Math 3331 Differential Equations

9.4 The (T, D)-Plane

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9.4 The (T, D)-Plane

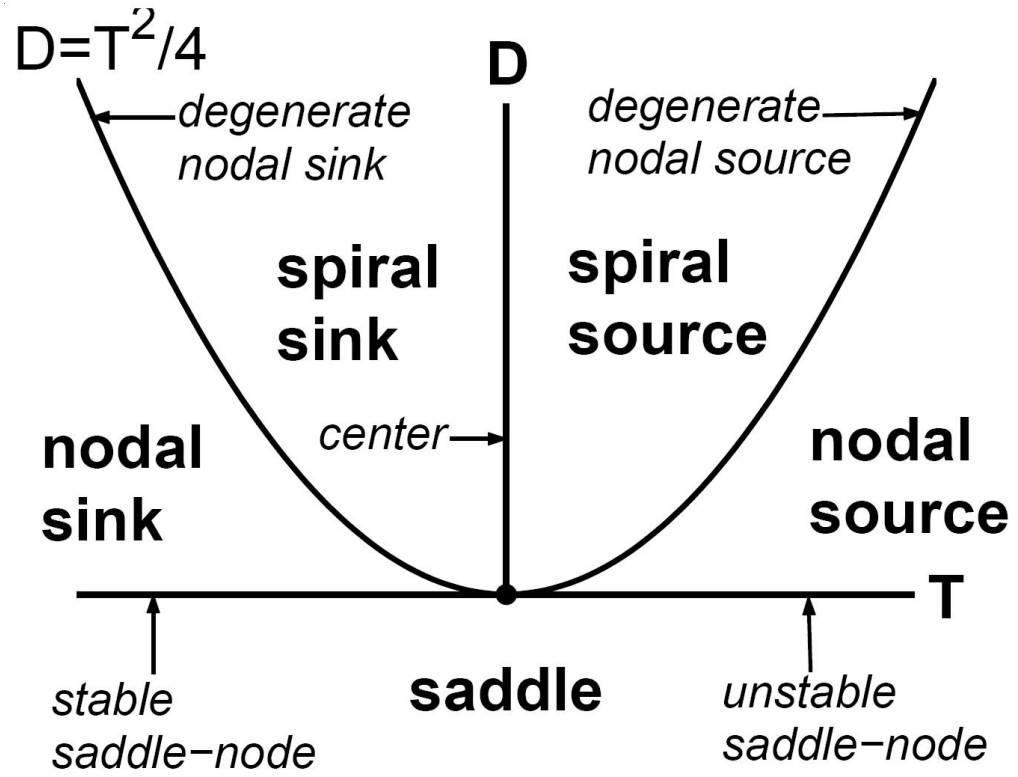
- Five Generic Cases
- Borderline Cases
- Other Special Case
- Example: Saddle
- Example: Nodal Sink
- Example: Center

Five Generic Cases

The (T, D)-Plane: $\lambda = T/2 \pm \sqrt{T^2 - 4D}/2$

Five Generic Cases:

- if $D < 0$ $\Rightarrow$ saddle
- if $D > 0$ and
  - $T > 0$ $\Rightarrow$ source
  - $T < 0$ $\Rightarrow$ sink
  - $T^2 > 4D$ $\Rightarrow$ node
  - $T^2 < 4D$ $\Rightarrow$ spiral

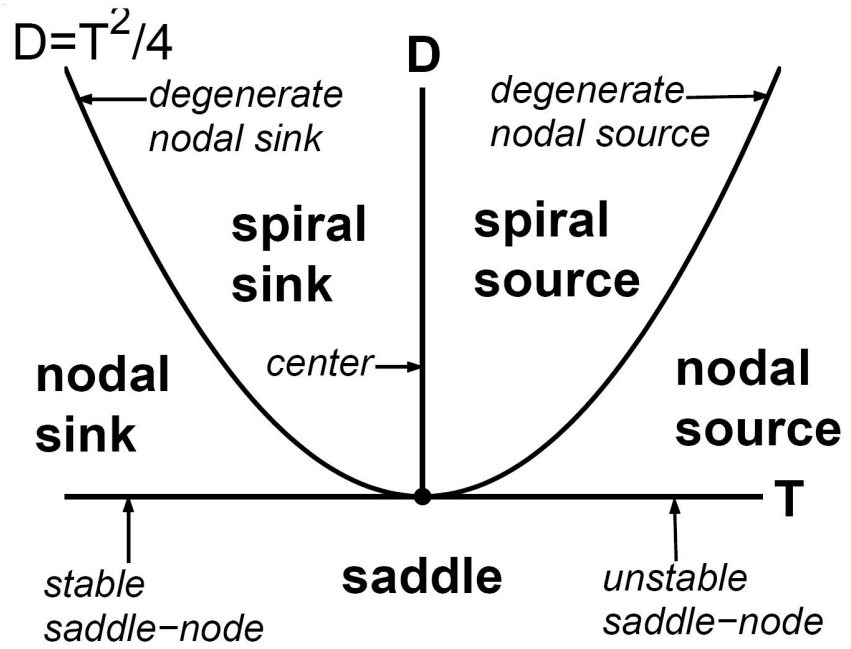


Borderline Cases

The (T, D)-Plane: $\lambda = T/2 \pm \sqrt{T^2 - 4D}/2$

Borderline Cases:

- if $T = 0$ and $D > 0 \Rightarrow$ center
- if $D = 0$, $T \neq 0 \Rightarrow$ saddle-node
  - if $T > 0 \Rightarrow$ unstable
  - if $T < 0 \Rightarrow$ stable
- if $T^2 = 4D$, $A \neq (T/2)I$, and
  - $T > 0 \Rightarrow$ d. nodal source
  - $T < 0 \Rightarrow$ d. nodal sink

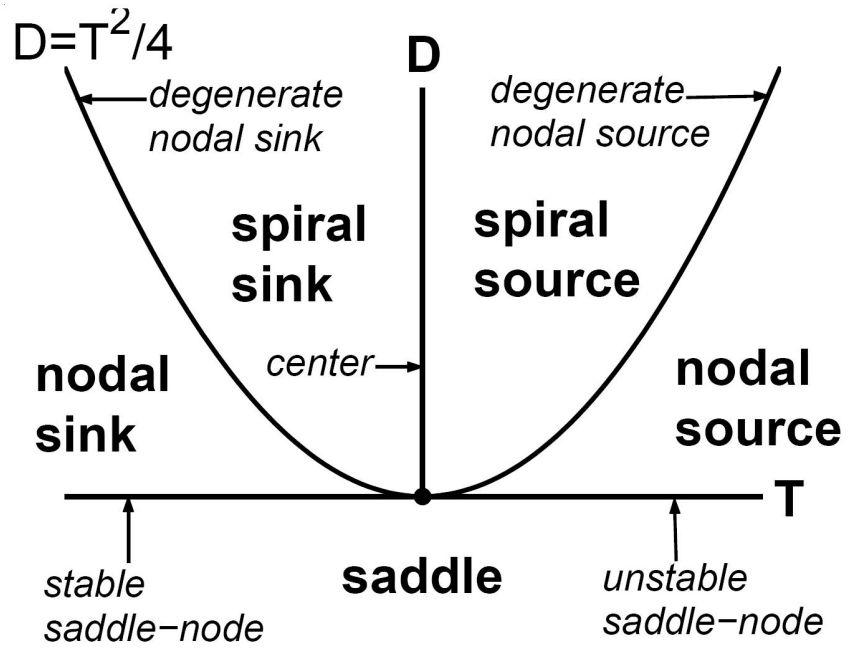


Other Special Case

The (T, D)-Plane: $\lambda = T/2 \pm \sqrt{T^2 - 4D}/2$

Other Special Case: $A = \lambda I$, $\lambda \neq 0$

- only half line solutions from origin
- Name: $\left\{\begin{matrix}\text{unstable}\\ \text{stable}\end{matrix}\right\}$ star if $\left\{\begin{matrix}\lambda > 0\\ \lambda < 0\end{matrix}\right\}$

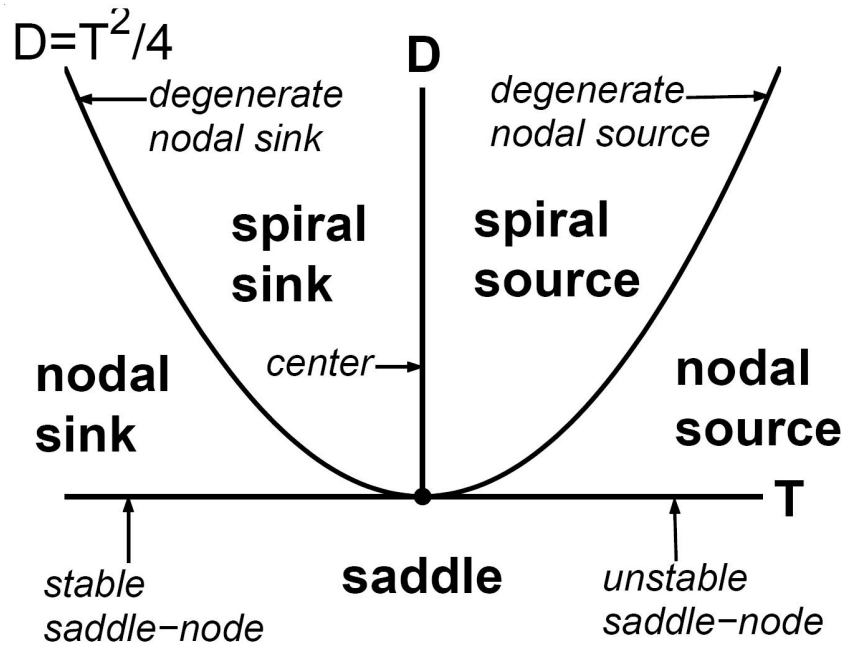


Example: Saddle

The (T, D)-Plane: $\lambda = T/2 \pm \sqrt{T^2 - 4D}/2$

Ex.: $A = \begin{bmatrix} 8 & 5 \\ -10 & -7 \end{bmatrix}$ $\{ D = -6 \}$

$\Rightarrow$ saddle

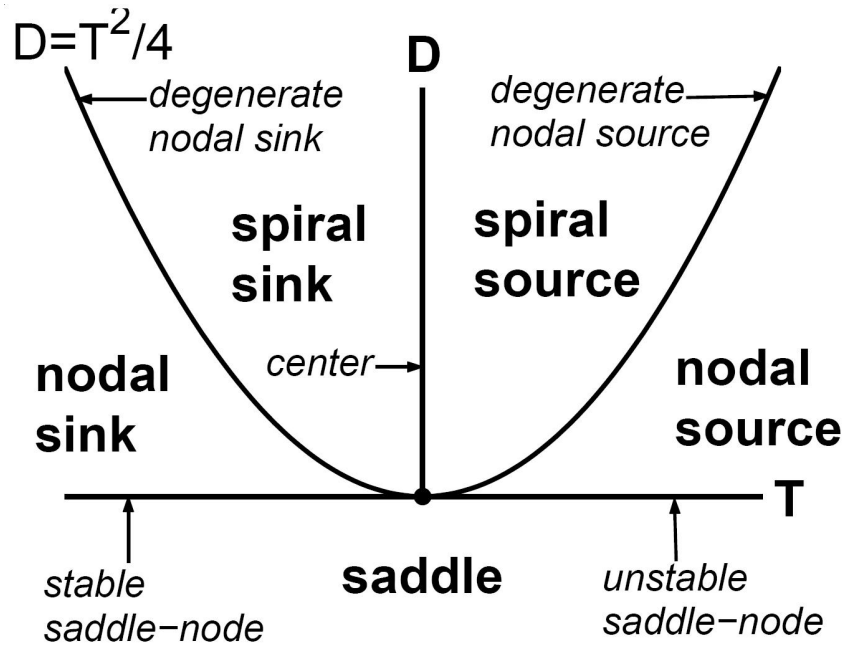


Example: Nodal Sink

The (T, D)-Plane: $\lambda = T/2 \pm \sqrt{T^2 - 4D}/2$

Ex.: $A = \begin{bmatrix} -2 & 0 \\ 1 & -1 \end{bmatrix}$ $\begin{Bmatrix} D = 2, T = -3 \\ T^2 - 4D = 1 \end{Bmatrix}$

$\Rightarrow$ nodal sink



Example: Center

The (T, D)-Plane: $\lambda = T/2 \pm \sqrt{T^2 - 4D}/2$

Ex.: $A = \begin{bmatrix} -10 & -25 \\ 5 & 10 \end{bmatrix} \begin{Bmatrix} D = 25 \\ T = 0 \end{Bmatrix}$

$\Rightarrow$ center

$c = 5 > 0 \Rightarrow$ counterclockwise direction of rotation