

## Homework #3

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**TRANSITION TO ADVANCED MATHEMATICS**  
**HOMEWORK#3 – DUE THURSDAY, 02/15**

*Problem 1.* Let  $x$  and  $y$  be integers. Prove that

- (a) if  $x$  and  $y$  are even, then  $x^2y$  is divisible by 8.
- (b) if  $x$  is odd, then  $x^2 + 15$  is divisible by 4.
- (c)  $x(x + 3)$  is even.

*Problem 2.* Prove that if  $a$  is a positive integer, then  $a^2 + 3a + 7$  is odd.

*Problem 3.* Prove that if  $x$  is a positive real number, then  $4x + \frac{1}{x} \geq 4$ .

*Problem 4.* Let  $x$  and  $y$  be integers. Prove the following by contraposition.

- (a) If  $x^2$  is not divisible by 4, then  $x$  is odd.
- (b) If  $xy$  is odd, then  $x$  and  $y$  are odd.

*Problem 5.* A circle in the plane has center  $(4, 5)$ .

- (a) Prove that  $(1, 2)$  and  $(-1, 0)$  are not both on the circle.
- (b) Prove that if  $(1, 1)$  is not inside the circle, then  $(2, -2)$  is not inside the circle .

*Problem 6.* Suppose  $a, b, c$ , and  $d$  are positive integers. Prove that

- (a)  $a$  is odd if and only if  $a + 1$  is even.
- (b)  $a + c = b$  and  $2b - a = d$  if and only if  $a = b - c$  and  $b + c = d$ .

*Problem 7.* Provide a proof or a counterexample for each statement below:

- (a) For all positive integers  $x$ ,  $x^2 + x + 41$  is a prime.
- (b) For integers  $a, b, c$ , if  $a$  divides  $bc$ , then  $a$  divides  $b$  or  $a$  divides  $c$ .
- (c) For all positive real numbers  $x$ ,  $x^2 - x \geq 0$ .
- (d) For all positive real numbers  $x$ ,  $2^x \geq x + 1$ .
- (e) For any two irrational numbers  $x$  and  $y$ ,  $xy$  is also irrational.

*Problem 8.* Prove that for all integers  $a, b$  and  $c$ , if  $a$  divides  $b - 1$  and  $a$  divides  $c - 1$ , then  $a$  divides  $bc - 1$ .

*Problem 9.* Let  $x$  and  $y$  be real numbers. If  $x + y$  is irrational, then either  $x$  or  $y$  is irrational.

*Problem 10.* Prove that if  $n$  is an integer and  $3n - 1$  is odd, then  $4n + 8$  is divisible by 8.