**Math 1330 - Section 2.1**
**Linear and Quadratic Functions**

**Recall: Equation of a line:** $2x+3y=4 \Rightarrow 3y=-2x+4$, $y=\frac{-2}{3}x+\frac{4}{3}$

General form: $ax+by=c$. (slope is: $m=-\frac{a}{b}$)

Slope-intercept form: $y=mx+b$ ( $m$ is the slope and $b$ is the $y$-intercept)

Point-slope form: $y-y_1=m(x-x_1)$

If two points on the line are given, then the slope is: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{\Delta y}{\Delta x}$ $(x_1,y_1)$ $(x_2,y_2)$

Vertical lines are of the form: $x=c$. slope undefined
Horizontal lines are of the form: $y=c$. slope $=0$

$y=\frac{1}{2}$ $x=1$

Two lines are parallel if they have the same slope. $m_1=m_2$
Two lines are perpendicular if their slopes are negative reciprocals of each other. $m_2=-\frac{1}{m_2}$

---

A **linear function** is a function of the form $f(x)=mx+b$, where $m$ is the **slope** and $b$ is the ***y*-intercept**.

**Example 1:** Write an equation of the linear function for which $f(2)=5$ and $f(-1)=2$.

Need two points $(2,5)$ $(-1,2)$

slope $=\frac{5-2}{2-(-1)}=\frac{3}{3}=1$

$y-5=1(x-2)$

$\boxed{y=x+3}$

**Example 2:** Write an equation of the linear function which contains the point (2, -5) and whose inverse contains the point (-1, 6).

$f^{-1}(-1)=6 \Leftrightarrow f(6)=-1$

Need two points $(2,-5)$ $(6,-1)$

Slope $m=\frac{-5+1}{2-6}=\frac{-4}{-4}=1 \Rightarrow y+5=1\cdot(x-2)$

$\boxed{y=x-7}$

**Example 3:** Write an equation of the linear function which is parallel to the line $2x-5y=10$ and which passes through the point (-1, -4).

gives the slope

$2x-5y=10$

$\frac{5y}{5}=\frac{2x-10}{5}$

$\Rightarrow y=\frac{2}{5}x-\frac{10}{5}$

being parallel, we get slope $=\frac{2}{5}$

$\left.\begin{array}{l} m=\frac{2}{5} \\ \text{point } (-1,-4) \end{array}\right\} \Rightarrow y+4=\frac{2}{5}(x+1)$

$\Rightarrow \boxed{y=\frac{2}{5}x-\frac{18}{5}}$

$f(x)=2(x+1)^2-1 \Leftarrow f(x)=2x^2+4x+1$

A **quadratic function** is a function of the form $f(x)=ax^2+bx+c,\ a\neq 0$.
The graph of a quadratic function is called a parabola. You should be able to identify the following features of the graph of a quadratic function:

- direction the graph opens (upward or downward) $a>0$ $a<0$
- whether the function has a maximum or a minimum
- $y$ intercept ( $f(0)$. ) $y=f(0)$
- coordinates of the vertex
- equation of the axis of symmetry
- maximum or minimum value

$a>0$ min $\quad$ max $a<0$

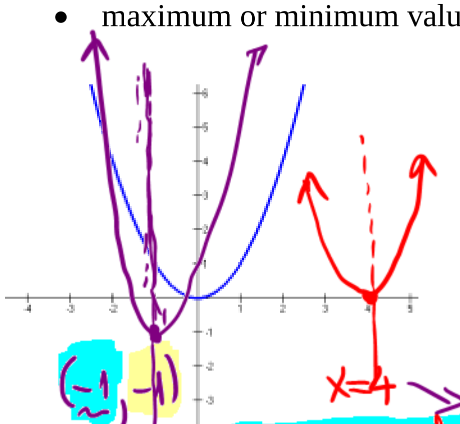
occur at vertex.

ex $f(x)=2(x+1)^2-1$

vertex shifted 1 left, 1 down

$(-1,-1)$

min. value $f(-1)=-1$




$x=-1 \rightarrow$ axis of symmetry = horizontal shiftment

If $a>0$, the parabola will open upward. In this case, the function has a minimum value.
If $a<0$, the parabola will open downward. In this case, the function has a maximum value.

**The standard form of a quadratic function:**

we like this form $f(x)=a(x-h)^2+k$ is in the standard form. $\quad$ vertex $(h,k)$

The vertex is $(h,k)$ and the axis of symmetry is the line $x=h$.

The maximum or minimum value of the function is the number $k$ (the $y$-coordinate of the vertex).

because we can find vertex, axis of symmetry $x=h$ and max/min value of function.
$f(h)=k$

remember $\begin{cases} (x+a)^2 = x^2 + 2a x + a^2 \\ (x-a)^2 = x^2 - 2a x + a^2 \end{cases}$ Need to find a, so we can add $a^2$.

**Example 4:** Given the function $f(x) = -2x^2 - 10x + 6$. $= -2(x^2 + 5x) + 6$

factor $-2$ $\Rightarrow = -2\left(x^2 + 5x + \left(\frac{5}{2}\right)^2\right) + 6 + 2\left(\frac{5}{2}\right)^2$

$a = \frac{5}{2}$

$-2 \cdot \left(\frac{5}{2}\right)^2$

$6 + 2 \cdot \frac{25}{4}$

$6 + \frac{25}{2}$

Find the standard form:

$\Rightarrow f(x) = -2\left(x + \frac{5}{2}\right)^2 + \frac{37}{2}$

Find the vertex:

$\Rightarrow \left(-\frac{5}{2}, \frac{37}{2}\right)$

Find the axis of symmetry:

$x = -\frac{5}{2}$

$f(x) = -2\left(x + \frac{5}{2}\right)^2 + \frac{37}{2}$.

max. value

State the max/min value:

$f\left(-\frac{5}{2}\right) = \frac{37}{2}$

NOTE: If you are not asked to write the function in standard form, you can find the vertex using a different method. The coordinates of the vertex of the graph of the function

$f(x) = ax^2 + bx + c,\ a \neq 0$ is the ordered pair $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

$\Rightarrow a(x-h)^2 + k$ $\qquad h = \frac{-b}{2a}, \quad k = f\left(-\frac{b}{2a}\right)$

If you are given the vertex of the graph of a function and another point, you can find the quadratic function equation.

$f(x) = -2x^2 - 10x + 6 \qquad h = \frac{-(-10)}{2(-2)} = \frac{-5}{2}$

**Example 5:** Write the equation of the quadratic function which passes through the point (0, 7) and whose vertex is (-2, 10).

Solution: $f(x) = a(x-h)^2 + k$ ← standard form

vertex $(-2, 10)$

$\Rightarrow f(x) = a(x+2)^2 + 10$

$x = 0$

$f(0) = a(0+2)^2 + 10 = 7$

$4a = -3 \Rightarrow a = -\frac{3}{4}$

$\Rightarrow f(x) = -\frac{3}{4}(x+2)^2 + 10$

Standard form $\to f(x) = a(x-h)^2 + k$ ← $k=0$

**Example 6:** Find the quadratic function that satisfies:

The axis of symmetry is: $x = -4$

The y-intercept is: (0,80)

There is only one *x*-intercept.

$$\underbrace{f(x) = a(x+4)^2}\ + k \leftarrow k=0$$

$$f(0) = a\cdot(0+4)^2 = 80$$

$$16a = 80 \Rightarrow a = 5$$

$$\Rightarrow \boxed{f(x) = 5(x+4)^2} \text{ answer}$$

Exercise

**Example 7:** A rocket is fired directly upwards with a velocity of 80 ft/sec. The equation for its height, *H*, as a function of time, *t*, is given by the function $H(t) = -16t^2 + 80t$ .

a. Find the time at which the rocket reaches its maximum height.

$h(t) = -16t^2 + 80t \Rightarrow$ it reaches max. height at vertex value

b. Find the maximum height of the rocket.

$$t = \frac{-80}{2(-16)} = \boxed{\frac{5}{2}} \text{ sec}$$

max height

$$= f\left(\frac{5}{2}\right) = -16\cdot\left(\frac{5}{2}\right)^2 + 80\cdot\frac{5}{2}$$

$$= -16\cdot\frac{25}{4} + 200 = \boxed{100 \text{ ft.}}$$