Math 3331 Differential Equations 8.4 Linear Systems

Blerina Xhabli

Department of Mathematics, University of Houston

blerina@math.uh.edu math.uh.edu/~blerina/teaching.html



8.4 Linear Systems

- Linear Systems: General Form
- Linear Systems: Matrix-Vector Notation
- Initial Value Problem
- Examples
- Worked out Examples from Exercises:
 - 13, 14



Linear Systems: General Form

General Form:

- $\begin{array}{rcl} x'_1 &=& a_{11}(t)x_1 + \dots + a_{1n}(t)x_n + f_1(t) \\ x'_2 &=& a_{21}(t)x_1 + \dots + a_{2n}(t)x_n + f_2(t) \\ \vdots & \vdots & \vdots \end{array}$
- $x'_n = a_{n1}(t)x_1 + \cdots + a_{nn}(t)x_n + f_n(t)$
 - $a_{ij}(t), f_i(t)$: known functions on interval *I*: $\alpha < t < \beta$



Linear Systems: Matrix-Vector Notation

Matrix-vector notation:

$$\mathbf{x} = [x_1, \dots, x_n]^T$$

$$\mathbf{f}(t) = [f_1(t), \dots, f_n(t)]^T$$

$$A(t) = [a_{ij}(t)]_{nn}$$

$$\mathbf{x}' = A(t)\mathbf{x} + \mathbf{f}(t) \qquad (1)$$

- (1) is homogeneous if f(t) = 0
- (1) is nonhomogeneous if ${f f}(t)
 eq 0$
- (1) has constant coefficients if $a_{ij}(t) = a_{ij}$ are constants

Initial Value Problem

Initial Value Problem:

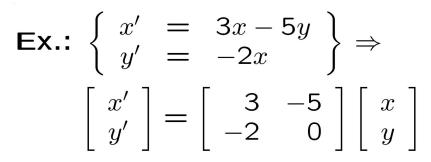
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$$\begin{array}{rcl} \mathbf{x}' &=& A(t)\mathbf{x} + \mathbf{f}(t) \\ \mathbf{x}(t_0) &=& \mathbf{x}_0 \end{array} \right\} (2)$$

Thm.: If $a_{ij}(t)$, $f_i(t)$ are continuous on I and $t_0 \in I$, then (2) has a unique solution on I.



Example 1



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is hom., constant coefficients.



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Example 2

Ex.:
$$\begin{cases} u' = \cos(t)v \\ v' = u + \sin t \end{cases}$$
is nonhom., non-constant coefs.:
$$A(t) = \begin{bmatrix} 0 & \cos t \\ 1 & 0 \end{bmatrix}, f(t) = \begin{bmatrix} 0 \\ \sin t \end{bmatrix}$$



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Example 3

Ex.: x' = xy, y' = x is nonlinear



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Math 3331 Differential Equation

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Exercise 8.4.13

Ex. 8.4.13: If possible, place system in form (1), if not possible explain why.

$$\left\{\begin{array}{rrr} x_1' &=& -2x_1+x_2^2\\ x_2' &=& 3x_1-x_2 \end{array}\right\} \text{ cannot be placed because it is nonlinear}.$$



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Exercise 8.4.14

Ex. 8.4.14: Same as Ex. 8.4.13

$$\begin{cases}
x'_1 = -2x_1 + 3tx_2 + \cos t \\
tx'_2 = x_1 - 4tx_2 + \sin t
\end{cases} \rightarrow \begin{bmatrix}
x'_1 \\
x'_2
\end{bmatrix} = \begin{bmatrix}
-2 & 3t \\
1/t & -4
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
\cos t \\
(\sin t)/t
\end{bmatrix}$$



Blerina Xhabli, University of Houston

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