QUADRATIC SYSTEMS OF CONSERVATION LAWS WITH GENERIC BEHAVIOR AT INFINITY

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Abstract. We identify quadratic systems of conservation laws with "generic" behavior at infinity, where the genericity conditions derive naturally when studying weak solutions of conservation laws. Namely, we identify quadratic models for which the vector field associated with the viscosity admissiblity criterion has properties at infinity that are true for an open and dense subset of the set of all planar quadratic vector fields in the metric associated with the Euclidian space of coefficients. We determine the boundaries of the regions containing "generic" models in the parameter space of coefficients of quadratic models. We show that when crossing the boundaries of non-generic models transversally, the Poincaré compactification of the corresponding vector field undergoes either a saddle node, or a transcritical bifurcation at infinity.

For quadratic models with a bounded elliptic region we calculate the loci of non-generic models assuming the viscosity matrix to be the identity. We obtain a two–parameter normal form for such models and show that the boundaries that determine generic models in the two–dimensional parameter space correspond to the Schaeffer–Shearer classification of models with an isolated umbilic point. Since the loci of non-generic models are invariant under the equivalence transformations that preserve weak solutions of conservation laws, understanding their behavior at infinity promises to provide an insight into a general classification of quadratic conservation laws.

1. Introduction

We identify quadratic models for which the vector field associated with the viscosity admissibility criterion is structurally stable in some neighborhood of infinity.

The motivation for this work is two–fold. The first is related to the problem of identifying all viscous admissible shock waves for a quadratic conservation law with a given viscosity matrix. As shown in Refs. [3, 4, 5] this can be accomplished by studying structural stability of the associated family of planar quadratic vector fields under the perturbations of the left state in a shock wave and of the shock speed. These parameters appear in the lower order terms of a quadratic vector field. In order to study vector fields that are structurally stable under perturbations of the corresponding lower order terms, we would like to identify vector fields whose homogeneous quadratic part is "generic". Since second order terms determine behavior at infinity, the goal is to identify quadratic models with "generic" properties at infinity. In this paper we derive the appropriate "genericity" conditions for a quadratic model with a given viscosity matrix. These conditions are associated with the invariants of quadratic flux functions; invariants under the equivalence transformations that preserve

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