Homework 3: January 24, 2017

1. Read from the textbook Section 1.4: "Equilibrium Temperature Distribution" (pages 14-18).

2. (Problem 1.4.1. in the text book) Determine the equilibrium temperature distribution for a one-dimensional rod with constant thermal properties with the following sources and boundary conditions:

(a) $Q = 0, \quad u(0) = 0, \quad u(L) = T,$

(b) $Q = 0, \quad u(0) = T, \quad u(L) = 0,$

(c) $Q = 0, \quad u_x(0) = 0, \quad u(L) = T,$

(d) $Q = 0, \quad u(0) = T, \quad u_x(L) = \alpha,$

(e) $Q/K_0 = 1, \quad u(0) = T_1, \quad u(L) = T_2,$

(f) $Q = x^2, \quad u(0) = T, \quad u_x(L) = 0,$

(g) $Q = 0, \quad u(0) = T, \quad u_x(L) + u(L) = 0,$

(h) $Q = 0, \quad u_x(0) - [u(0) - T] = 0, \quad u_x(L) = \alpha.$

In these you may assume that $u(x, 0) = f(x)$.

3. (Problem 1.4.4.) If both ends of the rod are insulated, derive using the partial differential equation, that the total thermal energy in the rod is constant.

4. (Problems 1.4.7 (a)–(c), pages 18-19.) For the following problems, determine an equilibrium temperature distribution (if one exists). For what values of β are there solutions? Explain physically.

(a) $u_t = u_{xx} + 1, \quad u(x, 0) = f(x), \quad u_x(0, t) = 1, \quad u_x(L, t) = \beta,$

(b) $u_t = u_{xx}, \quad u(x, 0) = f(x), \quad u_x(0, t) = 1, \quad u_x(L, t) = \beta,$

(c) $u_t = u_{xx} + x - \beta, \quad u(x, 0) = f(x), \quad u_x(0, t) = 0, \quad u_x(L, t) = 0.$

5. (Problem 1.4.10.) Suppose $u_t = u_{xx} + 4$, $u(x, 0) = f(x)$, $u_x(0, t) = 5$, $u_x(L, t) = 6$. Calculate the total thermal energy in the one-dimensional rod (as a function of time).

Hint: use an approach similar to solving Problem 3 above.

I will ask a volunteer in the class to solve this problem on the board for extra credit.