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MATH 1310

Session 1

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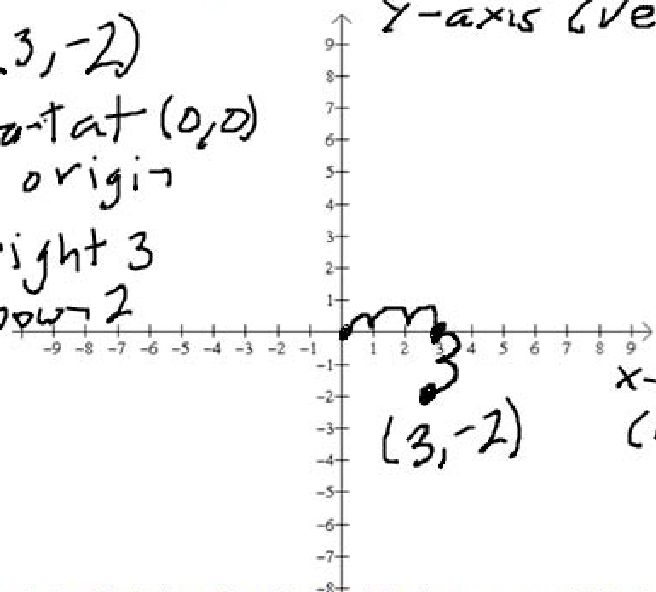
Section 1.1: Points, Regions, Distance and Midpoints

Graphing Points and Regions

Here's the coordinate plane:

(3, -2)
- start at (0,0) origin
- right 3
- down 2



(3, -2)

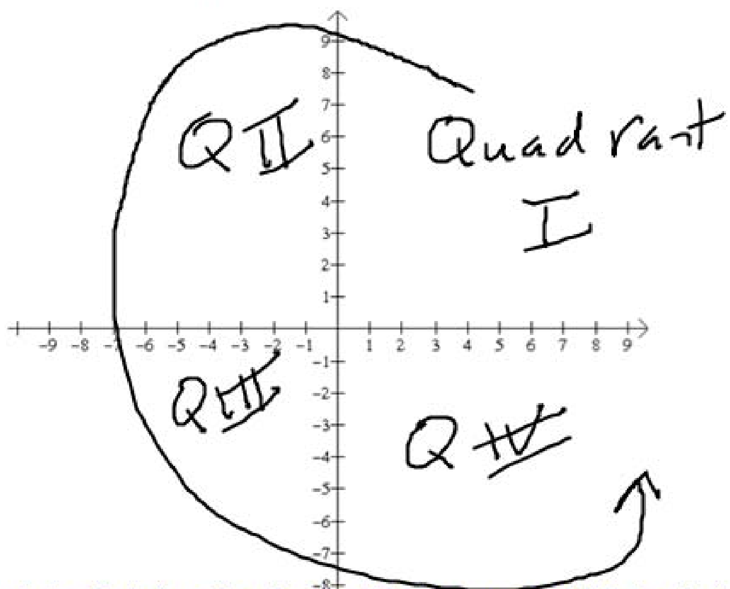
x-value: how far left/right
(neg) (positive)

y-value: how far up/down
(pos) (neg)

Section 1.1: Points, Regions, Distance and Midpoints

Graphing Points and Regions

Here's the coordinate plane:



As we see the plane consists of two perpendicular lines, the x-axis and the y-axis. These two lines separate them into four regions, or quadrants.

The pair, (x, y), is called an ordered pair. It corresponds to a single unique point in the coordinate plane. The first number is called the x coordinate, and the second number is called the y coordinate.

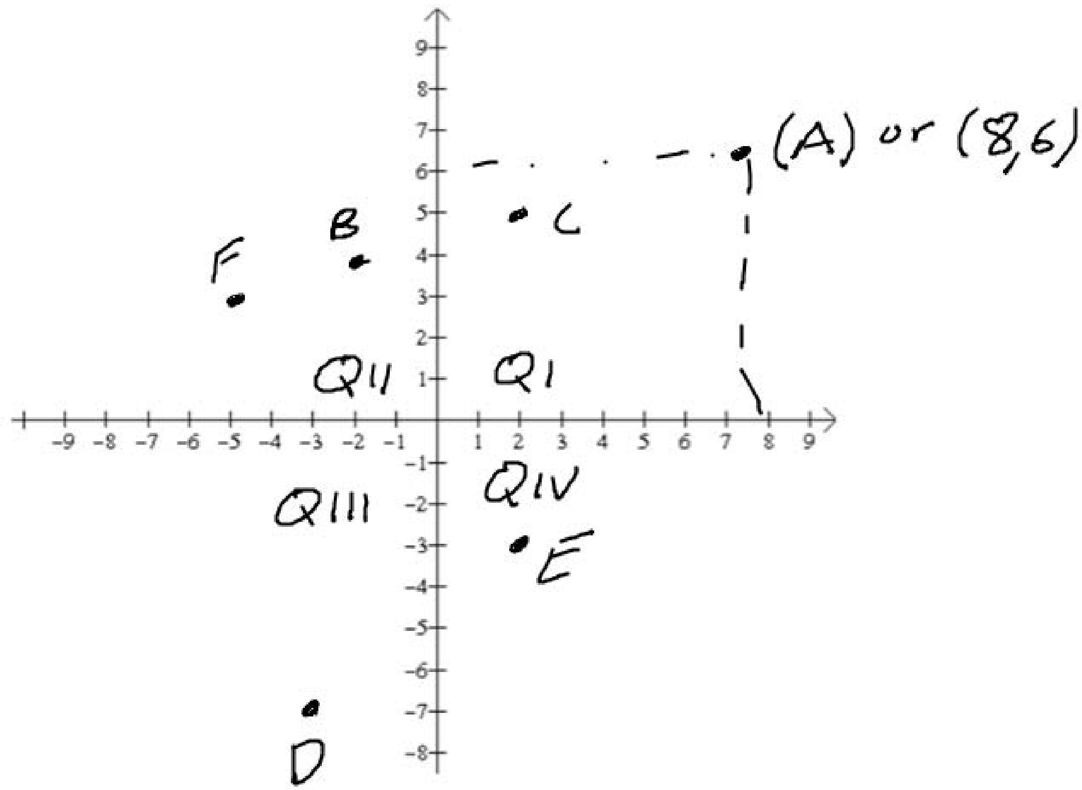
The ordered pair (0, 0) is referred to as the origin.
(where the axis intersect)

The x coordinate tells us the horizontal distance a point is from the origin. The y coordinate tells us the vertical distance a point is from the origin. You'll move right or up for positive coordinates and left or down for negative coordinates.

Section 1.1: Points, Regions, Distance and Midpoints

Example: Plot the following points.

A. (8,6) R8 U6 QI
B. (-2,4) L2 U4 QII
C. (2,5) R2 U5 QI
D. (-3,-7) L3 D7 QIII
E. (2,-3) R2 D3 QIV
F. (-5,3) L5 U3 QII



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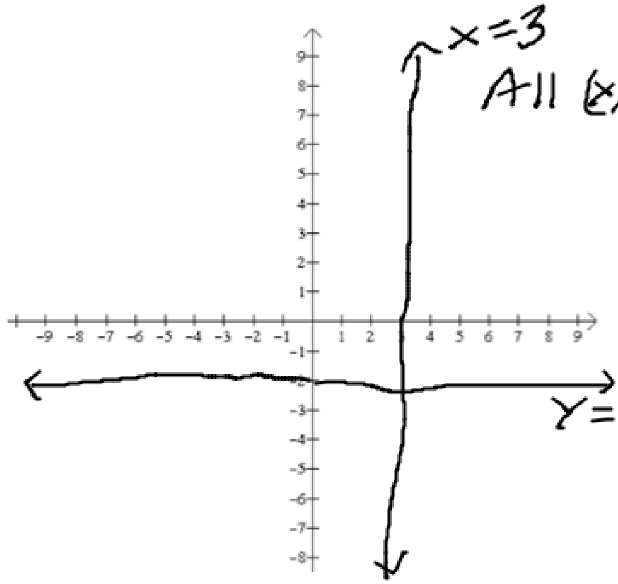
Graphing Regions in the Coordinate Plane

The set of all points in the coordinate plane with y coordinate k is the **horizontal line $y = k$** $x = 3$
The set of all points in the coordinate plane with x coordinate k is the **vertical line $x = k$**

$Y = -2$



All (x, y) would be (3, y)

$Y = -2$: All (x, y) would be (x, -2)

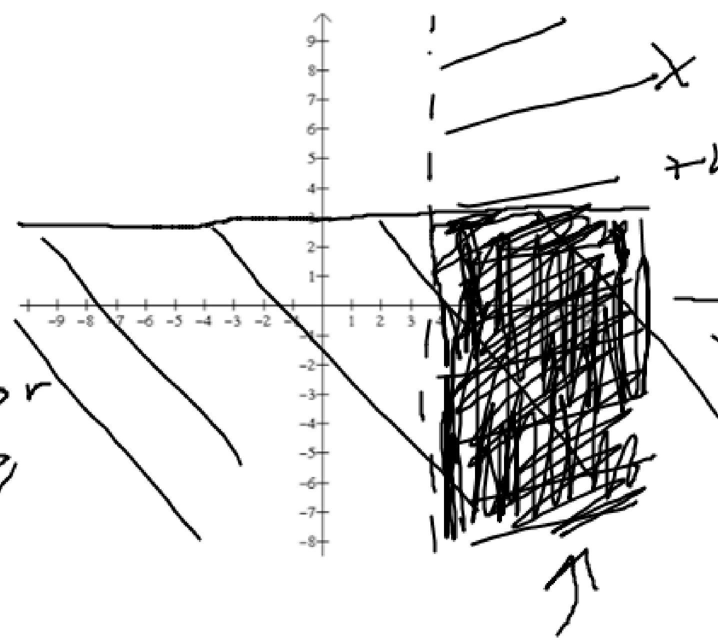
Graphing Regions in the Coordinate Plane

Graphing Regions in the Coordinate Plane

Example: Graph $\{(x, y) \mid x > 4 \text{ and } y \leq 3\}$.

Dotted Lines: < or >
Boundarys
But Not
equality

Solid Line: ≤ or ≥
Boundaries
and equality



$x > 4$:
x is greater than (Not equal to) 4

$y \leq 3$
y is less or equal to 3

S (Solution)

Graphing Regions in the Coordinate Plane

The Distance Formula

For any two points (x_1, y_1) and (x_2, y_2), the distance between them is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Square root of the difference of x-vals^2 and difference of y-vals^2.

Example: Find the distance between the following pair of points.

a) $(-3,1)$ & $(1,3)$

(x_1, y_1) (x_2, y_2)

$x_1 = -3$ $\quad x_2 = 1$

$y_1 = 1$ $\quad y_2 = 3$

$$d = \sqrt{(1 - {-3})^2 + (3-1)^2}$$

$1+3$

$$d = \sqrt{4^2 + 2^2} = \sqrt{16+4} = \sqrt{20}$$

$\sqrt{4}\ \sqrt{5}$

$\boxed{2\sqrt{5}}$

b) $(2\sqrt{3}, 5\sqrt{6})$ & $(-\sqrt{3}, \sqrt{6})$

$$d = \sqrt{(-1\sqrt{3} - 2\sqrt{3})^2 + (\sqrt{6} - 5\sqrt{6})^2}$$

$$= \sqrt{(-3\sqrt{3})^2 + (-4\sqrt{6})^2}$$

$$= \sqrt{(9 \cdot 3) + (16 \cdot 6)} = \sqrt{27 + 96} = \boxed{\sqrt{123}}$$

The Midpoint Formula

Averaging the x-vals and averaging the y-values

Midpoint Formula

The midpoint of the line segment joining the two points (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Example: Find the midpoint between the following pair of points.

a) $(-3,1)$ & $(1,3)$

$$M = \left(\frac{1 + {}^-3}{2}, \frac{1+3}{2}\right) = \left(\frac{-2}{2}, \frac{4}{2}\right) = (-1, 2)$$

b) $(2\sqrt{3}, 5\sqrt{6})$ & $(-\sqrt{3}, \sqrt{6})$

$$M = \left(\frac{-\sqrt{3} + 2\sqrt{3}}{2}, \frac{\sqrt{6} + 5\sqrt{6}}{2}\right) = \left(\frac{\sqrt{3}}{2}, \frac{6\sqrt{6}}{2}\right) = \left(\frac{\sqrt{3}}{2}, 3\sqrt{6}\right)$$

The Midpoint Formula

Lines

In this section, we'll review slope and different equations of lines. We will also talk about x-intercept and y-intercept, parallel and perpendicular lines.

Straight Lines

Horizontal Lines: $y = k$

Vertical Lines: $x = k$

Diagonal Lines: $y = mx + b$ (slope-int)

$y - y_1 = m(x - x_1)$ (point-slope)

$ax + by = c$ (Standard)

$\frac{x}{a} + \frac{y}{b} = 1$ (int-int)

Slope

Definition: The **slope** of a line measures the steepness of a line or the rate of change of the line.

To find the slope of a line you need two points. You can find the slope of a line between two points (x_1, y_1) and (x_2, y_2) by using this formula.

$$slope\ (m) = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \frac{\text{change of } y}{\text{change of } x}$$

Note:

-Lines with positive slope rise to the right.

-Line with negative slope fall to the right.

-Lines with slope equal to 0 are horizontal lines.

-Lines with undefined slope are vertical lines

Example 1: Find the slope of the line containing the following points

a. (4, -3) and (-2, 1)

$$m = \frac{1 - {}^{-}3}{-2 - 4} = \frac{4}{-6} = \frac{-2}{3}$$: Downhill

b. (-3, 1) and (-3, -2)

$$m = \frac{-2 - 1}{-3 - {}^{-}3} = \frac{-3}{0} = \text{undefined}$$

vertical line

Slope

Finding the Equation of a Line

Three usual forms:

1. **Point-Slope Form**
   $(y-y_1) = m(x-x_1)$ where (x_1, y_1) is a point on the line and m is the slope.

2. **Slope-Intercept Form**
   $y = mx + b$ where m is the slope and b is the y-intercept of the line.

3. **Standard Form**
   $Ax + By + C = 0$ where A and B are not both equal to 0.

4. $\frac{x}{a} + \frac{y}{b} = 1$ a is the x-int. b is the y-int

Finding the Equation of a Line

Example 2: Write the following equation in slope-intercept form and identify the slope and y-intercept.

$2x - 4y = 5$

$$y = \underline{m}x + \underline{b}$$

$$\begin{aligned} \cancel{2x} - 4y &= 5 \\ -2x \quad &\quad -2x \end{aligned}$$

$$\frac{-4y}{-4} = \frac{-2x}{-4} + \frac{5}{-4}$$

$$y = \frac{-2}{-4}x + \frac{5}{-4}$$

$$y = \underbrace{\frac{1}{2}}_{m}x - \underbrace{\frac{5}{4}}_{b}$$

slope: $m = \frac{1}{2}$

y-int: $b = \frac{-5}{4}$

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Example 3: Write an equation of the line that satisfies the given conditions.

a. $m = \frac{1}{2}$ and the y-intercept is 3.

$$y = mx + b$$

$$y = \frac{1}{2}x + 3$$

b. $m = -3$ and the line passes through (-2, 1).

$$(y - y_1) = m(x - x_1)$$

$$y - 1 = -3(x - {-2})$$

$$y - 1 = -3(x + 2)$$

$$y - 1 = -3(x + 2)$$

$$y - 1 = -3x - 6$$

$$+1 \qquad\qquad +1$$

$$y = -3x - 5$$

c. line passes through (-6, 10) and (-2, 2).

$$m = \frac{2 - 10}{-2 - {-6}} = \frac{-8}{4} = -2$$

$y - 10 = -2(x + 6)$ using $(-6, 10)$

or

$y - 2 = -2(x + 2)$ using $(-2, 2)$

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$$Y-10=-2(x+6)$$
$$Y-\cancel{10}=-2x-12$$
$$\underline{\quad+\cancel{10}\qquad\qquad+10\quad}$$
$$Y=-2x-2$$

$$Y-2=-2(x+2)$$
$$Y-\cancel{2}=-2x-4$$
$$\underline{\quad+\cancel{2}\qquad\qquad+2\quad}$$
$$Y=-2x-2$$

Parallel and Perpendicular Lines

Definition: Parallel lines are lines with slopes m_1 and m_2 such that they are equal, in other words

$$m_1 = m_2$$

Never intersect

Definition: Perpendicular lines are lines in which the product of the slopes equal -1.

$$m_1 m_2 = -1$$

Lines that intersect at right angles

Also known as the negative reciprocal. $m_2 = \frac{-1}{m_1}$

$y = 5x - 3$

and

$y = 5x + 2$

are parallel

$\parallel$

$y = 5x - 3$

and

$y = \frac{-1}{5}x + 2$

are perpendicular

$\perp$

Parallel and Perpendicular Lines

Example 4: Write an equation of the line that passes through the points (-3, 8) and parallel to $y = -2x + 4$

parallel to $y = -2x + 4$ $m = -2$

$y = mx + b$

$y = -2x + b$

$8 = -2(-3) + b$

$8 = \cancel{6} + b$

$-6 \quad \cancel{-6}$

$2 = b$

$\boxed{y = -2x + 2}$

Example 5: Write an equation of the line that passes through the points (1, 2) and perpendicular to $y = -2x + 4$.

perpendicular to $y = -2x + 4$

(new line) $m = \frac{1}{2}$ (Negative Reciprocal of -2)

$\boxed{y - 2 = \frac{1}{2}(x - 1)}$

$y - 2 = \frac{1}{2}x - \frac{1}{2}$

$+2 \qquad +2$

$\boxed{y = \frac{1}{2}x + \frac{3}{2}}$

$-\frac{1}{2} + 2$

$-\frac{1}{2} + \frac{2}{1}$

$-\frac{1}{2} + \frac{4}{2}$

$\frac{3}{2}$

x-intercept and y-intercept

When graphing an equation, it is usually very helpful to find the ***x* intercept(s)** and the ***y* -intercepts** of the graph. An x intercept is the first coordinate of the ordered pair of a point where the graph of the equation crosses the x axis. To find an x intercept, let $y = 0$ and solve the equation for x.

The ***y*-intercept** is the second coordinate of the ordered pair of a point where the graph of the equation crosses the y axis. To find a y intercept, let $x = 0$ and solve the equation for y.

General Rule for finding intercepts:

Set the opposite variable equal to zero.

Solve for the remaining variable.

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Example 5: Find the x and y intercepts of the graph of the equation $3x - 4y = 8$.

Question 1:
x-intercept: a. (-2,0) b. (0.5, 0) c. (8/3, 0) d. (3/8, 0)

Question 2:
y-intercept: a. (0, 2) b. (0, -2) c. (0, 4) d. (0, -4)

when you take a square root, you get ± answers.

Example 6: Find the x and y intercepts of the graph of the equation $y = x^2 - 9$.

Question 3:
x-intercept: a. (3,0) b. (4.5, 0) c. (±3, 0) d. (±4.5, 0)

Question 4:
y-intercept: a. (0, -9) b. (0, 9) c. (0, ±9) d. (0, ±2)

Graphing Equations

Yes No No

Example 1: Determine which of the points (3, 2), (-1, 3) and (0, 2) are on the graph of the equation $4x - 3y = 6$.

$4x - 3y = 6$

Test (3,2)

$4(3) - 3(2) = 6$

$12 - 6 = 6$

$6 = 6$ ✓

Test (-1,3)

$4(-1) - 3(3) = 6$

$-4 - 9 = 6$

$-13 \neq 6$

Test (0,2)

$4(0) - 3(2) = 6$

$0 - 6 = 6$

$-6 \neq 6$

✓ ✓ ✗

Example 2: Determine which of the points (-1, 1), (2, -1) and (-2, -1) are on the graph of the equation $x^2 + 3xy + 2 = 0$

(-1, 1)

$(-1)^2 + 3(-1)(1) + 2 = 0$

$1 - 3 + 2 = 0$

$0 = 0$ ✓

(2, -1)

$2^2 + 3(2)(-1) + 2 = 0$

$4 - 6 + 2 = 0$

$0 = 0$ ✓

(-2, -1)

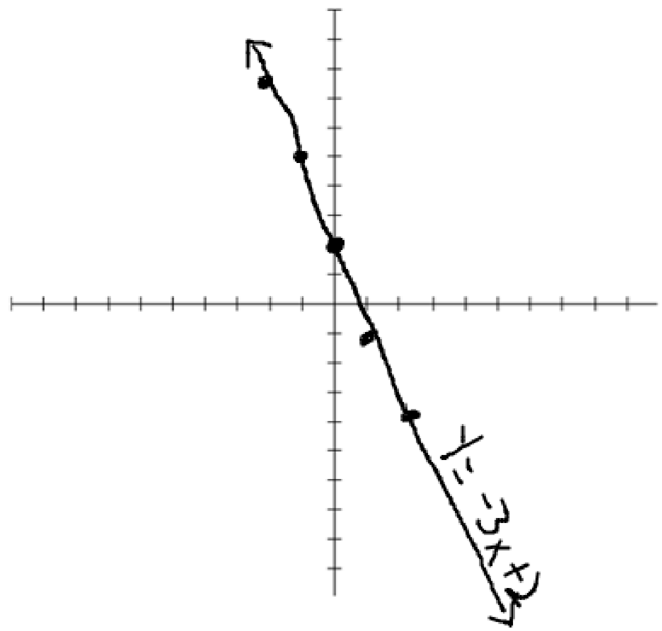
$(-2)^2 + 3(-2)(-1) + 2 = 0$

$4 + 6 + 2 = 0$

$12 \neq 0$

When we graphing an equation, it will be helpful to have more points than just the x and y intercepts of the graph. We can create a table of values with more choices for x and find the corresponding y values.

Example 3: Sketch the graph of the equation by plotting points: $y = -3x + 2$.



x	Y = -3x + 2	Y	(x, y)
-2	-3(-2)+2	8	(-2, 8)
-1	-3(-1)+2	5	(-1, 5)
0	-3(0)+2	2	(0, 2)
1	-3(1)+2	-1	(1, -1)
2	-3(2)+2	-4	(2, -4)

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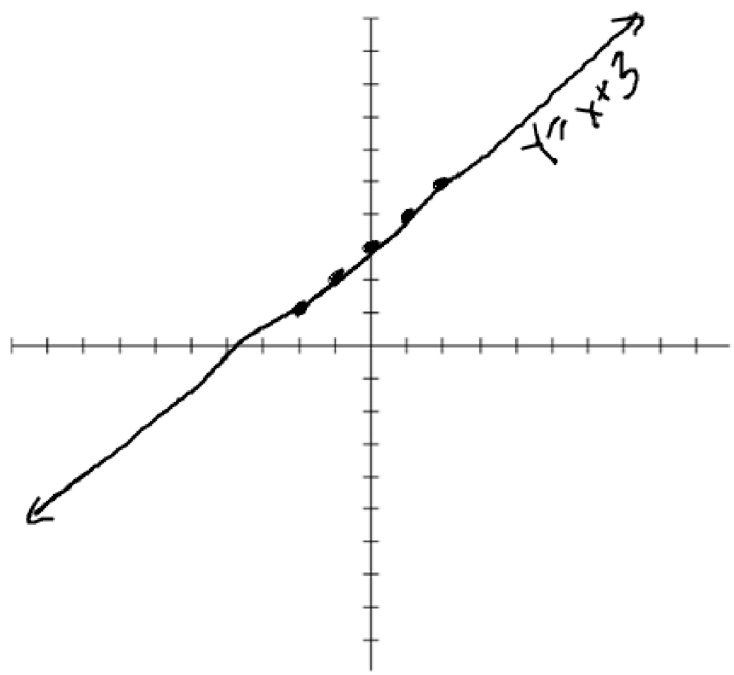
Example 4: Sketch the graph of the equation by plotting points: $y = x + 3$



x	$y = x + 3$	y	(x, y)
-2	$-2 + 3$	1	$(-2, 1)$
-1	$-1 + 3$	2	$(-1, 2)$
0	$0 + 3$	3	$(0, 3)$
1	$1 + 3$	4	$(1, y)$
2	$2 + 3$	5	$(2, 5)$

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