1. Read from the textbook Section 1.4: “Equilibrium Temperature Distribution” (pages 14-18).

2. (Problem 1.4.1. in the text book) Determine the equilibrium temperature distribution for a one-dimensional rod with constant thermal properties with the following sources and boundary conditions:

   (a) \( Q = 0, \ u(0) = 0, \ u(L) = T, \)
   (b) \( Q = 0, \ u(0) = T, \ u(L) = 0, \)
   (c) \( Q = 0, \ u_x(0) = 0, \ u(L) = T, \)
   (d) \( Q = 0, \ u(0) = T, \ u_x(L) = \alpha, \)
   (e) \( Q/K_0 = 1, \ u(0) = T_1, \ u(L) = T_2, \)
   (f) \( Q = x^2, \ u(0) = T, \ u_x(L) = 0, \)
   (g) \( Q = 0, \ u(0) = T, \ u_x(L) + u(L) = 0, \)
   (h) \( Q = 0, \ u_x(0) - [u(0) - T] = 0, \ u_x(L) = \alpha. \)

   In these you may assume that \( u(x,0) = f(x). \)

3. (Problem 1.4.4.) If both ends of the rod are insulated, derive using the partial differential equation, that the total thermal energy in the rod is constant.

4. (Problems 1.4.7 (a)-(c), pages 18-19.) For the following problems, determine an equilibrium temperature distribution (if one exists). For what values of \( \beta \) are there solutions? Explain physically.

   (a) \( u_t = u_{xx} + 1, \ u(x,0) = f(x), \ u_x(0,t) = 1, \ u_x(L,t) = \beta, \)
   (b) \( u_t = u_{xx}, \ u(x,0) = f(x), \ u_x(0,t) = 1, \ u_x(L,t) = \beta, \)
   (c) \( u_t = u_{xx} + x - \beta, \ u(x,0) = f(x), \ u_x(0,t) = 0, \ u_x(L,t) = 0. \)

5. (Problem 1.4.10.) Suppose \( u_t = u_{xx} + 4, \ u(x,0) = f(x), \ u_x(0,t) = 5, \ u_x(L,t) = 6. \) Calculate the total thermal energy in the one-dimensional rod (as a function of time).

   Hint: use an approach similar to solving Problem 3 above.

   I will ask a volunteer in the class to solve this problem on the board for extra credit.