# Homework #10

Last Name: _____

Name : _____

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**TRANSITION TO ADVANCED MATHEMATICS**
**HOMEWORK #10 – DUE THURSDAY, 04/26**

*Problem* 1. Let $A$, $B$, $C$ be sets and let $f : A \to B$ and $g : B \to C$ be two functions. Prove or find a counterexample to each of the following statements:

(a) If $g \circ f : A \to C$ is one-to-one, then $f$ is one-to-one.
(b) If $g \circ f : A \to C$ is one-to-one, then $g$ is one-to-one.
(c) If $g \circ f : A \to C$ is onto, then $f$ is onto.
(d) If $g \circ f : A \to C$ is onto, then $g$ is onto.

*Problem* 2. Exercise 4.5: Problem 4(a,d).

*Problem* 3. Exercise 4.5: Problem 10(a,e).

*Problem* 4. Exercise 4.5: Problem 16.

*Problem* 5. The Pigeonhole Principle is surprisingly powerful. We use it to show that a finite set is never equivalent to any of its proper subsets.

(i) State the Pigeonhole Principle.
(ii) The Italian village of Solomeo, near Perugia, has a population of 400. Prove that there are at least two village residents with the same birthday. (Prove by using the Pigeonhole Principle.)

*Problem* 6. The set $\mathbb{N}$ of natural numbers is infinite.

*Problem* 7. A set $S$ is called denumerable if $S \approx \mathbb{N}$.

(i) The set $\mathbb{Z}$ is denumerable.
(ii) The set $\mathbb{N} \times \mathbb{N}$ is denumerable.

*Problem* 8. The open interval $(0, 1)$ is uncountable.

**Important Note:** Problems 5,6,7,8 are from Section 5.1/5.2. On Tuesday, I will continue with those two sections and you will be able to finish these problems for Thursday. I will give helpful hints on how to approach these problems.