# Math 3331 Differential Equations

## 4.2 Second-Order Equations and Systems

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# 4.2 Second-Order Equations and Systems

- Second-Order Equations
- Planar Systems
  - *yv*-Phase Plane Plot
  - Phase Plane Portrait

# Second-Order Equations and Planar Systems

## Second-order DE

$$y'' + ay' + by = 0 \qquad (1)$$

$$p(\lambda) = \lambda^2 + a\lambda + b = 0$$

## planar system

$$x_1 = y, \quad x_2 = v = y'$$

$$\mathbf{x}' = A\mathbf{x}, \quad A = \begin{pmatrix} 0 & 1 \\ -b & -a \end{pmatrix} \qquad (2)$$

$$\det(A - \lambda I) = p(\lambda)$$

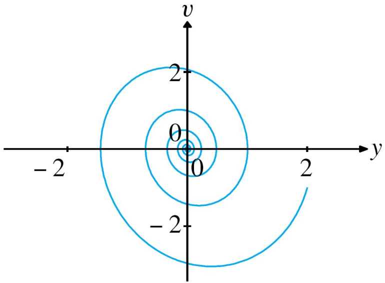
(Chapter 9)

## *yv*-Phase Plane Plot

A damped unforced spring:

$$my'' + \mu y' + ky = 0$$

with $m = 1$, $\mu = 0.4$, and $k = 3$.




(Spiral sink)

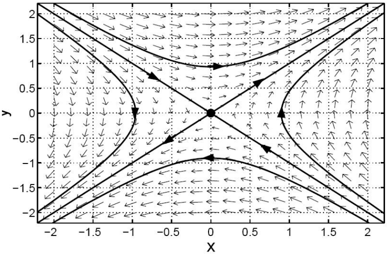
# Phase Plane Portrait

**Ex.:** $y'' - y = 0$ $(a = 0, b = -1)$

$$p(\lambda) = \lambda^2 - 1 \Rightarrow \lambda = \pm 1 \text{ (saddle)}$$

General solution: $y(t) = c_1 e^t + c_2 e^{-t}$

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \begin{cases} \lambda_1 = 1 & \leftrightarrow \mathbf{v}_1 = [1, 1]^T \\ \lambda_2 = -1 & \leftrightarrow \mathbf{v}_2 = [-1, 1]^T \end{cases}$$




Phase plane portrait for DE (1) = Phase plane portrait for (2)

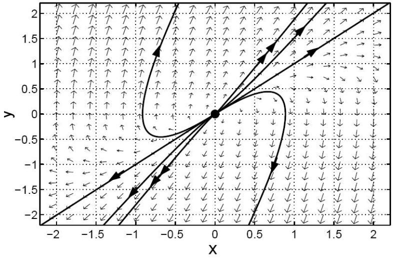
# Phase Plane Portrait

**Ex.:** $y'' - 3y' + 2y = 0$

$$p(\lambda) = \lambda^2 - 3\lambda + 2 = (\lambda - 1)(\lambda - 2)$$

$$\Rightarrow \text{source: } \; y(t) = c_1 e^{t} + c_2 e^{2t}$$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \rightarrow \begin{cases} \lambda_1 = 1 \leftrightarrow \mathbf{v}_1 = [1, 1]^T \\ \lambda_2 = 2 \leftrightarrow \mathbf{v}_2 = [1, 2]^T \end{cases}$$

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(Chapter 9)