NAME (please print): Answer Key

Quiz 1

1. Write an Initial Boundary Value problem modeling heat transfer in a rod of length L, with thermal diffusivity $k = 3$, and insulated end points.

IBV Problem
$$\begin{cases} u_t = 3u_{xx}, & x \in (0,L),\ t>0 \\ u(x,0) = f(x), & x \in (0,L) \\ \frac{\partial u}{\partial x}(0,t) = 0, & \frac{\partial u}{\partial x}(L,t) = 0,\ t>0 \end{cases}$$

2. What is Newton's Law of Cooling? State it for both ends of the rod (i.e., at $x = 0$ and $x = L$).

At $x=0$: $\quad -K_0 \frac{\partial u}{\partial x}(0,t) = -H\left[u(0,t) - u_B(t)\right]$

At $x=L$: $\quad -K_0 \frac{\partial u}{\partial x}(L,t) = H\left[u(0,t) - u_B(t)\right]$

3. Calculate the total energy as a function of time for a rod of length 2 with density 1, specific heat 2, and cross-sectional area 4, for which the temperature distribution $u(x,t)$ at any time $t > 0$ and $x \in (0,2)$ is given by $u(x,t) = e^{4t}(xt+4)$.

$$E(t) = 2\int_0^2 e^{4t}(xt+4)\cdot 4\,dx = 8\int_0^2 e^{4t}(xt+4)\,dx =$$
$$= 8e^{4t}\left(t\frac{x^2}{2} + 4x\right)\Big|_{x=0}^{x=2} = 8e^{4t}\left(t\cdot\frac{4}{2} + 8 - 0\right) = \underline{8e^{4t}(2t+8)}$$

4. Which of these PDEs is a heat equation (circle the equation):

- $u_t = 2u_{xx}$,
- $u_{tt} = 3u_{xx}$,
- $2u_t = 3u_x$,
- $u_t = -2u_x$.

5. (a) Let α and β be some positive constants. Find the constant C such that $w(x,t) = e^{-\alpha t}\cos\beta x + Ct$, satisfies the following heat equation:

$$u_t = \frac{\alpha}{\beta^2}u_{xx} + 10, \quad t>0, x\in\mathbf{R}.$$

(b) What is the initial condition satisfied by w?

(a) $w_t = -\alpha e^{-\alpha t}\cos\beta x + C$

$w_x = -\beta e^{\alpha t}\sin\beta x$

$w_{xx} = -\beta^2 e^{\alpha t}\cos\beta x$

$u_t = \frac{\alpha}{\beta^2}u_{xx} + C$

$\Rightarrow \boxed{C = 10}$

(b) For $t=0$: $\boxed{w(x,0) = \cos\beta x}$ INITIAL CONDITION