Popper # 27

1. $\sin 100^\circ \cos 10^\circ - \cos 100^\circ \sin 10^\circ = \sin(100^\circ - 10^\circ) = \sin(90^\circ) = 1$

A. 1 B. 0 C. $\sin(110^\circ)$ D. none

2. $(1-\cos^2\theta)\cot^2\theta = \frac{\cancel{\sin^2\theta}}{1} \cdot \frac{\cos^2\theta}{\cancel{\sin^2\theta}} = \cos^2\theta$

A. $\tan^2\theta$ B. $\cos^2\theta$ C. $\sec^2\theta$ D. none

3. $\dfrac{\sin(-\theta)}{\cos(-\theta)} = \tan(-\theta) = -\tan\theta$

A. $\tan\theta$ B. $-\tan\theta$ C. $\cot\theta$ D. none

4. $\dfrac{1-\cos^2\theta}{\cos^2\theta} = \dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

A. 1 B. 0 C. $\tan^2\theta$ D. $\cot^2\theta$

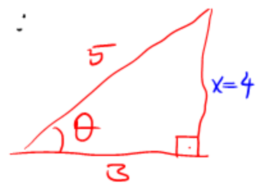
Popper # 28

$1 + \tan^2\theta = \sec^2\theta$

① $(\sec\theta - 1)(\sec\theta + 1) = \sec^2\theta - 1 = \tan^2\theta$

A. $\sec^2\theta$ B. $\tan^2\theta$ C. $\sin^2\theta$ D. $\cos^2\theta$ E. $\cot^2\theta$

→ Given $\cos\theta = \frac{3}{5}$; $0 < \theta < \frac{\pi}{2}$, find :



$\cos\theta = \frac{3}{5}$

$\sin\theta = \frac{4}{5}$

② $\sin(2\theta) = 2\sin\theta \cdot \cos\theta = 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \frac{24}{25}$

A. $\frac{6}{25}$ B. $\frac{8}{25}$ C. $\frac{24}{25}$ D. $\frac{4}{25}$ E. none

③ $\cos(2\theta) = 2\cos^2\theta - 1 = 2\left(\frac{3}{5}\right)^2 - 1 = 2 \cdot \frac{9}{25} - \frac{25}{25} = \frac{-7}{25}$

A. $\frac{7}{25}$ B. $-\frac{7}{25}$ C. $\frac{18}{25}$ D. $\frac{-18}{25}$ E. none

④ $\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{24/25}{-7/25} = -24/7$

A. $\frac{24}{7}$ B. $\frac{-24}{7}$ C. $\frac{4}{3}$ D. $\frac{16}{9}$ E. none.

Popper #29

Make sure you recognize formulas:

$$\sin^2\Big(\underbrace{\tfrac{3\pi}{7}}_{x}\Big) + \cos^2\Big(\underbrace{\tfrac{3\pi}{7}}_{x}\Big) = 1 \Leftarrow \text{No Hesitation}$$

match!

(1) $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right) = \cos\left(2\times\frac{\pi}{8}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

A. 0 B. 1 C. $\frac{1}{2}$ D. $\frac{\sqrt{2}}{2}$ E. none

Given $\sin(x) = \frac{3}{5}$, $\cos(y) = \frac{12}{13}$, $0 < x < 90^\circ$, $270^\circ < y < 360^\circ$.

$\cos x = \frac{4}{5}$ $\sin(y) = -\frac{5}{13}$

(2) $\sin(x+y) = \sin x \cos y + \cos x \sin y = \frac{3}{5}\cdot\frac{12}{13} + \frac{4}{5}\cdot\left(-\frac{5}{13}\right) = \frac{16}{65}$

A. $\frac{36}{65}$ B. $\frac{16}{65}$ C. $-\frac{16}{65}$ D. $\frac{-20}{65}$

(3) $\tan\left(\frac{y}{2}\right) = \frac{1-\cos y}{\sin y} = \frac{1-\frac{12}{13}}{\frac{-5}{13}} = \frac{\frac{1}{\cancel{13}}}{\frac{-5}{\cancel{13}}} = -\frac{1}{5}$

A. $-\frac{12}{5}$ B. $-\frac{1}{5}$ C. $\frac{5}{13}$ D. none

(4) $\cos(2x) = \cos^2 x - \sin^2 x = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{7}{25}$

A. $\frac{7}{25}$ B. $\frac{18}{25}$ C. $\frac{-7}{25}$ D. none

Wednesday, 11/11

Linear Equation: $2x + 3 = 9 \leftarrow$ Solve!
$\quad -3 \quad -3$

"Goal is to leave x alone"

$\frac{2x}{2} = \frac{6}{2} \implies \boxed{x=3}$

Section 6.3 - Solving Trigonometric Equations

Next, we'll use all of the tools we've covered in our study of trigonometry to solve some equations. An equation that involves a trigonometric function is called a trigonometric equation. Since trigonometric functions are periodic, there may be infinitely solutions to some trigonometric equations.

- Trigonometric Equation $\leftarrow$ Unit Circle

Let's say we want to solve the equation: $\sin(x) = \frac{1}{2}$

Ask yourself: Which angle(s) have sine $= \frac{1}{2}$?

The first angles that come to mind are: $\boxed{x = \frac{\pi}{6} \text{ and } x = \frac{5\pi}{6}}$. $\leftarrow$ Answer.

in one period

$x = 30°$ or $150°$

(convert in radians)

Remember that the period of the sine function is 2π; sine function repeats itself after each rotation.

The solutions of unit circle repeat themselves in every periodic rotation.

Therefore, the solutions of the equation are: $x = \underbrace{\frac{\pi}{6}}_{\text{unit circle}} + \underbrace{2k\pi}_{2\pi \cdot k}$, $x = \underbrace{\frac{5\pi}{6}}_{\text{unit circle}} + \underbrace{2k\pi}_{2\pi \cdot k}$, where k is any integer.

Recall: For sine and cosine functions, the period is 2π. For tangent and cotangent functions, the period is π.

Do NOT FORGET:

General (ALL) Solutions $=$ $\underbrace{\text{Special Solutions}}_{\text{unit circle}}$ $+$ Period $\cdot$ k

one rotation

Example 1: a) Solve the equation in the interval $[0,2\pi)$: $2\cos x = -1$

$2\cos x = -1$

Q.II Q III

$\cos x = -\frac{1}{2} \Longrightarrow x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$

period $= 2\pi$

b) Find all solutions to the equation: $2\cos x = -1$

From part (a), $\cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$

all $\boxed{x = \frac{2\pi}{3} + 2\pi \cdot k \quad \text{or} \quad x = \frac{4\pi}{3} + 2\pi \cdot k,}$ k integer

one period

Example 2: a) Solve the equation in the interval $[0,\pi)$: $\tan x = -1$

$\tan x = -1$

From 0 to π,

only $\boxed{x = \frac{3\pi}{4}}$ in Quadrant II, gives $\tan\left(\frac{3\pi}{4}\right) = -1$

b) Find all solutions to the equation: $\tan x = -1$

period $= \pi$

$\Longrightarrow \boxed{x = \frac{3\pi}{4} + \pi \cdot k,}$ k integer one period

Example 3: Solve the equation in the interval $[0,\pi)$: $2\sin(2x)=1$

$2\sin(2x)=1$

$\sin(2x)=\frac{1}{2} \Rightarrow \frac{2x}{2}=\frac{\pi}{6}\Big/2$ or $\frac{2x}{2}=\frac{5\pi}{6}\Big/2$

$\text{period}=\frac{2\pi}{2}=\pi$

$\Rightarrow \boxed{x=\frac{\pi}{12} \quad \text{or} \quad x=\frac{5\pi}{12}}$

Friday 11/13

Example 4: Solve the equation in the interval $[0,2\pi)$: $\csc^2 x=4$ $\leftarrow$ In one period

$\csc^2 x=4 \Longleftrightarrow$ $\csc x=+2$ or $\csc x=-2$

$\csc x=\pm 2$

$\frac{1}{\sin x}=2$ | $\frac{1}{\sin x}=-2$

$\sin x=\frac{1}{2}$ | $\sin x=\frac{-1}{2}$

$\boxed{x=\frac{\pi}{6},\ \frac{5\pi}{6} \quad \text{or} \quad x=\frac{7\pi}{6},\ \frac{11\pi}{6}}$

Example 5: Find all solutions to the equation: $\cos(2x)=0$

need to find solutions in one period first and add repetitions of periods.

$\rightarrow \cos(2x)=0 \Rightarrow \frac{2x}{2}=\frac{\pi}{2}\Big/2$ or $\frac{2x}{2}=\frac{3\pi}{2}\Big/2$

$\Downarrow$

$x=\frac{\pi}{4}$ or $x=\frac{3\pi}{4}$

$\text{period}=\frac{2\pi}{2}=\pi$

All solutions:

$$\boxed{\begin{aligned} x&=\frac{\pi}{4}+\pi\cdot k \quad , \quad k \text{ integer.} \\ x&=\frac{3\pi}{4}+\pi\cdot k \end{aligned}}$$

one period

Example 6: Solve the equation in the interval $[0,2\pi)$: $2\sin^2 x - 5\sin x - 3 = 0$

$$2\sin^2 x - 5\sin x - 3 = 0$$

$$\underbrace{(\sin x - 3)}_{0}\underbrace{(2\sin x + 1)}_{0} = 0$$

$\sin x - 3 = 0$ or $2\sin x + 1 = 0$

$\sin x = 3$ Can't happen

$2\sin x = -1$

$\sin x = -\frac{1}{2}$ $\Longrightarrow$ one period $\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

Example 7: Solve the equation in the interval $[0,2\pi)$: $\cos^2 x - 3\sin x - 3 = 0$

$\cos^2 x - 3\sin x - 3 = 0$ (Transform in an equation with just one trig. function)

$$1 - \sin^2 x - 3\sin x - 3 = 0$$

$$-\sin^2 x - 3\sin x - 2 = 0$$

$$\sin^2 x + 3\sin x + 2 = 0$$

$$\underbrace{(\sin x + 1)}_{0} \text{ or } \underbrace{(\sin x + 2)}_{0} = 0$$

$\sin x + 1 = 0$ or $\sin x + 2 = 0$

$\sin x = -1$ $\sin x = -2$ Can't happen

$\boxed{x = \frac{3\pi}{2}}$

Example 8: Solve the equation in the interval $[0,2\pi)$: $\cos(2x) = 5\sin^2 x - \cos^2 x$

$\underbrace{\cos(2x)}_{2\cos^2 x - 1} = 5\underbrace{\sin^2 x}_{1-\cos^2 x} - \cos^2 x$ (Always, keep just one trig. expression.) (We'll do everything with cos x)

$$\Longrightarrow 2\cos^2 x - 1 = 5(1 - \cos^2 x) - \cos^2 x$$

$$2\cos^2 x - 1 = 5 - 5\cos^2 x - \cos^2 x = 5 - 6\cos^2 x$$

$+6\cos^2 x$ $+1$ $+1$ $+6\cos^2 x$

$$8\cos^2 x = 6 \Longleftrightarrow \cos^2 x = \frac{6}{8} \Longleftrightarrow \cos^2 x = \frac{3}{4}$$

$$\cos x = \pm\sqrt{\frac{3}{4}} \longrightarrow \cos x = +\frac{\sqrt{3}}{2} \text{ or } \cos x = -\frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{11\pi}{6} \text{ or } x = \frac{5\pi}{6}, \frac{7\pi}{6}}$$

one period

Example 9: Find all solutions to the equation: $\sin^2 x \cos x = \cos x$

(bring everything in one side)

$$\sin^2 x \cdot \cos x = \cos x$$

$$\sin^2 x \cos x - \cos x = 0$$

$$\underbrace{\cos x}_{0} \underbrace{(\sin^2 x - 1)}_{0} = 0$$

$\cos x = 0$ or $\sin^2 x = 1 \Rightarrow \sin x = \pm 1$

$$\underbrace{x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or} \quad x = \frac{\pi}{2}, \frac{3\pi}{2}}_{\text{in one period}}$$

All Solutions:

$$\boxed{\begin{aligned} x &= \frac{\pi}{2} + 2\pi \cdot k, \\ x &= \frac{3\pi}{2} + 2\pi \cdot k, \end{aligned}}$$

k integer

exercise

Example 10: Find all solutions: $\sec^2 x + 2\tan x = 0$ (use $\sec^2 x = 1 + \tan^2 x$)

$$\underbrace{\sec^2 x} + 2\tan x = 0$$

$$\tan^2 x + 1 + 2\tan x = 0 \iff (\tan x + 1)(\tan x + 1) = 0$$

i.e. $\tan x + 1 = 0$

$\tan x = -1 \Rightarrow x = -\frac{\pi}{4}$ in one period $= \pi$

$$\Rightarrow \boxed{x = -\frac{\pi}{4} + \pi \cdot k,} \quad k \text{ integer.}$$

Do not forget $x = -\frac{\pi}{4}$ is the same as $x = \frac{7\pi}{4}$, so

the answer $\boxed{x = \frac{7\pi}{4} + \pi \cdot k}$, they are the same exercise

there are two periodical intervals

Example 11: Solve the equation in the **interval** $[0,2)$: $\cot(\pi x) = -1$

$\cot(\pi x) = -1$

$\text{period} = \frac{\pi}{\pi} = 1$

$\frac{\cancel{\pi} x}{\cancel{\pi}} = \frac{3\cancel{\pi}}{4\cancel{\pi}}$ ⟵ over one period

$x = \frac{3}{4} \Rightarrow x = \frac{3}{4} + 1 = \frac{7}{4}$

next period.

$\Rightarrow \boxed{x = \frac{3}{4}, \frac{7}{4}}$

exercise

Example 12: Find all solutions of the equation in the interval $[0,4\pi)$: $2\sin\left(\frac{x}{2}\right) = 1$

one period

$2\sin\left(\frac{x}{2}\right) = 1$

$\Rightarrow \sin\left(\frac{x}{2}\right) = \frac{1}{2}$ ⟶ Think, in one full rotation,

$\text{period} = \frac{2\pi}{1/2} = 4\pi$

$2 \times \frac{x}{2} = \frac{\pi}{6} \times 2$ or $2 \times \frac{x}{2} = \frac{5\pi}{6} \times 2$

$x = \frac{\cancel{2}\pi}{\cancel{6}} = \frac{\pi}{3}$, $x = \frac{\cancel{10}^{5}\pi}{\cancel{6}_{3}} = \frac{5\pi}{3}$

$\Rightarrow \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

Always, use identities (if possible) to simplify!

exercise

Example 13: Find all solutions of the equation in the interval $\underbrace{[0,2\pi)}_{\text{period }=2\pi}$: $\sec(x+2\pi)=2$

Hence, $\sec(x+\underbrace{2\pi}_{\text{period}}) = \sec(x)$

Thus, $\sec(x) = 2$

$\frac{1}{\cos x} = 2$

$\cos x = \frac{1}{2} \Longrightarrow \boxed{x = \frac{\pi}{3}, \frac{5\pi}{3}}$

Example 14: Find all solutions of the equation in the interval $\underbrace{[0,\pi)}_{\text{one period}}$: $2\sin\left(2x-\frac{3\pi}{2}\right)=\sqrt{2}$

$\Rightarrow 2\sin\left(2x-\frac{3\pi}{2}\right) = \sqrt{2}$

$\sin\left(2x-\frac{3\pi}{2}\right) = \frac{\sqrt{2}}{2}$ (there is no identity to apply, hence go to unit circle.)

$\frac{\pi}{4}$ or $\frac{3\pi}{4}$

In one full rotation, this expression

$2x-\frac{3\pi}{2} = \frac{\pi}{4}$ or $2x-\frac{3\pi}{2} = \frac{3\pi}{4}$

$2x = \frac{\pi}{4} + \frac{3\pi}{2}\cdot\frac{2}{2}$ $\qquad$ $2x = \frac{3\pi}{4} + \frac{3\pi}{2}\cdot\frac{2}{2}$

$\frac{2x}{2} = \frac{\frac{7\pi}{4}}{2}$ $\qquad$ $\frac{2x}{2} = \frac{\frac{9\pi}{4}}{2}$

$\rightarrow \boxed{x = \frac{7\pi}{8}}$ ← Answer. $\qquad$ $x = \frac{9\pi}{8}$ not in $[0,\pi)$