Math 3331 Differential Equations

6.3 Numerical Error Comparisons

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6.3 Numerical Error Comparisons

- Numerical Error Comparisons
  - Examples
  - Least Square Fit
- Worked out Examples from Exercises:
  - 3, 7,

Example

Ex. $y' = t - y,\ y(0) = 0.5$

$$y(1) \approx y_m \rightarrow E(h) = |y(1) - y_m|$$

$$h = 1/m, \quad m = 1, 2, 4, 8, 16, 32$$

h	EM	RKM2	RKM4
1	0.5518	0.198181	0.010680838
1/2	0.1768	0.034118	0.000437105
1/4	0.0772	0.006974	0.000022137
1/8	0.0364	0.001581	0.000001246
1/16	0.0177	0.000377	0.000000074
1/32	0.0087	0.000092	0.000000005

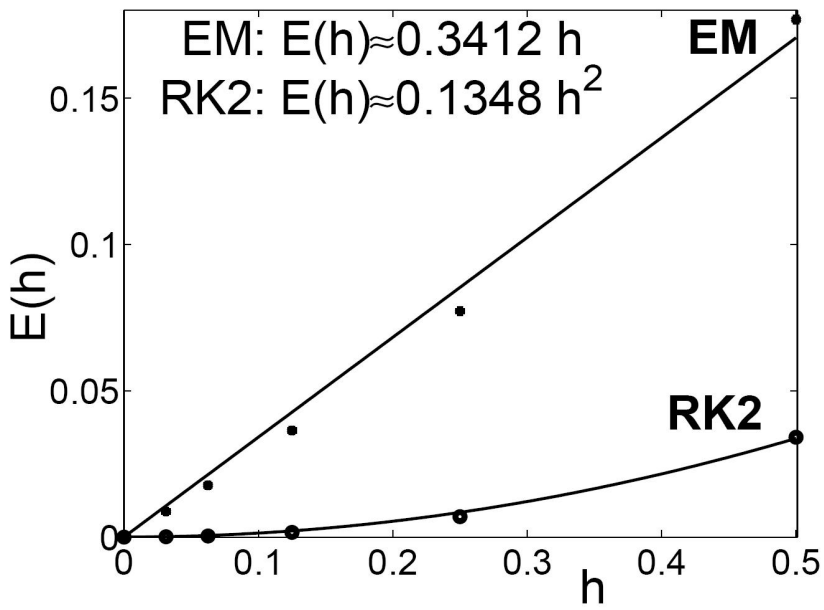
$E(h)$ for

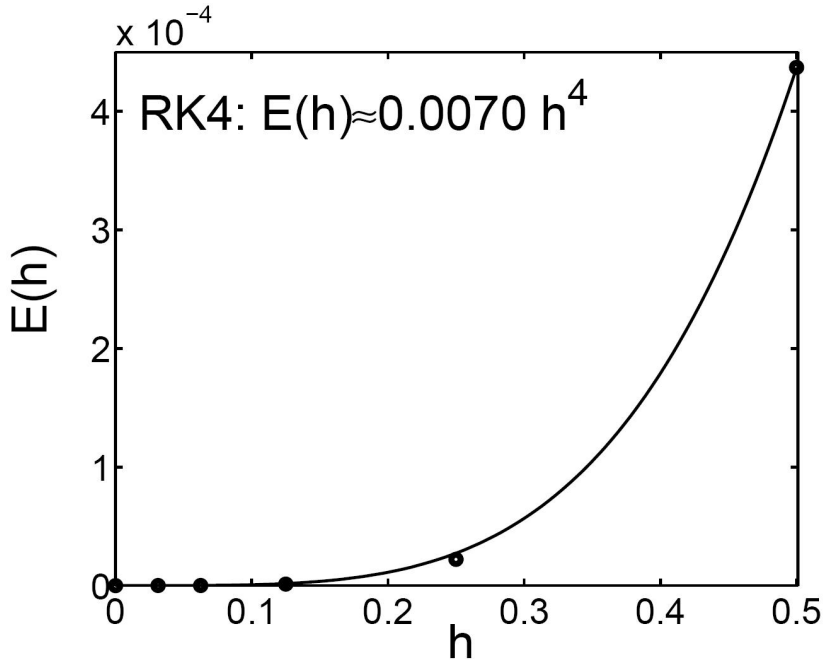
EM: Euler Method

RKM2: 2nd order RKM

RKM4: 4th order RKM

Example (cont.): Least Square Fit of $E(h)$





Exercise 6.2.3

Ex. 3: $y' = ty,\ y(0) = 1.$

Compute five RK2-iterates for $h = 0.1$.
Arrange computation and results in a table.

k	t_k	y_k	s_l	s_r	h	$h(s_l + s_r)/2$
0	0	1	0	0.1	0.1	0.005
1	0.1	1.0050	0.1005	0.2030	0.1	0.0152
2	0.2	1.0202	0.2040	0.3122	0.1	0.0258
3	0.3	1.0460	0.3138	0.4309	0.1	0.0372
4	0.4	1.0832	0.4333	0.5633	0.1	0.0498
5	0.5	1.1331	0.5665	0.7138	0.1	0.0640

Exercise 6.2.7 (i)

Ex. 7: $z' + z = \cos x,\ z(0) = 1$

(i) Compute RK2-approximations in $0 \leq x \leq 1$ for $h = 0.2$, $h = 0.1$, $h = 0.05$.

(ii) Find exact solution

(iii) Plot exact solution as curve and RK2 approximations as points.

(i) In Matlab, the RK2 approximation for $h = 0.2$ is computed and stored in arrays *x0_2*, *z0_2* via

Analogously for $h = 0.1$ and $h = 0.05$ (arrays *x0_1*, *z0_1* and *x0_05*, *z0_05*).

```
h=0.2;
m=1/h;x=0;z=1;
xv=x;zv=z;
for k=1:m
    sl=cos(x)-z;
    sr=cos(x+h)-(z+sl*h);
    z=z+h*(sl+sr)/2;zv=[zv z];
    x=x+h;xv=[xv x];
end
x0_2=xv;z0_2=zv;
```

Exercise 6.2.7 (ii)

Ex. 7: $z' + z = \cos x,\ z(0) = 1$

(i) Compute RK2-approximations in $0 \le x \le 1$ for $h = 0.2$, $h = 0.1$, $h = 0.05$.
(ii) Find exact solution
(iii) Plot exact solution as curve and RK2 approximations as points.

(ii) Variation of Parameter:

$$z_h' = -z \ \Rightarrow\ z_h(x) = e^{-x}$$

$$\begin{aligned} z(x) &= e^{-x} + \int_0^x e^{\xi} \cos(\xi)\, d\xi \\ &= (\cos x + \sin x + e^{-x})/2 \end{aligned}$$

Exercise 6.2.7 (iii)

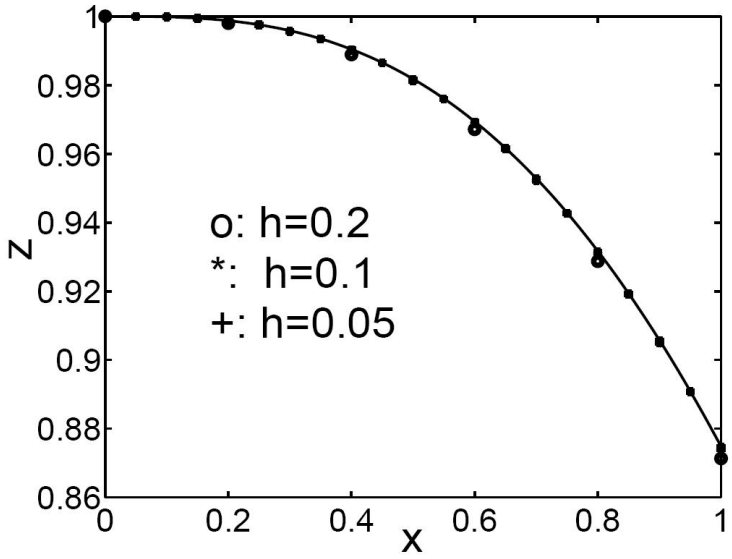
Ex. 7: $z' + z = \cos x,\ z(0) = 1$

(i) Compute RK2-approximations in $0 \le x \le 1$ for $h = 0.2$, $h = 0.1$, $h = 0.05$.

(ii) Find exact solution

(iii) Plot exact solution as curve and RK2 approximations as points.

(iii) Plot:
(see CN Sec. 6.1 for commands)



Exercise 6.2.7a (i)

Ex. 7a: $z' + z = \cos x,\ z(0) = 1$

(i) Compute RK4-approximation in $0 \le x \le 1$ for $h = 0.2$.

(iii) Plot exact solution as curve and RK4 approximation as points.

(i) RK4 approximation for $h = 0.2$ is computed and stored in arrays *xv*, *zv*:

```
h=0.2;
m=1/h;x=0;z=1;
xv=x;zv=z;
for k=1:m
    s1=cos(x)-z;
    s2=cos(x+h/2)-(z+s1*h/2);
    s3=cos(x+h/2)-(z+s2*h/2);
    s4=cos(x+h)-(z+s3*h);
    z=z+h*(s1+2*s2+2*s3+s4)/6;
    zv=[zv z];
    x=x+h;xv=[xv x];
end
```

Exercise 6.2.7a (ii)

Ex. 7a: $z' + z = \cos x,\ z(0) = 1$

(i) Compute RK4-approximation in $0 \leq x \leq 1$ for $h = 0.2$.

(iii) Plot exact solution as curve and RK4 approximation as points.

(iii) Matlab plot commands:

```
x=linspace(0,1,100);
z=1/2*(cos(x)+sin(x)+exp(-x));
plot(xv,zv,'ko',x,z,'k'),
xlabel('x'),ylabel('z'),
axis([0 1 0.86 1])
```