Homework 7: Feb 16, 2017

1. (Problem 2.3.3.) Consider the heat equation $u_t = ku_{xx}$, subject to the boundary conditions $u(0, t) = 0$ and $u(L, t) = 0$. Solve the initial boundary value problem if the temperature is initially:

   (d) $u(x, 0) = \begin{cases} 1 & 0 < x \leq L/2, \\ 2 & L/2 < x \leq L \end{cases}$

   (e) $u(x, 0) = f(x)$ (the resulting integrals do not need to be evaluated).

2. (Problem 2.3.8) Consider $u_t = ku_{xx} - \alpha u$.

   This corresponds to a one-dimensional rod either with heat loss through the lateral sides with outside temperature 0°C, or with insulated lateral sides with a heat sink proportional to the temperature. Suppose that the boundary conditions are

   $u(0, t) = 0, u(L, t) = 0$.

   Solve the time dependent initial boundary value problem with $u(x, 0) = f(x)$ if $\alpha > 0$. Analyze the temperature for large time ($t \to \infty$).

3. (Problem 2.4.1) Solve the heat equation $u_t = ku_{xx}$ for $x \in (0, L), t > 0$ with the boundary conditions

   $u_x(0, t) = 0, \quad u_x(L, t) = 0,$

   and the following initial data:

   (a) $u(x, 0) = \begin{cases} 0 & x < L/2 \\ 1 & x > L/2 \end{cases}$

   (b) $u(x, 0) = 6 + 4 \cos \frac{3\pi x}{L}$,

   (c) $u(x, 0) = -2 \sin \frac{\pi x}{L}$,

   (d) $u(x, 0) = -3 \cos \frac{8\pi x}{L}$.

4. (Problem 2.4.3) Solve the eigenvalue problem

   $\frac{d^2 \phi}{dx^2} = -\lambda \phi,$

   subject to

   $\phi(0) = \phi(2\pi), \quad \frac{d\phi}{dx}(0) = \frac{d\phi}{dx}(2\pi).$