

# Math 3331 Differential Equations

## 2.5 Mixing Problems

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## 2..5 Mixing Problems

- Balance Law
- Mixture of Water and Salt
  - Example 5.1
  - Example 5.3



# Mixing Problems

## Solution of a mixture of water and salt

$x(t)$ : amount of salt

$V(t)$ : volume of the solution

$c(t)$ : concentration of salt

$$\Rightarrow c(t) = \frac{x(t)}{V(t)}$$

## Balance Law

$$\frac{dx}{dt} = \text{rate in} - \text{rate out}$$

rate = flow rate  $\times$  concentration



# Example 1

## See Text, Example 2.5.1

$V(t) = 100$  gal, kept constant

concentration in = 2 lb/gal

flow rate in = 3 gal/min

flow rate out = flow rate in

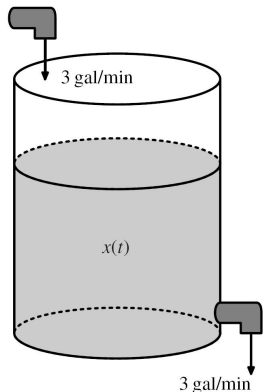
$$\Rightarrow c(t) = x(t)/100 \text{ lb/gal}$$

$$\Rightarrow \text{rate in} = 6 \text{ lb/min}$$

$$\Rightarrow \text{rate out} = 3x(t)/100 \text{ lb/min}$$

## Balance Law

$$\frac{dx}{dt} = 6 - 3x/100$$



example 1)

$$V(t) = \text{volume} = \text{constant} = 100$$

$$x(t) = \text{amount of salt}$$

$$\text{concentration} = \frac{\text{amount}}{\text{volume}} = \frac{x(t)}{100}$$

$$\frac{dx}{dt} = \text{rate of change of amount}$$

Balance Law = rate in - rate out

$$= 2 \times 3 - 3 \times \frac{x}{100}$$

}  $\Rightarrow x' = 6 - \frac{3}{100}x$   
linear diff. eqn.  
and  $x(0) = 0$

$\Rightarrow$  let's solve it

$$x' = -\frac{3}{100}x + 6$$

$$(*) \quad x' + \frac{3}{100}x = 6$$

integrating factor  $u(t) = e^{\int \frac{3}{100} dt} = e^{\frac{3}{100}t}$

→ Multiply both sides of (\*) by  $u(t)$ :

$$e^{\frac{3}{100}t} \cdot x' + \frac{3}{100} e^{\frac{3}{100}t} x = 6 e^{\frac{3}{100}t}$$

$$\Rightarrow [e^{\frac{3}{100}t} \cdot x]' = 6 e^{\frac{3}{100}t}$$

Integrate  
 $\Rightarrow$

$$e^{\frac{3}{100}t} \cdot x = 200 e^{\frac{3}{100}t} + C$$

$$\Rightarrow x(t) = 200 + C e^{-\frac{3}{100}t}, \quad x(0) = 0$$

$$\Rightarrow C = -200$$

Hence,

$$x(t) = 200 - 200 e^{-\frac{3}{100}t}$$

$$t=60$$

$$x(60) \approx 167 \text{ lbs of salt.}$$



# Example 2

## See Text, Example 2.5.2

concentration in = 1.5 lb/gal

flow rate in = 3 gal/min

flow rate out = 1 gal/min

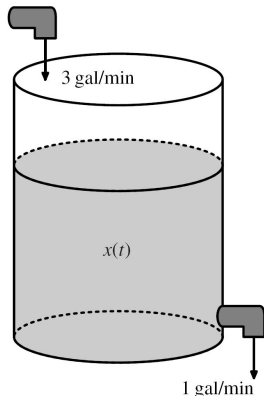
$$V(0) = 300 \text{ gal}$$

$$\Rightarrow V(t) = 300 + 2t \text{ lb/gal}$$

$$\Rightarrow c(t) = x(t)/(300 + 2t) \text{ lb/gal}$$

$$\Rightarrow \text{rate in} = 4.5 \text{ lb/min}$$

$$\Rightarrow \text{rate out} = x(t)/(300 + 2t) \text{ lb/min}$$



## Balance Law

$$\frac{dx}{dt} = 4.5 - x/(300 + 2t)$$

