Quiz 3

1. State the principle of superposition for homogeneous linear problems

\[ L(u) = 0. \]

2. (a) Find the eigenvalues and eigenfunctions of the eigenvalue problem

\[ \frac{d^2 \phi}{dx^2} = -\lambda \phi, \quad \lambda > 0, \]

\[ \phi(0) = 0, \]

\[ \phi(L) = 0. \]

(b) How many eigenvalues are there for this problem?

3. (a) Solve the following initial-boundary-value problem for the heat equation, where \( k > 0 \) is a given constant:

\[ \frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, \quad x \in (0, L), \, t > 0, \]

\[ u(0, t) = 0, \]

\[ u(L, t) = 0, \]

\[ u(x, 0) = 4 \sin \frac{5\pi x}{L}. \]

(b) What is the behavior of the solution as \( t \to \infty \)?