Practice Quiz 8

1. (a) What does the mean value property for the Laplace’s equation state? Prove the mean value property.

(b) What is does the Maximum Principle for the Laplace’s equation state?

2. How do you find the general solution $G(r)$ of the following ODE, for all $n = 0, 1, 2, \ldots$:

$$r^2G''(r) + rG'(r) = n^2G(r).$$

Show your work.

3. Solve the Laplace’s equations inside a circular disk of radius 1, with the prescribed boundary data equal to:

$$u(1, \theta) = 2 + \sin(3\theta), \quad -\pi \leq \theta < \pi.$$ 

Recall: Laplace’s equation in polar coordinates is given by:

$$\Delta u = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial u}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u}{\partial \theta^2} = 0.$$ 

Show all your work.