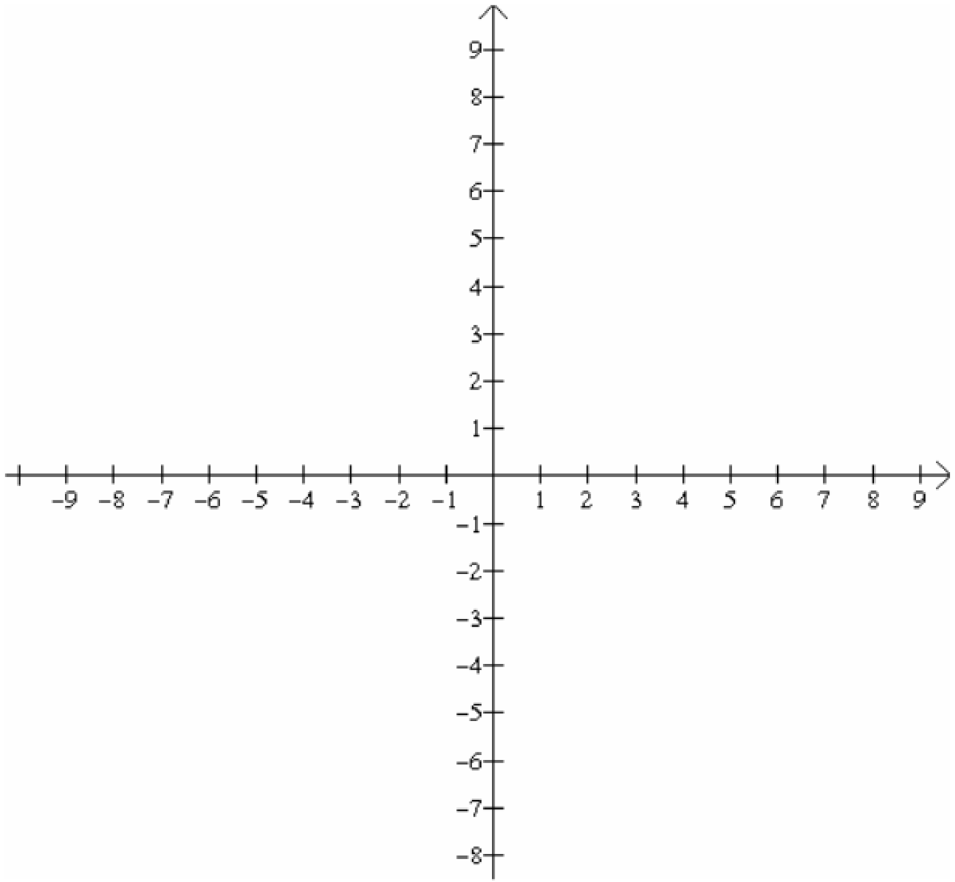
MATH 1310

Session 1

Section 1.1: Points, Regions, Distance and Midpoints

Graphing Points and Regions

Here's the coordinate plane:



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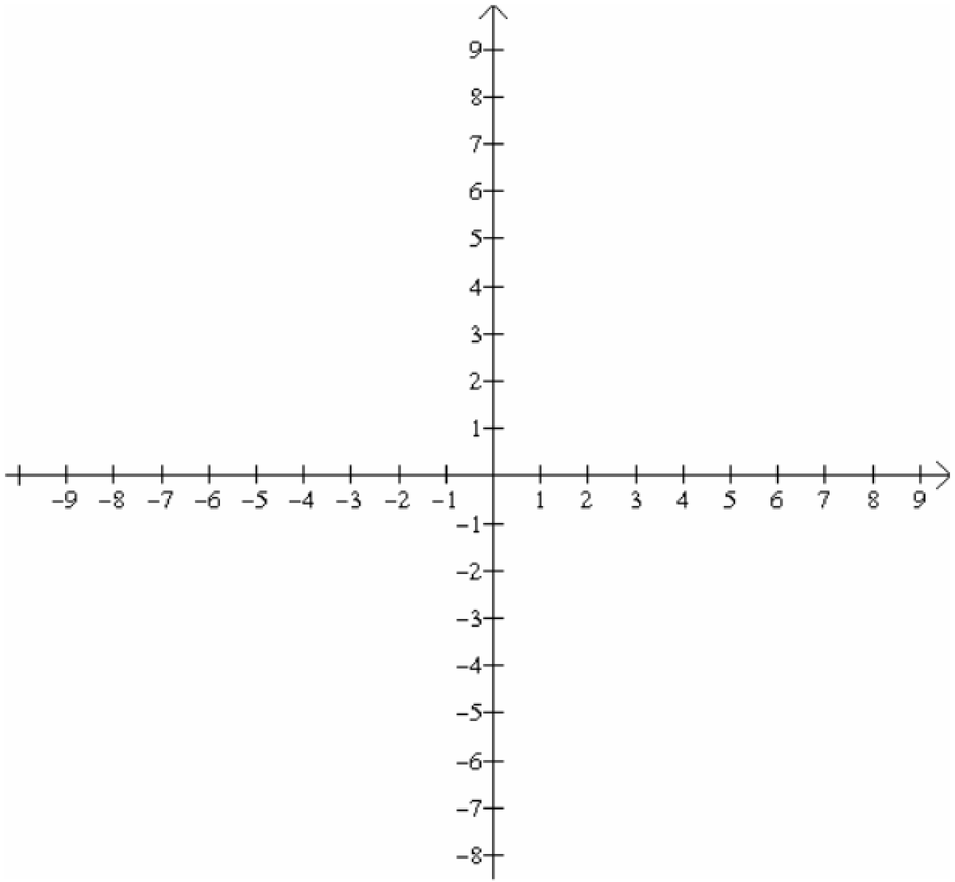


As we see the plane consists of two perpendicular lines, the x-axis and the y-axis. These two lines separate them into four regions, or quadrants.

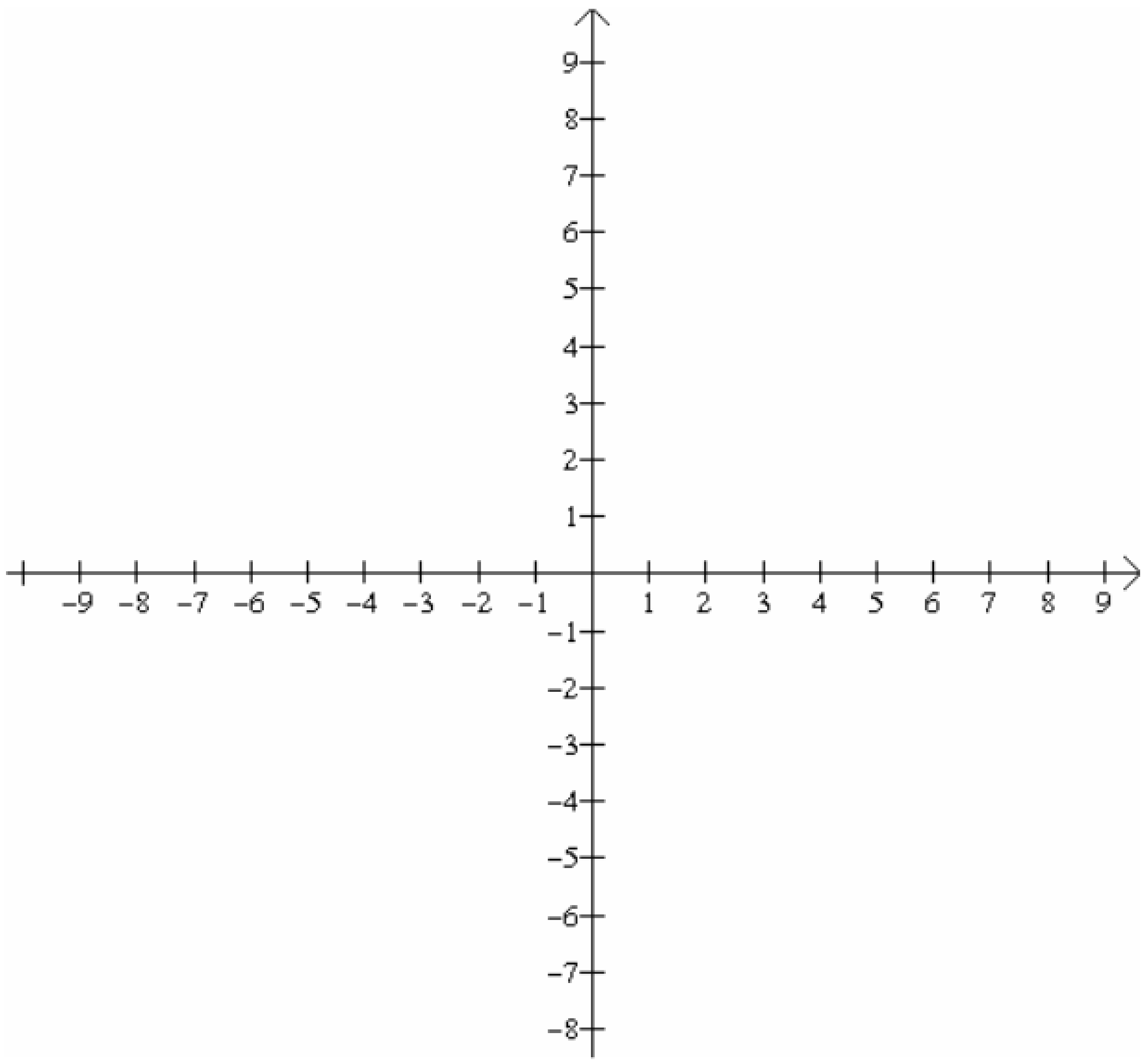
The pair, (x, y), is called an ordered pair. It corresponds to a single unique point in the coordinate plane. The first number is called the x coordinate, and the second number is called the y coordinate.

The ordered pair (0, 0) is referred to as the origin.

The x coordinate tells us the horizontal distance a point is from the origin. The y coordinate tells us the vertical distance a point is from the origin. You'll move right or up for positive coordinates and left or down for negative coordinates.

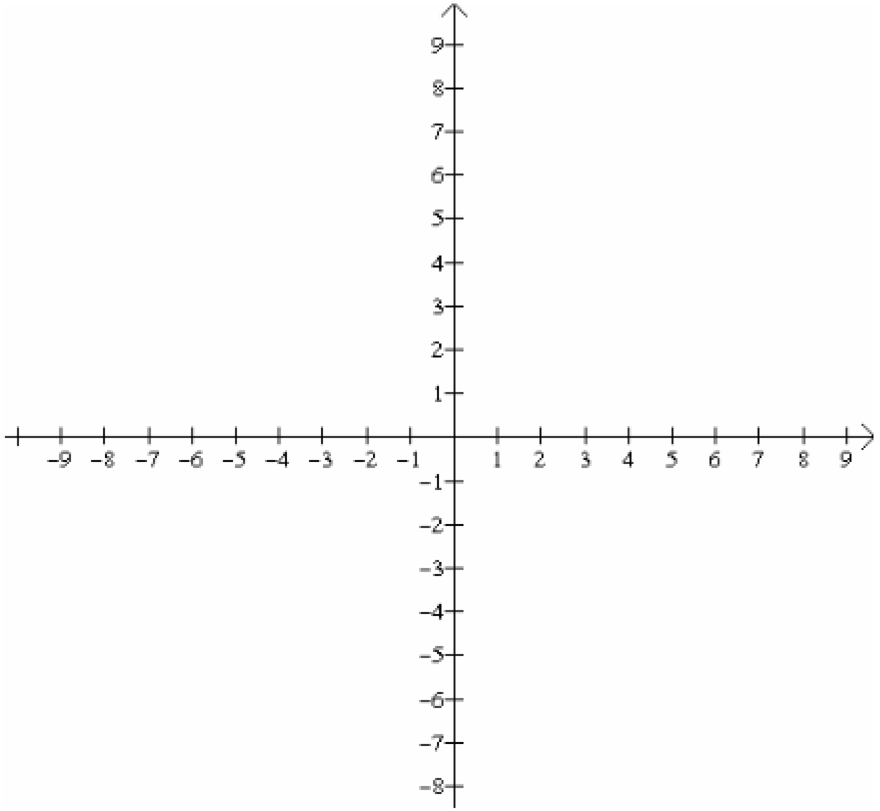
Example: Plot the following points.

A. (8,6)
B. (-2,4)
C. (2,5)
D. (-3,-7)
E. (2,-3)
F. (-5,3)



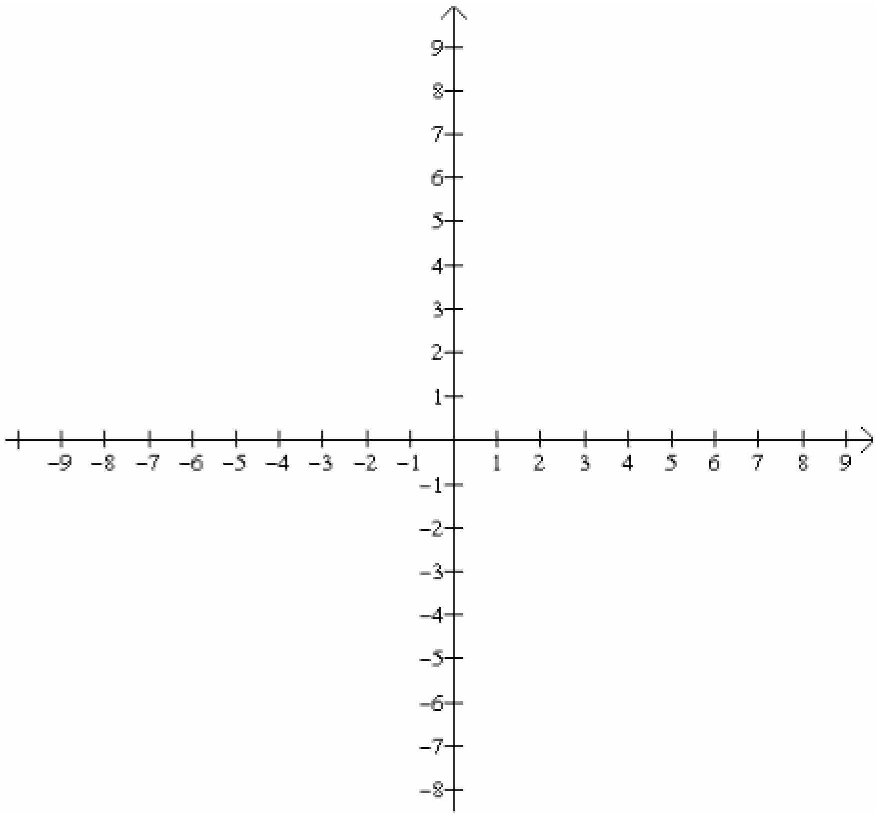
Graphing Regions in the Coordinate Plane

The set of all points in the coordinate plane with y coordinate k is the **horizontal line** $\boldsymbol{y = k}$.
The set of all points in the coordinate plane with x coordinate k is the **vertical line** $\boldsymbol{x = k}$.



Graphing Regions in the Coordinate Plane

Example: Graph $\{(x, y) \mid x > 4 \text{ and } y \leq 3\}$.



The Distance Formula

For any two points (x_1, y_1) and (x_2, y_2), the distance between them is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Find the distance between the following pair of points.

a) $(-3,1)$ & $(1,3)$

b) $(2\sqrt{3}, 5\sqrt{6})$ & $(-\sqrt{3}, \sqrt{6})$

The Midpoint Formula

Midpoint Formula

The midpoint of the line segment joining the two points (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

Example: Find the midpoint between the following pair of points.

a) $(-3,1)$ & $(1,3)$

b) $(2\sqrt{3}, 5\sqrt{6})$ & $(-\sqrt{3}, \sqrt{6})$

Lines

In this section, we'll review slope and different equations of lines. We will also talk about x-intercept and y-intercept, parallel and perpendicular lines.

Slope

Definition: The **slope** of a line measures the steepness of a line or the rate of change of the line.

To find the slope of a line you need two points. You can find the slope of a line between two points (x_1, y_1) and (x_2, y_2) by using this formula.

$$slope\ (m) = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 1: Find the slope of the line containing the following points

a. (4, -3) and (-2, 1)

b. (-3, 1) and (-3, -2)

Note:

-Lines with positive slope rise to the right.

-Line with negative slope fall to the right.

-Lines with slope equal to 0 are horizontal lines.

-Lines with undefined slope are vertical lines

Finding the Equation of a Line

Three usual forms:

1. **Point-Slope Form**
   $(y-y_1) = m(x-x_1)$ where (x_1, y_1) is a point on the line and m is the slope.

2. **Slope-Intercept Form**
   $y = mx + b$ where m is the slope and b is the y-intercept of the line.

3. **Standard Form**
   $Ax + By + C = 0$ where A and B are not both equal to 0.

Example 2: Write the following equation in slope-intercept form and identify the slope and y-intercept.

$2x - 4y = 5$

Example 3: Write an equation of the line that satisfies the given conditions.

a. $m = \frac{1}{2}$ and the y-intercept is 3.

b. $m = -3$ and the line passes through (-2, 1).

c. line passes through (-6, 10) and (-2, 2).

Parallel and Perpendicular Lines

Definition: Parallel lines are lines with slopes m_1 and m_2 such that they are equal, in other words

$$m_1 = m_2$$

Definition: Perpendicular lines are lines in which the product of the slopes equal -1.

$$m_1 m_2 = -1$$

Also known as the negative reciprocal. $m_2 = \frac{-1}{m_1}$

Example 4: Write an equation of the line that passes through the points (-3, 8) and parallel to $y = -2x + 4$

Example 5: Write an equation of the line that passes through the points (1, 2) and perpendicular to $y = -2x + 4$.

x-intercept and y-intercept

When graphing an equation, it is usually very helpful to find the ***x* intercept(s)** and the ***y* -intercepts** of the graph. An *x* intercept is the first coordinate of the ordered pair of a point where the graph of the equation crosses the *x* axis. To find an *x* intercept, let $y = 0$ and solve the equation for *x*.

The ***y*-intercept** is the second coordinate of the ordered pair of a point where the graph of the equation crosses the *y* axis. To find a *y* intercept, let $x = 0$ and solve the equation for *y*.

Popper 1:

Example 5: Find the x and y intercepts of the graph of the equation $3x - 4y = 8$.

Question 1:
x-intercept: a. (-2,0) b. (0.5, 0) c. (8/3, 0) d. (3/8, 0)

Question 2:
y-intercept: a. (0, 2) b. (0, -2) c. (0, 4) d. (0, -4)

Example 6: Find the x and y intercepts of the graph of the equation $y = x^2 - 9$.

Question 3:
x-intercept: a. (3,0) b. (4.5, 0) c. (±3, 0) d. (±4.5, 0)

Question 4:
y-intercept: a. (0, -9) b. (0, 9) c. (0, ±9) d. (0, ±2)

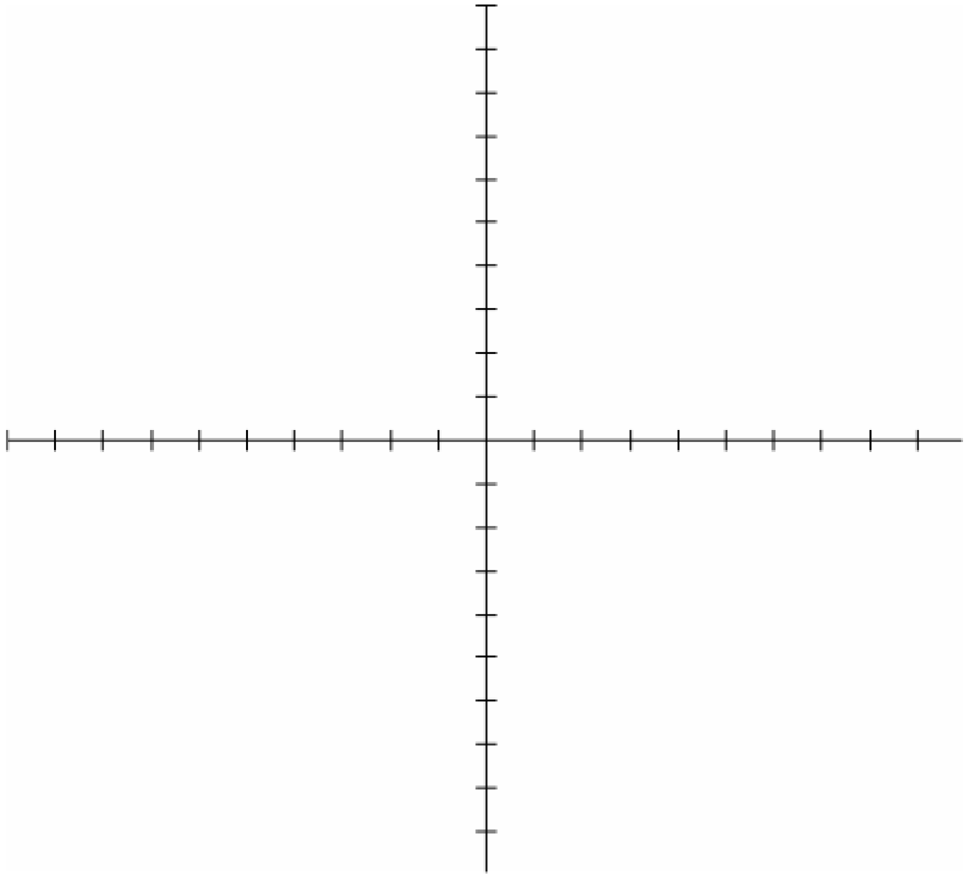
Graphing Equations

Example 1: Determine which of the points (3, 2), (-1, 3) and (0, 2) are on the graph of the equation $4x - 3y = 6$.

Example 2: Determine which of the points (-1, 1), (2, -1) and (-2, -1) are on the graph of the equation $x^2 + 3xy + 2 = 0$

When we graphing an equation, it will be helpful to have more points than just the *x* and *y* intercepts of the graph. We can create a table of values with more choices for *x* and find the corresponding *y* values.

Example 3: Sketch the graph of the equation by plotting points: $y = -3x + 2$.



Example 4: Sketch the graph of the equation by plotting points: $y = x + 3$