1. \( f(x) = \sec x \), \( 0 \leq x \leq 2\pi \). Find \( x \) where \( f(x) \) is undefined.

\[ \frac{1}{\cos x} \text{, } \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{3\pi}{2} \]

A. \( \frac{\pi}{2}, \frac{3\pi}{2} \)  
B. 0, \( \pi \)  
C. none of them

2. \( g(x) = \csc x \), \( 0 \leq x \leq 2\pi \). Find \( x \) where \( g(x) \) is undefined.

\[ \frac{1}{\sin x} \text{, } \sin x = 0 \Rightarrow x = 0, \pi, 2\pi \]

A. \( \frac{\pi}{2}, \frac{3\pi}{2} \)  
B. 0, \( \pi, 2\pi \)  
C. none of them

Popper 19 — 10/21

1. A
2. B
3. A
\[ f(x) = 2 \sin \left( \frac{3x - \pi}{3} \right) + 1 \]

\[ y = \sin x \]

\\

- **A = 2 → Vertical Stretching**

- **B = 3 → Period = \frac{2\pi}{3} \iff Horizontal Shrinking**

- **C = \pi → Phase Shift = \frac{\pi}{3} \iff Horizontal Shift to Right**

- **D = 1 → Vertical Shift 1 up.**
- It is cosine function, upside down

- Amplitude: \( \frac{4}{2} = 2 \) \( \Rightarrow \boxed{A = -2} \)

- Period: \( \frac{5\pi}{4} - \frac{\pi}{4} = \frac{4\pi}{4} = \pi \)

\[ \frac{2\pi}{B} = \pi \Rightarrow \boxed{B = 2} \]

- It is shifted \( \frac{\pi}{4} \) units right: \( \frac{C}{B} = \frac{\pi}{4} \)

\[ \frac{C}{2} = \frac{\pi}{4} \Rightarrow \boxed{C = \frac{\pi}{2}} \]

- It is shifted 1 unit up: \( \boxed{D = 1} \)

- Hence, \( f(x) = -2 \cos(2x - \frac{\pi}{2}) + 1 \)
Section 5.3a - Graphs of Secant and Cosecant Functions

Using the identity \( \csc(x) = \frac{1}{\sin(x)} \), you can conclude that the graph of \( g \) will have a vertical asymptote whenever \( \sin(x) = 0 \). This means that the graph of \( g \) will have vertical asymptotes at \( x = 0, \pm \pi, \pm 2\pi, \ldots \). The easiest way to draw a graph of \( g(x) = \csc(x) \) is to draw the graph of \( f(x) = \sin(x) \), sketch asymptotes at each of the zeros of \( f(x) = \sin(x) \), then sketch in the cosecant graph.

\[
g(x) = \csc(x) = \frac{1}{\sin(x)}; \quad \text{if } \sin(x) = 0, \text{ then } g(x) \text{ has a vertical asymptote.}
\]

Here’s the graph of \( f(x) = \sin(x) \) on the interval \( \left( -\frac{5\pi}{2}, \frac{5\pi}{2} \right) \).

Next, we’ll include the asymptotes for the cosecant graph at each point where \( \sin(x) = 0 \).

Recall:
\[
\sin\left(\frac{\pi}{6}\right) = \sin\left(\frac{\pi}{2}\right) = \frac{1}{2}.
\]
\[
\sin\left(\frac{5\pi}{6}\right) = \sin\left(\frac{3\pi}{2}\right) = -\frac{1}{2}.
\]
Now we’ll include the graph of the cosecant function.

Period: $2\pi$

Vertical Asymptote: $x = k\pi$, $k$ is an integer

$x$-intercepts: None

$y$-intercept: None

Domain: $x \neq k\pi$, $k$ is an integer

Range: $(-\infty, -1] \cup [1, \infty)$

Typically, you’ll just graph over one period $(0, 2\pi)$.

To graph $y = A\csc(Bx - C) + D$, first graph, THE HELPER GRAPH: $y = A\sin(Bx - C) + D$.

$\begin{equation}
\text{Always} \quad \frac{\csc x}{y = 2\csc(2x-\pi) + 1}
\end{equation}$

Graph $y = 2\sin(2x-\pi) + 1$

then put "the parabola" shapes on top of sine function.

Do not forget $V.A$. 

2
You’ll also be able to take advantage of what you know about the graph of \( f(x) = \cos(x) \) to help you graph \( g(x) = \sec(x) \). Using the identity \( \sec(x) = \frac{1}{\cos(x)} \), you can conclude that the graph of \( g \) will have a vertical asymptote whenever \( \cos(x) = 0 \).

This means that the graph of \( g \) will have vertical asymptotes at \( x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \ldots \). The easiest way to draw a graph of \( g(x) = \sec(x) \) is to draw the graph of \( f(x) = \cos(x) \), sketch asymptotes at each of the zeros of \( f(x) = \cos(x) \), then sketch in the secant graph.

\[
g(x) = \sec(x) = \frac{1}{\cos(x)}; \quad \text{if} \quad \cos(x) = 0, \quad \text{then} \quad g(x) \quad \text{has a vertical asymptote.}
\]

Here’s the graph of \( f(x) = \cos(x) \) on the interval \( (-\frac{5\pi}{2}, \frac{5\pi}{2}) \).

\[
\cos x = 0 \quad \Rightarrow \quad x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \ldots
\]

There are \( V.A. \) for \( f(x) = \cos(x) \).  
\[
\cos x = 0 \quad \Rightarrow \quad x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \ldots
\]

Next, we’ll include the asymptotes for the secant graph.

\[
\cos 0 = 1
\]
\[
\cos \frac{\pi}{2} = \frac{1}{2}
\]
\[
\cos \frac{\pi}{3} = -\frac{1}{2}
\]
\[
\cos \frac{\pi}{6} = -\frac{1}{2}
\]
\[
\cos \frac{\pi}{3} = 1
\]
\[
\cos \pi = -1
\]
\[
\cos 2\pi = 1
\]
Now we’ll include the graph of the secant function.

Period: $2\pi$

Vertical Asymptote: $x = \frac{k\pi}{2}$ is an odd integer

$x$-intercepts: None

$y$-intercept: (0, 1)

Domain: $x \neq \frac{k\pi}{2}$, $k$ is an odd integer

Range: $(-\infty, -1] \cup [1, \infty)$

Typically, you’ll just graph over one period $(0, 2\pi)$.

To graph $y = A\sec(Bx - C) + D$, first graph, THE HELPER GRAPH: $y = A\cos(Bx - C) + D$. 

$y = A\sec(Bx - C) + D$ Always

Graph $y = A\cos(Bx - C) + D$ and then put "parabola" shapes on top of cosine!

Do not forget $\text{V.A.}$.
Example 1: Sketch $f(x) = 4 \sec \left( \frac{x}{2} \right)$

**Helper graph:**

$y = 4 \cos \left( \frac{x}{2} \right)$

Vertical Stretch

$B = \frac{1}{2}$

Horizontal Stretch

Period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

Note $f(x) = 4 \sec \left( \frac{x}{2} \right)$ has V. A. at

$x = \pi, \, 3\pi, \, 5\pi, \ldots, \, -\pi, \, -3\pi, \, -5\pi$

because $\cos \left( \frac{x}{2} \right) = 0$ at those points.
Example 2: Sketch $f(x) = -2 \csc \left( \frac{\pi x}{2} - \frac{\pi}{2} \right)$

**Helper graph:**

- $A = -2 \Rightarrow$ vertical stretching and reflection
- $B = \frac{\pi}{2} \Rightarrow$ horizontal shrinking

- period $= \frac{2\pi}{\frac{\pi}{2}} = 4$

- $C = \frac{\pi}{2} \Rightarrow$ horizontal shift

  by $\frac{C}{B} = \frac{\frac{\pi}{2}}{\frac{\pi}{2}} = 1$ unit right.

$\Rightarrow$ Then put $y = -2 \csc \left( \frac{\pi x}{2} - \frac{\pi}{2} \right)$.

Note $f(x) = -2 \csc \left( \frac{\pi x}{2} - \frac{\pi}{2} \right)$ has V.A.

At $x = 1, 3, 5, 7, \ldots$

- $-1, -3, -5, -7, \ldots$

because $\sin \left( \frac{\pi x}{2} - \frac{\pi}{2} \right) = 0$ at these values.
Example 3: Give an equation of the form \( y = A \csc(Bx - C) + D \) and \( y = A \sec(Bx - C) + D \) that could describe the following graph.

Exercise: Give an equation of the form \( y = A \csc(Bx - C) + D \) and \( y = A \sec(Bx - C) + D \) that could describe the following graph.
Exercise: Find the vertical asymptotes of:

a) \( f(x) = 2 \sec \left( x - \frac{\pi}{2} \right) \)

\[ \cos \left( \frac{x}{2} - \frac{\pi}{2} \right) = 0 \implies \frac{x}{2} - \frac{\pi}{2} = k\pi \text{, } k \text{ odd} \]

b) \( f(x) = 2 \csc \left( x - \frac{\pi}{4} \right) \)

\[ \sin \left( x - \frac{\pi}{4} \right) = 0 \]

\[ \frac{x}{4} = 0 \implies x = \frac{\pi}{4} \]
\[ \frac{x}{4} = \pi \implies x = \frac{5\pi}{4} \]
\[ \frac{x}{4} = 2\pi \implies x = \frac{9\pi}{4} \]