A function is **one-to-one** if no two elements in the domain have the same image.

**The Horizontal Line Test:** A function is one-to-one if any horizontal line intersects the graph of the function in no more than one point.

**Example 1:** Determine if the functions graphed are one-to-one.

**Example 2:** Determine if $f(x) = x^2 + 3$ is one-to-one.

**Example 3:** Determine if $f(x) = x^3 - 2$ is one-to-one.
If a function is one-to-one then there is an associated function called “the inverse”.

The inverse function of a one-to-one function is a function \( f^{-1}(x) \) such that \( (f \circ f^{-1})(x) = (f^{-1} \circ f)(x) = x \).

Note: \( f^{-1}(x) \neq \frac{1}{f(x)} \)

To determine if two functions are inverses of one another, you need to compose the functions in both orders. Your result should be \( x \) in both cases. That is, given two functions \( f \) and \( g \), the functions are inverses of one another if and only if \( f(g(x)) = g(f(x)) = x \).

Example 4: Determine if \( f(x) = 5x - 2 \) and \( g(x) = \frac{x - 2}{5} \) are inverses of one another.

Note: The inverse function reverses what the function did. Therefore, the domain of \( f \) is the range of \( f^{-1} \) and the range of \( f \) is the domain of \( f^{-1} \).

Example 5: If \( f(-1) = 2 \), \( f^{-1}(-1) = 0 \) and \( f(2) = 5 \), find \( f(0) \) and \( f^{-1}(5) \).

Example 6: Find the linear function \( f \) if \( f^{-1}(4) = 0 \) and \( f^{-1}(2) = 1 \).
You need to be able to find the inverse of a function. Follow this procedure to find an inverse function:

1. Rewrite the function as \( y = f(x) \).
2. Interchange \( x \) and \( y \).
3. Solve the equation you wrote in step 2 for \( y \).
4. Rewrite the inverse using inverse notation, \( f^{-1}(x) \).

**Example 7:** You know that \( f(x) = 4x - 7 \) is a one-to-one function. Find its inverse.

**Example 8:** Determine if \( f(x) = (x - 5)^2, \ x \geq 5 \) is a one-to-one function. If it is, find its inverse.

**Example 9:** \( f(x) = \frac{1 + x}{2 - x} \) is a one-to-one function. Find its inverse.

**(Extra) Example 10:** Find the inverse of the function \( f(x) = 5 + \sqrt{4x + 1} \).