Homework 7: Feb 16, 2017

1. (Problem 2.3.3.) Consider the heat equation $u_t = ku_{xx}$, subject to the boundary conditions $u(0,t) = 0$ and $u(L,t) = 0$. Solve the initial boundary value problem if the temperature is initially:

(d) $u(x,0) = \begin{cases} 1 & 0 < x \leq L/2, \\ 2 & L/2 < x \leq L \end{cases}$

(e) $u(x,0) = f(x)$ (the resulting integrals do not need to be evaluated).

2. (Problem 2.3.8) Consider

$$u_t = ku_{xx} - \alpha u.$$

This corresponds to a one-dimensional rod either with heat loss through the lateral sides with outside temperature 0°, or with insulated lateral sides with a heat sink proportional to the temperature. Suppose that the boundary conditions are

$$u(0,t) = 0, u(L,t) = 0.$$

Solve the time dependent initial boundary value problem with $u(x,0) = f(x)$ if $\alpha > 0$. Analyze the temperature for large time ($t \to \infty$).

3. (Problem 2.4.1) Solve the heat equation $u_t = ku_{xx}$ for $x \in (0,L), t > 0$ with the boundary conditions

$$u_x(0,t) = 0, \quad u_x(L,t) = 0,$$

and the following initial data:

(a) $u(x,0 = \begin{cases} 0 & x < L/2 \\ 1 & x > L/2. \end{cases}$

(b) $u(x,0) = 6 + 4\cos\frac{3\pi x}{L}$,

(c) $u(x,0) = -2\sin\frac{\pi x}{L}$,

(d) $u(x,0) = -3\cos\frac{8\pi x}{L}$.

4. (Problem 2.4.3) Solve the eigenvalue problem

$$\frac{d^2\phi}{dx^2} = -\lambda\phi,$$

subject to

$$\phi(0) = \phi(2\pi), \quad \frac{d\phi}{dx}(0) = \frac{d\phi}{dx}(2\pi).$$