Section 7.2 - Area of a Triangle

In this section, we’ll use a familiar formula and a new formula to find the area of a triangle.

You have probably used the formula \( K = \frac{1}{2}bh \) to find the area of a triangle, where \( b \) is the length of the base of the triangle and \( h \) is the height of the triangle. We’ll use this formula in some of the examples here, but we may have to find either the base or the height using trig functions before proceeding.

Here’s another approach to finding area of a triangle. Consider this triangle:

The area of the triangle ABC is: \( K = \frac{1}{2}bc \sin(A) \)

It is helpful to think of this as \( \text{Area} = \frac{1}{2} \text{side} \times \text{side} \times \text{sine of the included angle.} \)

Example 1: Find the area of the triangle.
Example 2: Find the area of the triangle.

Example 3: Find the area of an isosceles triangle with legs measuring 12 inches and base angles measuring 52 degrees each. Round to the nearest hundredth.

Example 4: In triangle ABC; $a = 12, b = 20$ and $\sin(C) = 0.42$. Find the area of the triangle.
Example 5: In triangle KLM, k = 10 and m = 8. Find all possible measures of the angle L if
a) the area of the triangle is 20 unit squares.

b) the area of the triangle is 25 unit squares.

c) the area of the triangle is 80 unit squares.
Formula for Area of a Regular Polygon Given a Side Length

\[ A = \frac{S^2 N}{4 \tan \left( \frac{\pi}{N} \right)} \], where \( S \) = length of a side, \( N \) = number of sides.

Example 6: A regular hexagon is inscribed in a circle of radius 12. Find the area of the hexagon.

For reference, a pentagon has 5 sides, a hexagon has 6 sides, a heptagon has 7 sides, an octagon has 8 sides, a nonagon has 9 sides and a decagon has 10 sides.
**Area of a segment of a circle**

You can also find the area of a segment of a circle. The shaded area of the picture is an example of a segment of a circle.

To find the area of a segment, find the area of the sector with central angle $\theta$ and radius $OA$. Then find the area of $\Delta OAB$. Then subtract the area of the triangle from the area of the sector.

Area of segment $= \text{Area of sector AOB} - \text{Area of } \Delta OAB$

$$= \frac{1}{2} r^2 \theta - \frac{1}{2} r^2 \sin(\theta)$$

Example 7: Find the area of the segment of the circle with radius 8 inches and central angle measuring $\frac{\pi}{4}$.