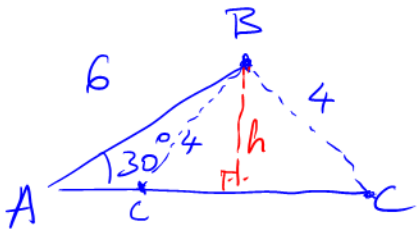
adj opposite



angle A > 90°
opposite < adjacent
} ⇒ Not possible

0 Choices

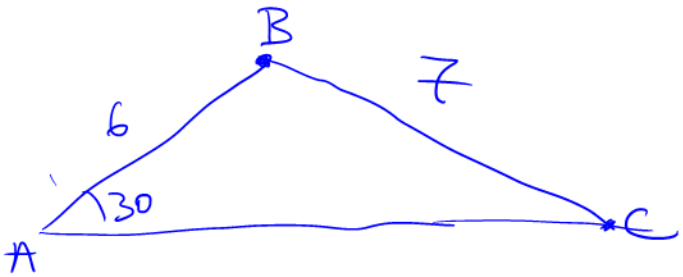
b) Given a triangle ABC with AB = 6 cm and BC = 4, angle A measures 30^0.
How many choices are there for the measure of angle C?



$h = \sin 30^\circ \cdot 6 = 3$

adj > opp > h ⇒ **2 Choices**

c) In triangle ABC with AB = 6, BC = 7, the measure of angle A is 30^o.
How many choices are there for the measure of angle C?

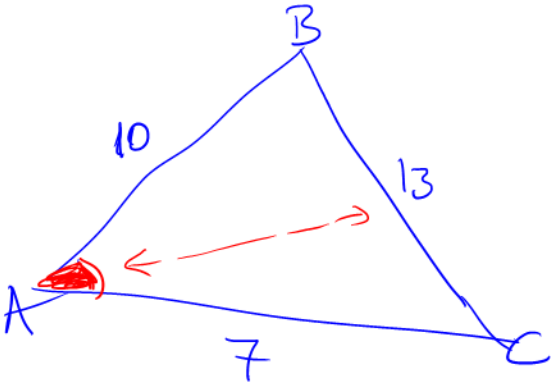


If opposite > adjacent
then 1 triangle.

1 choice

Question #10 – Law of Cosine

11) ABC is a triangle with AB = 10, BC = 13, and AC = 7. Find cos(A).



$13^2 = 10^2 + 7^2 - 2 \cdot 7 \cdot 10 \cdot \cos(A)$

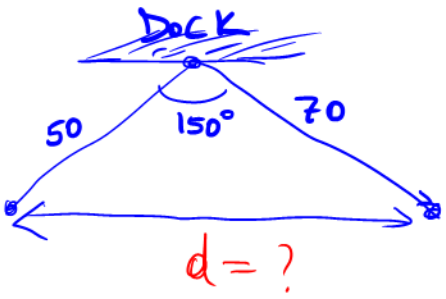
$169 = 100 + 49 - 140 \cdot \cos A$

$20 = -140 \cdot \cos A$

$\cos A = -\frac{20}{140} = \frac{-1}{7}$ ⇒ **$\cos A = -\frac{1}{7}$**

Question #10 – Law of Cosine

13) Two boats leave the dock at the same time and they travel with an angle of 150^0 between them. What is the distance between them after they each travel 50 meters and 70 meters respectively?



$$d^2 = 50^2 + 70^2 - 2 \cdot 50 \cdot 70 \cdot \cos(150^\circ)$$

$$d^2 = 2500 + 4900 - 7000 \cdot \left(-\frac{\sqrt{3}}{2}\right)$$

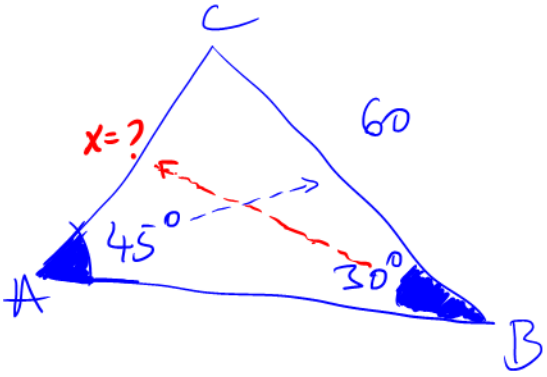
$$d^2 = 7400 + 3500\sqrt{3}$$

$$d^2 = 100\left(74 + 35\sqrt{3}\right) \Rightarrow d = \sqrt{100(74+35\sqrt{3})}$$

$$\boxed{d = 10\sqrt{74+35\sqrt{3}}}$$

Question #11 – Law of Sines

14) Given a triangle ABC, A = 45^0, B = 30^0, BC = 60 cm, find AC.



$$\frac{\sin(30^\circ)}{x} = \frac{\sin(45^\circ)}{60}$$

$$x \cdot \frac{\sin(45^\circ)}{\sin(45^\circ)} = \frac{60 * \sin(30^\circ)}{\sin(45^\circ)}$$

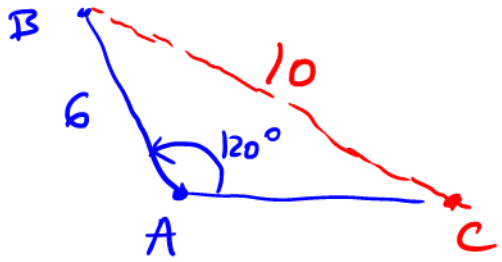
$$\Rightarrow x = \frac{60 \cdot \sin(30^\circ)}{\sin(45^\circ)} = \frac{60 \cdot \frac{1}{2}}{\frac{\sqrt{2}}{2}} = 30 \cdot \frac{2}{\sqrt{2}}$$

$$x = \frac{60}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{60\sqrt{2}}{2} = 30\sqrt{2}$$

$$\Rightarrow \boxed{x = 30\sqrt{2}}$$

• extra examples:

1. given a triangle ABC, $\hat{A} = 120^\circ$, $AB = 6$ (adj), $BC = 10$ (opp = bigger), how many choices are there for angle C?

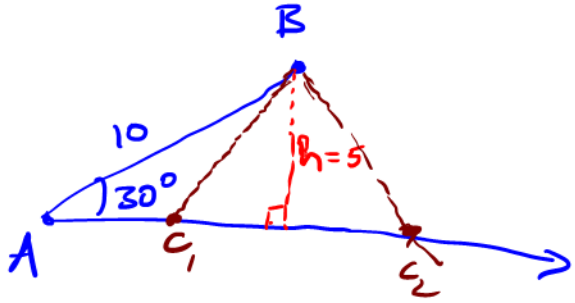


Always 1 triangle

$\Rightarrow$ **1 choice**

2. given a triangle ABC, $\hat{A} = 30^\circ$, $AB = 10$ (adj), $BC = 5\sqrt{3}$ (opp), how many choices are there for angle C?

Do the trick (Pull the height)



$h = 10 \cdot \sin 30^\circ = 10 \cdot \frac{1}{2} = 5$

$h < \text{opp} = BC = 5\sqrt{3}$

2 triangles $\Rightarrow$ **2 choices**