Homework #5

Last Name: ______________________________

Name : ______________________________

PSID: ______________________________

TRANSITION TO ADVANCED MATHEMATICS
HOMEWORK#5 – DUE TUESDAY, 02/27

Problem 1. Let A and B be nonempty subsets. Prove that $A \times B = B \times A$ if and only if $A = B$.

Problem 2. Exercise 2.3.1(b, f, j, l, m, n).

Problem 3. Exercise 2.4: Problem 6(c, e, j).

Problem 4. Exercise 2.4: Problem 7(c, m).

Problem 5. Exercise 2.4: Problem 8(b, d).

Below you will find the basic steps for mathematical induction methods.
It is always important to understand the statement $(\forall n \in \mathbb{N})\, P(n)$.
Follow these steps to do exercise 2.4:

Principal of Mathematical Induction – PMI:

Proof. Let $S = \{n \in \mathbb{N} : P(n) \text{ is true}\}$.

(i) Show $1 \in S$.
(ii) For all $n \in \mathbb{N}$, if $n \in S$, then $n + 1 \in S$.
In other words, assume $n \in S$ equivalently $P(n)$ is true, and show that $n + 1 \in S$ equivalently $P(n + 1)$ is true.
(iii) Therefore, by PMI, $S = \mathbb{N}$. Thus, $(\forall n \in \mathbb{N})\, P(n)$ is true.

□

Generalized Principal of Mathematical Induction – GPMI:

There are some statements which are not true for all natural numbers, but are true for numbers in some inductive subset of $\mathbb{N}$. To prove such statements, we need a slightly generalized form of the PMI, where the basis step starts at some number other than 1. The rest is the same as PMI.

Proof. Let $S = \{n \in \mathbb{N} : P(n) \text{ is true for } n \geq k\}$.

(i) Show $k \in S$.
(ii) For all $n \geq k$, if $n \in S$, then $n + 1 \in S$.
In other words, assume $n \geq k$ and $n \in S$ equivalently $P(n)$ is true, and show that $n + 1 \in S$ equivalently $P(n + 1)$ is true.
(iii) Therefore, by GPMI, S contains all natural numbers greater than or equal to k. Thus, $(\forall n \in \mathbb{N})\, P(n)$ is true for all $n \geq k$.

□