Homework 5: Feb 7, 2017

1. Read sections 2.3.1, 2.3.2, 2.3.3, 2.3.4, and 2.3.5 from the textbook.

2. (Problem 2.3.1. pg 51) For the following PDEs, what ordinary differential equation are implied by the method of separation of variables?

(a) $\frac{\partial u}{\partial t} = k\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)$ (solution in text book)

(b) $\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} - v_0\frac{\partial u}{\partial x}$ (solution in class)

(c) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ (solution in text book)

(e) $\frac{\partial u}{\partial t} = k\frac{\partial^4 u}{\partial x^4}$ (solution in text book)

(f) $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$ (solution in text book)

3. Consider the differential equation

$$\frac{d^2\phi}{dx^2} + \lambda\phi = 0.$$

Determine the eigenvalues λ and the corresponding eigenfunctions if ϕ satisfies the following boundary conditions:

(a) $\phi(0) = 0, \phi(\pi) = 0$

(b) $\phi(0) = 0, \phi(1) = 0$

(c) $\frac{d\phi}{dx}(0) = 0, \frac{d\phi}{dx}(L) = 0,$

(d) $\frac{d\phi}{dx}(0) = 0, \phi(L) = 0,$

(g) $\phi(0) = 0, \frac{d\phi}{dx}(L) + \phi(L) = 0.$

4. Solve the following initial-boundary value problem for the heat equation:

$$\begin{aligned}&\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2},\\&u(0,t) = 0,\\&u(L,t) = 0,\\&u(x,0) = 6\sin\frac{9\pi x}{L}.\end{aligned}$$