1. Find the Laplace Transform of the following function

$$f(x) = \begin{cases} \sin x, & 0 \le x < \pi/2 \\ 2\cos x + 1, & x \ge \pi/2 \end{cases}.$$

2. Let $F(s) = \dfrac{2 + se^{-3s}}{s^2 + 2s}$ be the Laplace transformation of a piecewise defined function $f(t)$. Find the piecewise defined function.

3. Use the Laplace Transform to find the solution of the following initial-value problems

   a. $y'' + y = \cos 2t, \quad y(0) = 0, \quad y'(0) = 1.$

   b. $y'' - y = e^t, \quad y(0) = 0, \quad y'(0) = 0.$

   c. $y'' + y = g(t), \quad y(0) = 0, \quad y'(0) = 1, \quad$ where $g(t) = \begin{cases} 2t, & \text{for } 0 \le t < 1 \\ 2, & \text{for } 1 \le t < \infty \end{cases}.$

4. Consider the initial value problem

$$x' = -x + t, \quad 0 \le t \le 1, \quad x(0) = 0.5. \tag{1}$$

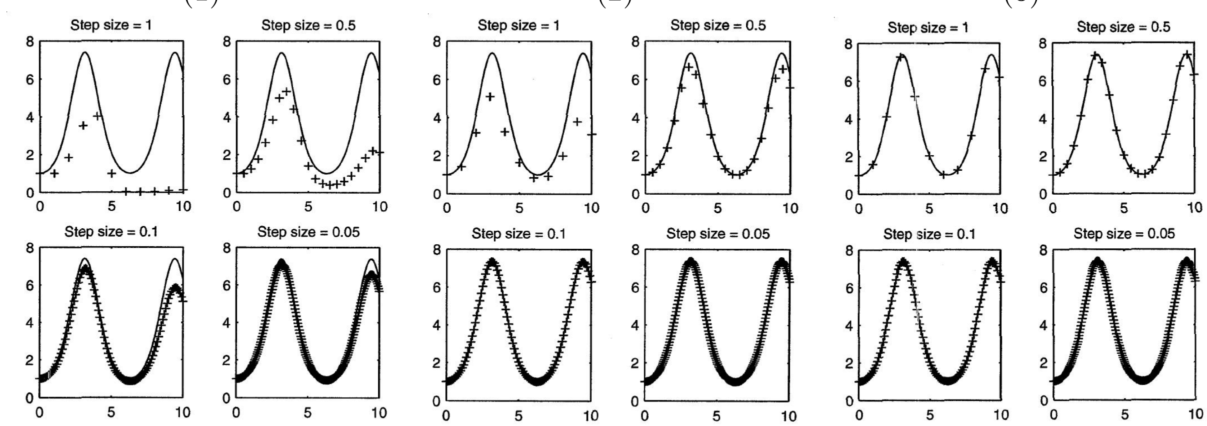
   Use the Euler, RK2 and RK4 methods to approximate the value of $x(1)$ for a step size $h = 0.5$ and compute the error of your numerical solution.

5. Consider the initial value problem

$$x' = x \sin t, \quad t \ge 0, \quad x(0) = 1. \tag{2}$$

   The equation is separable and the solution is $x(t) = e^{1-\cos t}$. The Euler method, RK2 and RK4 methods, with step sizes $h = 1$, 0.5, 0.1 and 0.05 produce the following results. Indicate each graph (1,2,3) by its corresponding numerical method and explain your answer.

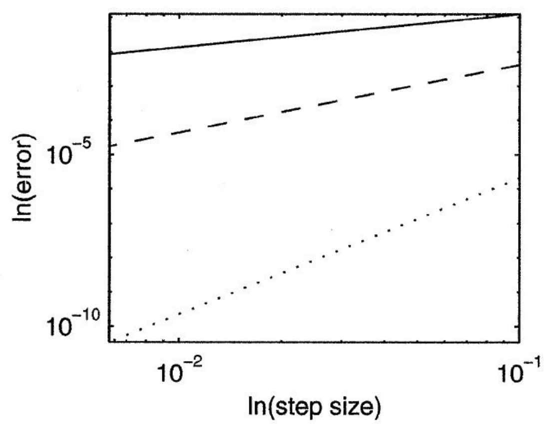
(1) (2) (3)



6. Consider the initial value problem

$$x' = x, \quad 0 \le t \le 1, \quad x(0) = 1. \tag{3}$$

The equation is separable and the solution is $x(t) = e^t$. We used the Euler method, RK2 and RK4 methods to compute the value of $x(1)$ and constructed a plot of the logarithm of the error versus the logarithm of the step size for each numerical method. The slope of the solid line is 0.9716, the slope of the dashed line is 1.9755, and the slope of the dotted line is 3.9730. Indicate each line by its corresponding numerical method and explain your answer.



7. Write each initial value problems as a system of the first-order equations using vector notation.

a. $x'' + \delta x' - x + x^3 = \gamma \cos \omega t, \quad x(0) = x_0, \; x'(0) = v_0$

b. $x'' + \mu(x^2 - 1)x' + x = 0, \quad x(0) = x_0, \; x'(0) = v_0$

8. Consider the predator-prey system

$$\begin{aligned} F' &= +0.2F - 0.1FS \\ S' &= -0.3S + 0.1FS \end{aligned}$$

Describe the behaviour of the system.
What happens to the solution that starts with $F(0) = 3$ and $S(0) = 2$?

9. Consider the system

$$\begin{aligned} x' &= 4x - 4x^2 - xy \\ y' &= 4y - xy - 2y^2 \end{aligned}.$$

Plot the nullclines for each equation of the given system. Calculate the equilibrium points and plot them in your sketch.

10. Consider the system

$$\begin{aligned} x' &= 1 - (y - \sin(x))\cos(x) \\ y' &= \cos(x) - y + \sin(x) \end{aligned} .$$

(i) Show that $x(t) = t, \quad y(t) = \sin(t)$ is a solution.

(ii) Plot the solution in a phase plane.

(iii) Consider the solution to the system with initial conditions $x(0) = \pi/2$ and $y(0) = 0$. shaow that $y(t) < \sin(x(t))$ for all t.

11. Write the given system of differential equations in matrix vector form. Show that the given vector-valued function is a solution of the system.

$$\begin{aligned} x_1' &= 8x_1 - 10x_2 \\ x_2' &= 5x_1 - 7x_2 \end{aligned} , \quad \mathbf{x} = (2e^{3t}, e^{3t})^t$$