

What is the best way to learn mathematical proofs?

A math-major student writes:

Discrete math is about proofs. In lecture, the professor would write a proposition on the board — e.g., *if n is a perfect square then it's also odd*— then walk through a proof. Proposition after proposition, proof after proof. As the class advanced, we learned increasingly advanced techniques for building these proofs. I soon developed a singular obsession: **I wanted to be able to recreate, with pencil and paper, and no helper notes, every single proof presented in class.** No exceptions. Lack of understanding of even one proof wouldn't be tolerated.

My Obsession in Practice

Here's how I learned every proof.

1. **I bought a package of white printer paper.**
2. As the term progressed, **I copied each proposition presented in class onto its own sheet of paper.** I would write the problem as the top of the sheet and recreate the proof, from my notes, below.
3. **I tried to do this every week** — copying the most recent material onto its own sheets — though I often got behind.
4. While doing this work I would sometimes — okay, *many* times — **realize I didn't quite understand the proof I had copied in my notes.** In these cases, I would break out the textbook, or do some web searching for the problem, to see if I could make sense of what I was writing down. This usually worked. In the worst case scenario, I would ask the professor or the TA for help. Not understanding the proof was not an option. I wasn't practicing transcription; I knew I had to learn these.
5. **About two weeks before each exam I started scheduling sessions to aggressively review my "proof guides."** I always worked on the second floor of the *Dana Biomedical Library* on the outskirts of campus. (Think: dark, concrete-floored stacks, with desks tucked away at the end of long rows, each illuminated by a single, bright incandescent bulb...study heaven.) I did standard Quiz and Recall: splitting the proofs between those I could replicate from scratch and those that gave me trouble, and then, in the next round, focusing only on those that gave me trouble, and so on, until every sheet had been conquered.

By the day of the exam, you could give me *any* problem from the course and I could rattle off the proof, without mistake and without hesitation.

Another student writes:

Writing proofs is one of the areas that need a lot of work, and I had a lot of difficulties with it when I first started. I think a good way to get better is to just get exposed to a lot of proofs and try to understand how mathematicians are thinking when they are writing them. It's important to write a lot of proofs yourself. If you get stuck, look it up, then try to prove it again yourself without peeking at the solution. A good way to test your understanding of a proof that I found to be helpful, is to try to write it again after a few days so that you'd have forgotten some details of the model answer. If you can write it perfectly, then you probably understood it. There are also a lot of books that can be useful during such a transition.

Some math professor advices:

Learning particular proofs that someone has told you is not particularly useful. It is akin to learning procedures for doing things without knowing the rationale for the procedure.

The **best** proofs are those you generate yourself from some underlying principles that you understand. That may be recreating a proof in a book or one that you have been told, but the important point is that you *understand* how the proof is put together. That way you can apply the principles more broadly, prove things you have never been told, and even, potentially, prove something that has never been proven before. This is very different from merely regurgitating some proof, although such regurgitation *may* be the best way to pass some exam.

It is the difference between a cook following a recipe and a chef creating a meal. Cooks are very useful, but chefs add the zest to life :-)