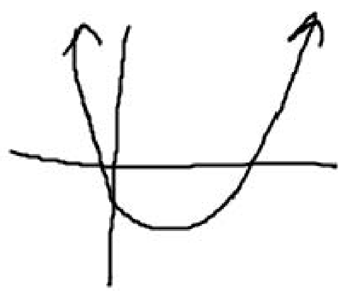
MATH 1310

Session 5

Maximum and Minimum Values

A quadratic equation is of the form $f(x) = ax^2 + bx + c$, where a, b, and $c$are real and $a \neq 0$

We have seen the graphs of **parabolas**. $D: (-\infty, \infty)$



Maximum and Minimum Values

Opening Up or Opening Down

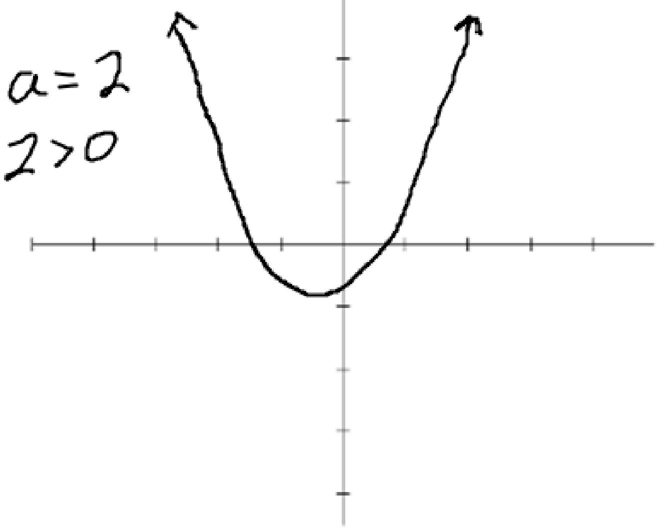
the sign of the x^2 term.

If $a > 0$ then the parabola will open upwards.

$y = 2x^2 + 3x - 1$ open upwards

$a = 2$

$2 > 0$



If $a < 0$ then the parabola will open downwards.

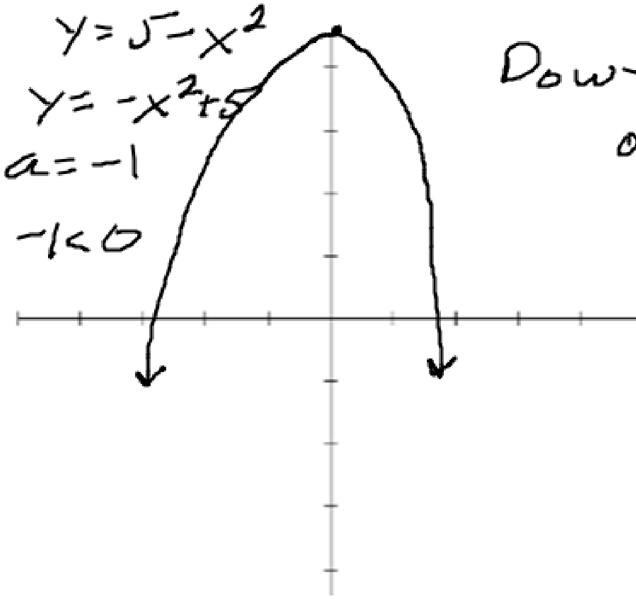
$y = 5 - x^2$

$y = -x^2 + 5$

$a = -1$

$-1 < 0$

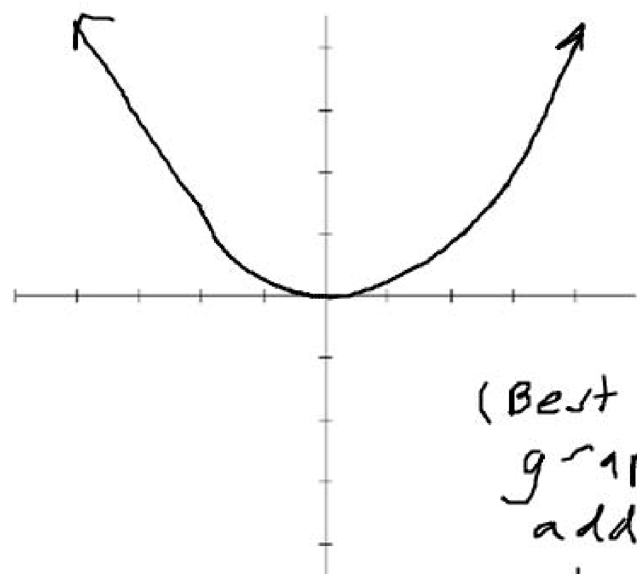
Downward opening



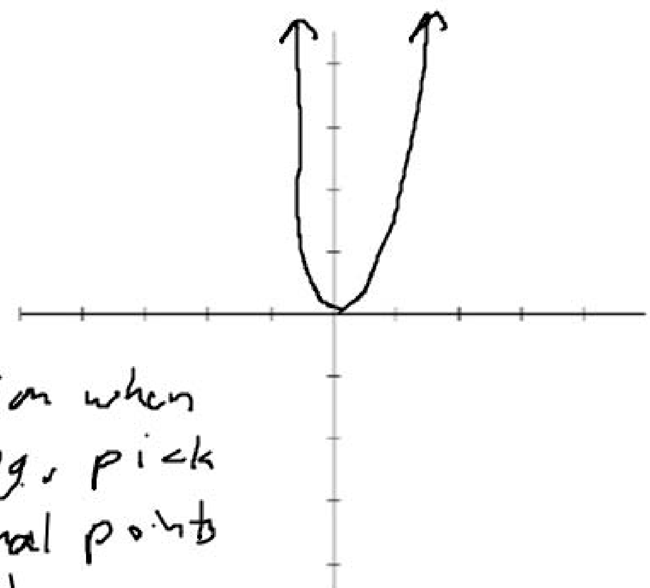
Opening Up or Opening Down

Note: The larger $|a|$, the narrower the parabola

$y = \frac{1}{2}x^2$



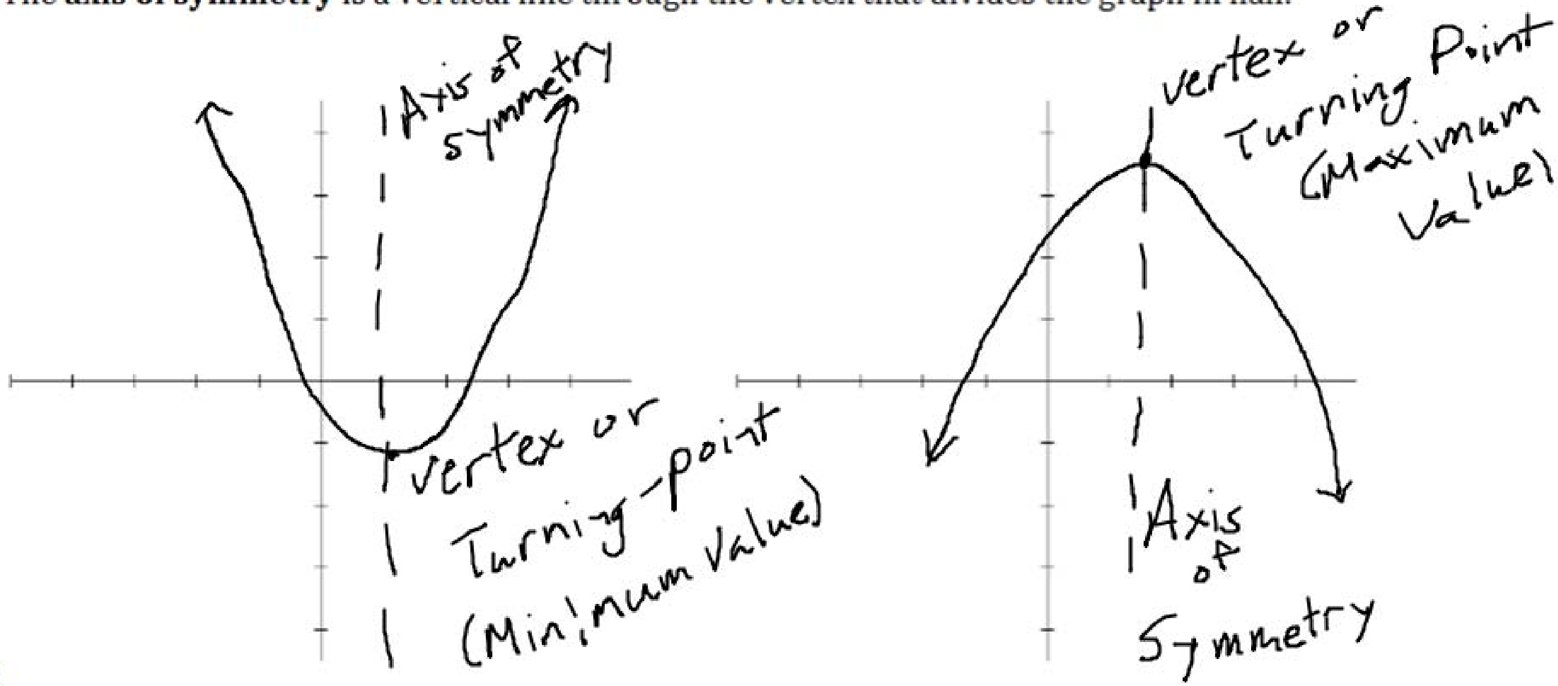
$y = 3x^2$



(Best option when graphing, pick additional points to plot)

The **vertex** is the turning point of the parabola and is the **minimum point** on the graph when it opens upward and the **maximum point** on the graph when it opens downward. Every parabola has a maximum or minimum, but **NOT** both.

The **axis of symmetry** is a vertical line through the vertex that divides the graph in half.



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The Standard form of a Quadratic Function

The quadratic function $f(x) = a(x-h)^2 + k$ is in **standard form**

The vertex is the point $(h,\ k)$ and the axis of symmetry is $x = h$

The domain is $(-\infty, \infty)$.

The range is $[k,\ \infty)$ if $a > 0$ or $(-\infty,\ k]$ if $a < 0$

$y = 2(x-3)^2 + 1$

$y = a(x-h)^2 + k$

$h = 3 \quad k = +1$

vertex: $(3, 1)$

Axis: $x = 3$

Range: $[1, \infty)$

$y = -3(x+2)^2 + 5$

$h = -2 \quad k = 5$

vertex: $(-2, 5)$

Axis: $x = -2$

Range: $(-\infty, 5]$

Our first task will be to change a given quadratic function from the form $f(x) = ax^2 + bx + c$ to standard form. We'll complete the square to do this. Once the function is in standard form, we can sketch a graph using transformations and then read off the maximum or minimum value

$$f(x) = 2x^2 + 12x + 5$$

$$f(x) = 2(x^2 + 6x) + 5$$

$$b = 6$$

$$\left(\frac{b}{2}\right) = 3$$

$$\left(\frac{b}{2}\right)^2 = 9$$

$$f(x) = \underline{2}(x^2 + 6x + \underline{9}) + 5 - 2(9)$$

$$f(x) = 2(x^2 + 6x + 9) - 13$$

$$f(x) = 2(x+3)^2 - 13$$

$h = -3$ $k = -13$

vertex: $(-3, -13)$

Axis: $x = -3$

Domain $(-\infty, \infty)$

Range: $[-13, \infty)$

Example 1: Write the following quadratic in standard form. Then find the vertex and the axis of symmetry.

a. $f(x) = (3x^2 - 12x) - 1$

$3(x^2 - 4x) - 1$

$b = -4$

$\frac{b}{2} = -2$

$\left(\frac{b}{2}\right)^2 = 4$

$3(x^2 - 4x + 4) - 1 - 3(4)$

$f(x) = 3(x-2)^2 - 13$

$h = 2$, $k = -13$

vertex: $(2, -13)$

Axis of symmetry: $x = 2$

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b. $f(x) = (-x^2 + 2x) + 3$

$-(x^2 - 2x) + 3$

$b = -2$

$\left(\frac{b}{2}\right) = -1$

$\left(\frac{b}{2}\right)^2 = 1$

$-(x^2 - 2x + 1) + 3 - (-1)(1)$

$-(x-1)^2 + 4$

$f(x) = -(x-1)^2 + 4$

vertex: $(1, 4)$

Axis of Symmetry: $x = 1$

c. $f(x) = -10x^2 + 60x$

$-10(x^2 - 6x)$

$b = -6$

$\frac{b}{2} = -3$

$\left(\frac{b}{2}\right)^2 = 9$

$-10(x^2 - 6x + 9) - (-10)(9)$

$f(x) = -10(x-3)^2 + 90$

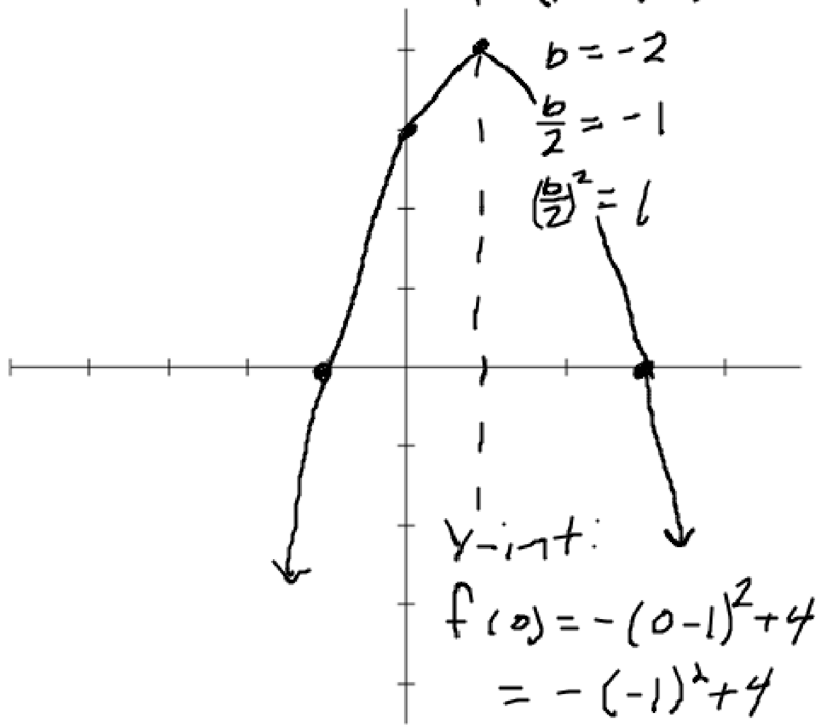
vertex: $(3, 90)$

Axis: $x = 3$

Graphing Quadratic Functions with Equations in Standard Form

1. Determine whether the parabola opens upward or downward. $a > 0$ or $a < 0$
2. Determine the vertex. (h, k)
3. Find any x-intercept by replacing $f(x)$ with 0 and then solving for x. $y = 0$, then solve.
4. Find the y-intercept by replacing x with 0. $x = 0$, then solve
5. Plot the intercept(s) and vertex, sketch the graph and draw the axis of symmetry.

Example 2: Sketch the graph of $f(x) = (-x^2 + 2x) + 3$



$-(x^2 - 2x) + 3$

$b = -2$

$\frac{b}{2} = -1$

$\left(\frac{b}{2}\right)^2 = 1$

Y-int:

$f(0) = -(0-1)^2 + 4$

$= -(-1)^2 + 4$

$= -1 + 4 = 3$

$(0, 3)$

$-(x^2 - 2x + 1) + 3 - (-1)(1)$

$f(x) = -(x-1)^2 + 4$

vertex: $(1, 4)$

Axis: $x = 1$

Domain: $(-\infty, \infty)$

opening down

Range: $(-\infty, 4]$

$0 = -(x-1)^2 + 4$

$-4 = -(x-1)^2$

$\pm\sqrt{4} = \sqrt{(x-1)^2}$

$\pm 2 = x - 1$

$x = 1 \pm 2$

$(3, 0), (-1, 0)$

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Shortcut:

For $f(x) = ax^2 + bx + c$, the vertex is $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$. So the axis of symmetry is $x = -\frac{b}{2a}$.

$f(x) = \underline{\underline{2}}x^2 - 24x + 5$

$a = 2$

$b = -24$

$c = 5$

vertex $(6, -67)$

$x = \frac{-b}{2a} = \frac{24}{4} = 6$

$f(x) = 2(x-6)^2 - 67$

$f(6) = 2(6)^2 - 24(6) + 5$

$= 2(36) - 24(6) + 5$

$= 72 - 144 + 5$

$= -72 + 5 = -67$

Example 3: Let $f(x) = 2x^2 + 4x + 7$. Determine, without graphing, whether the given quadratic function has a minimum or maximum value. Then find the coordinates of the minimum or maximum point

$a = 2$: opens upward : vertex is a minimum point

$$x = \frac{-b}{2a} = \frac{-4}{2(2)} = \frac{-4}{4} = -1$$

min point: $(-1, 5)$

$$f(-1) = 2(-1)^2 + 4(-1) + 7$$

$$= 2(1) - 4 + 7$$

$$= 2 - 4 + 7$$

$$f(-1) = 5$$

vertex: $(-1, 5)$

Example 4: Suppose $f(x) = 5x^2 - 30x + 41$. Write the equation in standard form. State the coordinates of the vertex. Determine, without graphing, whether the given quadratic function has a minimum or maximum value. Then find the coordinates of the minimum or maximum point.

$x = \frac{-b}{2a} = \frac{30}{10} = 3$

$f(3) = 5(3)^2 - 30(3) + 41$

$f(3) = 45 - 90 + 41$

$f(3) = -45 + 41 = -4$

vertex: $(3, -4)$

$f(x) = 5(x-3)^2 - 4$

$a = 5 > 0$: opens up

vertex is a min.

Finally, given the vertex of a quadratic function and one other point that lies on the graph of the quadratic function, you should be able to write the quadratic function.

Example 5: Find a quadratic function with vertex (2, 6) which passes through (-1, 4).

$h = 2$

$k = 6$

plug in for (x, y)

$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x-2)^2 + 6$$

$$4 = a(-1-2)^2 + 6$$

$$4 = a(-3)^2 + 6$$

$$4 = 9a + 6$$

$$4 = 9a + \cancel{6}$$

$$-6 \qquad \cancel{-6}$$

$$\frac{-2}{9} = \frac{\cancel{9}a}{\cancel{9}}$$

$$-2/9 = a$$

$$y = -\frac{2}{9}(x-2)^2 + 6$$

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Example 6: Find a quadratic function with vertex (3, -1) which passes through (5, 7).

$$Y = a(x-h)^2 + k$$

$$Y = a(x-3)^2 - 1 \qquad [\text{Plug in } h=3,\ k=-1]$$

$$7 = a(5-3)^2 - 1 \qquad [\text{Plug in } x=5,\ y=7]$$

$$7 = a(2)^2 - 1 \qquad [\text{Solve for } a]$$

$$7 = 4a \cancel{-1}$$

$$+1 \qquad \cancel{+1}$$

$$\frac{8}{4} = \frac{\cancel{4}a}{\cancel{4}}$$

$$2 = a$$

$$Y = 2(x-3)^2 - 1$$

Popper 5: $f(x) = x^2 + 2x - 3$

1. Determine the equation of the axis of symmetry:

    a. $x = -1$ b. $x = -4$ c. $x = 2$ d. $x = -½$

2. Determine the coordinates of the vertex:

    a. (-1, -3) b. (1, 4) c. (-1, -4) d. (2, -3)

3. Determine the coordinates of the y-intercept:

    a. (0,3) b. (0,2) c. (0,-3) d. (0,-4)

Popper 5: f(x) = x2 + 2x – 3

Popper 5: $f(x) = x^2 + 2x - 3$

4. Determine the coordinates of the left x-intercept:

   a. (3,0) b. (-3,0) c. (2,0) d. (-1,0)

5. Determine the coordinates of the right x-intercept:

   a. (3,0) b. (4,0) c. (1,0) d. (5,0)

6. Determine the direction of the parabola:

   a. Opening Up b. Opening Down

7. Is the vertex a minimum or a maximum?

   a. Minimum b. Maximum

Popper 5: f(x) = x2 + 2x – 3

Combining Functions:

Suppose we have two functions $f(x)$ and $g(x)$. The domain of $f(x)$ is the set A. The domain of $g(x)$ is the set B. We can combine these two functions together in five different ways:

Sum of Functions
Difference of Functions
Product of Functions
Quotient of Functions
Composition of Functions

Combining Functions:

Sum of Functions: $(f + g)(x) = f(x) + g(x)$ with domain $A \cap B$

Difference of Functions: $(f - g)(x) = f(x) - g(x)$ with domain $A \cap B$

$$f(x) = \frac{2}{x} \qquad g(x) = \frac{2}{x+1}$$

$$(f+g)(x) = f(x) + g(x) = \frac{2}{x} + \frac{2}{x+1} = \frac{2(x+1)}{x(x+1)} + \frac{2(x)}{x(x+1)}$$

$$= \frac{2x+2}{x(x+1)} + \frac{2x}{x(x+1)} = \frac{4x+2}{x^2+x}$$

$$(f-g)(x) = f(x) - g(x) = \frac{2}{x} - \frac{2}{x+1} = \frac{2x+2}{x(x+1)} - \frac{2x}{x(x+1)} = \frac{2}{x^2+x}$$

Product of Functions: $(fg)(x) = f(x)g(x)$ with domain $A \cap B$

$$f(x) = \frac{2}{x} \qquad g(x) = \frac{2}{x+1}$$

$$(fg)(x) = \frac{2}{x} \cdot \frac{2}{x+1} = \frac{4}{x^2+x}$$

Quotient of Functions: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with domain $\{x \in A \cap B \mid g(x) \neq 0\}$

$f(x) = \frac{2}{x} \qquad g(x) = 6 + x$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{\frac{2}{x} \cdot x}{(6+x) \cdot x} = \frac{2}{x^2 + 6x}$$

Example 1: Suppose $f\,(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

a. $(f+g)(x) = 2x - 5 + x^2 - 3x + 5 = x^2 - x$

$D: (-\infty, \infty)$

b. $(f-g)(x) = (2x-5) - (x^2 - 3x + 5) = 2x - 5 - x^2 + 3x - 5$

$= -x^2 + 5x - 10$

$D: (-\infty, \infty)$

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Example 1: Suppose $f(x) = 2x - 5$ and $g(x) = x^2 - 3x + 5$. Find each of the following and state the domain:

c. $(fg)(x) = (2x-5)(x^2-3x+5)$

$= 2x^3 - 6x^2 + 10x - 5x^2 + 15x - 25$

$= 2x^3 - 11x^2 + 25x - 25$ D: $(-\infty, \infty)$

d. $\left(\frac{f}{g}\right)(x) = \dfrac{2x-5}{x^2-3x+5}$ D: $(-\infty, \infty)$

$x^2 - 3x + 5 = 0$ $\rightarrow$ only needed to determine Domain

$x = \dfrac{3 \pm \sqrt{9-4(1)(5)}}{2(1)} = \dfrac{3 \pm \sqrt{-11}}{2}$ No Real Solutions

Example 2: Given $f(x) = 3x^4 + 2x^3 - 8$ and, $g(x) = -x^2$, find each of the following functions and its domain.

a. $(g - f)(x) = -x^2 - (3x^4 + 2x^3 - 8)$

$= -x^2 - 3x^4 - 2x^3 + 8$

$= -3x^4 - 2x^3 - x^2 + 8$ $\quad D: (-\infty, \infty)$

b. $\frac{f(x)}{g(x)} = \dfrac{3x^4 + 2x^3 - 8}{-x^2} = -3x^2 - 2x + \dfrac{8}{x^2}$

$D: (-\infty, 0) \cup (0, \infty)$

Example 3: Let $f(x) = x^2 - 3x - 1$ and $g(x) = -3x - 10$. Find

$(f+g)(1) = f(1) + g(1) = (-3) + (-13) = -16$

$f(1) = (1)^2 - 3(1) - 1 = 1 - 3 - 1 = -3$

$g(1) = -3(1) - 10 = -3 - 10 = -13$

$(gg)(-1) = g(-1) \cdot g(-1) = (-7) \cdot (-7) = 49$

$g(-1) = -3(-1) - 10 = 3 - 10 = -7$

Composition of Functions

$f \circ g$ "follows"

The composition of the function f with g is denoted $f \circ g$ by and is defined by the

$$(f \circ g)(x) = f(g(x))$$

The domain of the composition $f \circ g$ is the set of all x such that

1. x is in the domain of g (the "inside" function)
2. $g(x)$ is in the domain of f (the "outside" function)

$f(x) = \frac{5}{x}$

$g(x) = \sqrt{x+7}$

$$(f \circ g)(x) = f(g(x)) = f(\sqrt{x+7}) = \frac{5}{\sqrt{x+7}}$$

Inside: $g(x)$: $x+7 \geq 0$

$x \geq -7$

Outside: $f(x)$: $x \neq 0$

$g(x) \neq 0$

$\sqrt{x+7} \neq 0$

$x \neq -7$

Composition of Functions D: $(-7, \infty)$

Example 4: Let $f(x) = x^2 + 1$ and $g(x) = -2x + 5$, find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x)) = f(-2x+5) = (-2x+5)^2 + 1$$

$$(-2x+5)(-2x+5) + 1$$

$$4x^2 - 10x - 10x + 25 + 1$$

$$(f \circ g)(x) = 4x^2 - 20x + 26$$

Example 5: Let $f(x) = \frac{1}{x}$ and $g(x) = \frac{5}{x+4}$, find $(g \circ f)(x)$

$$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{x}\right) = \frac{5}{\left(\frac{1}{x} + 4\right)} \cdot \frac{x}{x}$$

$$\boxed{\frac{5x}{1+4x}}$$

Example 6: Let $f(x) = \sqrt{4 - x^2}$ and $g(x) = \sqrt{3 - x}$, find $(f \circ g)(x)$.

$$(f \circ g)(x) = f(g(x)) = f\left(\sqrt{3-x}\right) = \sqrt{4 - \left(\sqrt{3-x}\right)^2}$$

$$= \sqrt{4 - (3-x)} = \sqrt{4 - 3 + x}$$

$$\boxed{\begin{array}{l} (f \circ g)(x) = \sqrt{1 + x} \\ (f \circ g)(x) = \sqrt{x + 1} \end{array}}$$

Example 7: Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x + 3$. Find each of the following.

a. $(f \circ g)(2) = f(g(2)) = f(15) = 3(15) - 5 = 45 - 5 = \boxed{40}$

$g(2) = 2^2 + 4(2) + 3$

$= 4 + 8 + 3 = 15$

b. $(g \circ f)(-1)$

$g(f(-1)) = g(-8) = (-8)^2 + 4(-8) + 3 = \boxed{35}$

$f(-1) = 3(-1) - 5 = -3 - 5 = -8$

c. $(g \circ f)(x) = g(f(x))$

$g(3x-5) =$

$(3x-5)^2 + 4(3x-5) + 3$

$9x^2 - 30x + 25 + 12x - 20 + 3$

$\boxed{9x^2 - 18x + 8}$

d. $(g \circ g)(0) = g(g(0)) = g(3)$

$g(0) = 0^2 + 4(0) + 3 = 3$

$g(3) = 3^2 + 4(3) + 3 = 9 + 12 + 3 = \boxed{24}$

Determine the value of the difference quotient for $f(x) = 2x^2 + 3x - 1$

The difference quotient is:

$$\frac{f(x+h) - f(x)}{h}$$

$(x+h)^2$

$(x+h)(x+h)$

$x^2 + hx + hx + h^2$

$x^2 + 2xh + h^2$

$$f(x+h) = 2(x+h)^2 + 3(x+h) - 1 = 2(x^2 + 2xh + h^2) + 3(x+h) - 1$$

$$= 2x^2 + 4xh + 2h^2 + 3x + 3h - 1$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 + \cancel{3x} + 3h \cancel{- 1} - \cancel{2x^2} - \cancel{3x} \cancel{+ 1}}{h} = \frac{\cancel{h}(4x + 2h + 3)}{\cancel{h}}$$

$$= 4x + 2h + 3$$

Determine the value of the difference quotient for f(x) = 2x2 = 3x - 1

Steps for evaluating difference quotient:

1) Evaluate $f(x+h)$

2) plug $f(x+h)$ and $f(x)$ into the formula

3) cancel in your numerator

4) GCF factoring of the "h"

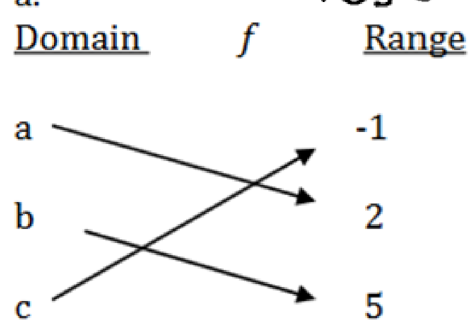
5) cancel with the h in the denominator

Inverse Functions

Let f be a function with domain A. f is said to be **one-to-one** if no two elements in A have the same image.

Example 1: Determine if the following function is one-to-one.

a. Yes (1-1)

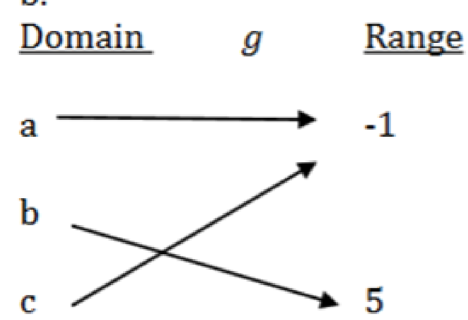


$a \to 2$

$b \to 5$

$c \to -1$

b. Not 1-1



$f(a) = f(c) = -1$

(However, it is still a function)

$a \to -1$

$b \to 5$

$c \to -1$

A one-to-one function has an inverse. The inverse function reverses whatever the first function did. These two statements mean exactly the same thing:

1. f is one-to-one (1-1)
2. f has an inverse function

The inverse of a function f is denoted by f^{-1} , read "f-inverse".

Note: $f^{-1}(x) \neq \frac{1}{f(x)}$ like $x^{-3} = \frac{1}{x^3}$

$f^{-1}(x)$ means the inverse of $f(x)$

It does not mean $\frac{1}{f(x)}$

Domain and Range

Suppose f is a one-to-one function with domain A and range B. The inverse function has domain B and range A.

f: $(3, -1)$, $(-1, 4)$ g: $(-1, 3)$, $(4, -1)$

Example 1: Suppose f and g are inverse functions. If $f(3) = -1$ and $f(-1) = 4$, then find $g(-1)$.

$g(-1) = 3$

If $f(x)$ contain the point (a, b)

then $f^{-1}(x)$ contains the point (b, a)

If $f(x)$ has domain: $[a, \infty)$, range: $(-\infty, b]$

Then $f^{-1}(x)$ has domain $(-\infty, b]$, range $[a, \infty)$

Property of Inverse Functions

Let f and g be two functions such that $(f \circ g)(x) = x$ for every x in the domain of g and $(g \circ f)(x) = x$ for every x in the domain of f then **f and g are inverses of each other.**

Example 2: Show that the following functions are inverses of each other.

$f(x) = 3x + 7$ and $g(x) = \frac{x}{3} - \frac{7}{3}$

f and g are inverses

$$f(g(x)) = f\left(\frac{x}{3} - \frac{7}{3}\right) = 3\left(\frac{x}{3} - \frac{7}{3}\right) + 7 = x - 7 + 7 = x$$

$$g(f(x)) = g(3x+7) = \frac{3x+7}{3} - \frac{7}{3} = \frac{\cancel{3}x}{\cancel{3}} + \frac{7}{3} - \frac{7}{3} = x$$

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Example 3: Determine whether the following pair of functions are inverses of each other.

$f(x) = 2x - 1$ and $g(x) = \frac{x}{2} + 1$

$$f(g(x)) = f\left(\frac{x}{2}+1\right) = 2\left(\frac{x}{2}+1\right) - 1 = x + 2 - 1 = x + 1 \neq x$$

f and g are <u>Not</u> inverses

How to find the equation of the inverse function of a one-to-one function:

1. Replace $f(x)$ by y.
2. Exchange x and y.
3. Solve for y.
4. Replace y by $f^{-1}(x)$
5. Verify.

$f(x) = \sqrt{2x+1}$ $D: [-1/2, \infty)$

$y = \sqrt{2x+1}$ $R: [0, \infty)$

$x = \sqrt{2y+1}$

$x^2 = 2y+1$ $f^{-1}(x) = \frac{1}{2}x^2 - \frac{1}{2}$

$x^2 - 1 = 2y$ $D: [0, \infty)$

$\frac{x^2-1}{2} = y$ $R: [-1/2, \infty)$

Example 4: Write the equation of the inverse function for $f(x) = 3x - 3$

$$y = 3x - 3$$

$$x = 3y - 3$$

$$x + 3 = 3y$$

$$\frac{x+3}{3} = y$$

$$f^{-1}(x) = \frac{1}{3}x + 1$$

$$f(f^{-1}(x)) = f\left(\frac{1}{3}x + 1\right) = 3\left(\frac{1}{3}x + 1\right) - 3$$

$$x + 3 - 3 = x$$

$$f^{-1}(f(x)) = f^{-1}(3x - 3) = \frac{1}{3}(3x - 3) + 1$$

$$x - 1 + 1 = x$$

Example 5: Write the equation of the inverse for $f(x) = \frac{6}{4-x}$

$$y = \frac{6}{4-x}$$

$$\frac{x}{1} = \frac{6}{4-y}$$

$$6 = x(4-y)$$

$$\frac{6}{x} = 4-y$$

$$\frac{6}{x} = \cancel{4} - y$$

$$-4 \qquad -\cancel{4}$$

$$-1\left(\frac{6}{x} - 4\right) = (-y)\cdot -1$$

$$\frac{-6}{x} + 4 = y$$

$$y = \frac{-6}{x} + 4 = 4 - \frac{6}{x} = \frac{4x-6}{x}$$

$$\frac{4x}{x}$$

$$f(f^{-1}(x)) = \frac{6}{4 - \frac{4x-6}{x}} \cdot \frac{x}{x}$$

$$\frac{6x}{4x-(4x-6)}$$

$$\frac{6x}{\cancel{4x}-\cancel{4x}+6}$$

$$\frac{\cancel{6}x}{\cancel{6}} = x$$

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$f(x) = \frac{6}{4-x}$ $\quad f^{-1}(x) = \frac{4x-6}{x}$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{6}{4-x}\right) = \frac{4\left(\frac{6}{4-x}\right)-6}{\frac{6}{4-x}} \cdot \frac{4-x}{4-x}$$

$$= \frac{4 \cdot 6 - 6(4-x)}{6} = \frac{\cancel{24} - \cancel{24} + 6x}{6} = \frac{6x}{6} = x$$

Popper 5: Question 8:

Example 6: Write the equation of the inverse for $f(x) = (x+1)^3 + 1$

a. $\sqrt[3]{x-2}$ b. $\sqrt[3]{x-1}+1$ c. $\sqrt[3]{x-1}-1$ d. $\sqrt[3]{x-1}$

Question 9:

Example 7: Write the equation of the inverse for $f(x) = \sqrt[3]{x+4}$

a. $\sqrt[3]{x-4}$ b. $x^3 - 4$ c. $x^3 + 4$ d. $\sqrt[3]{x} - 2$

Popper 5: Question 8:

It is easiest to determine if a function is one-to-one by looking at its graph. We can use the Horizontal Line Test to determine if a function is one-to-one.

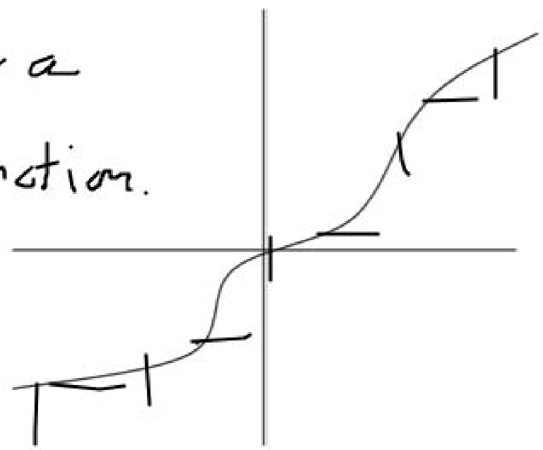
Horizontal Line Test: A function is one-to-one if no horizontal line intersects its graph in more than one point.

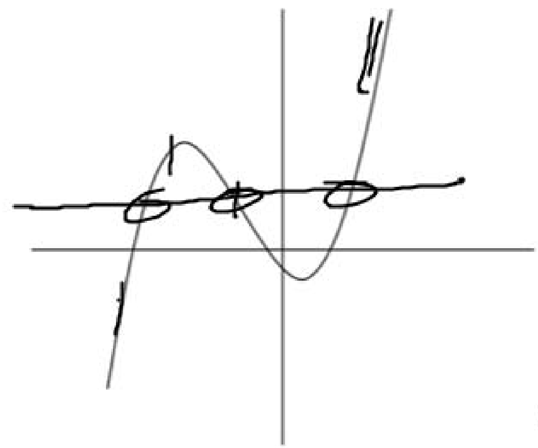
Vertical Line Test: Is it a function? Horizontal Line Test:

Is it 1-1?

This is a

1-1 function.





This is

a function,

but Not

1-1.

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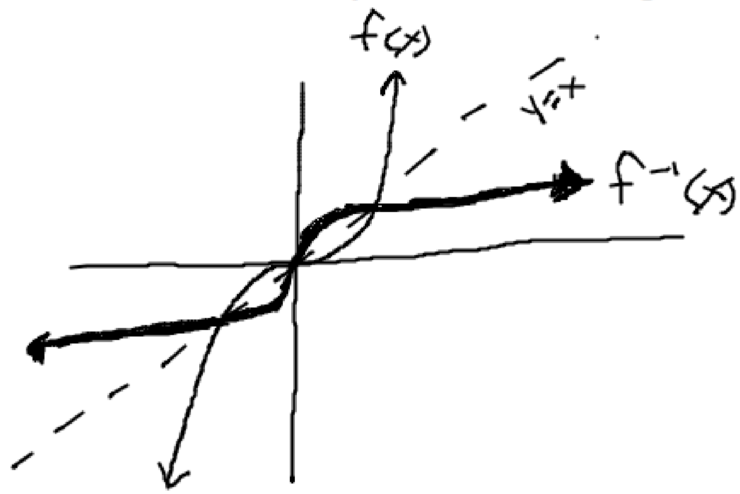
Graphing the Inverse Function

Given that f is 1-1, the graph of f^{-1} is a reflection of the graph of f about the line $y = x$

Remember:

1. The inverse function reverses whatever the first function did.
2. The Domain of f becomes the Range of and the Range of f becomes the Domain of f^{-1} .



If f is increasing, f^{-1} is increasing.

If f is more vertical, f^{-1} is more horizontal

If f contains (a, b), f^{-1} contains (b, a)

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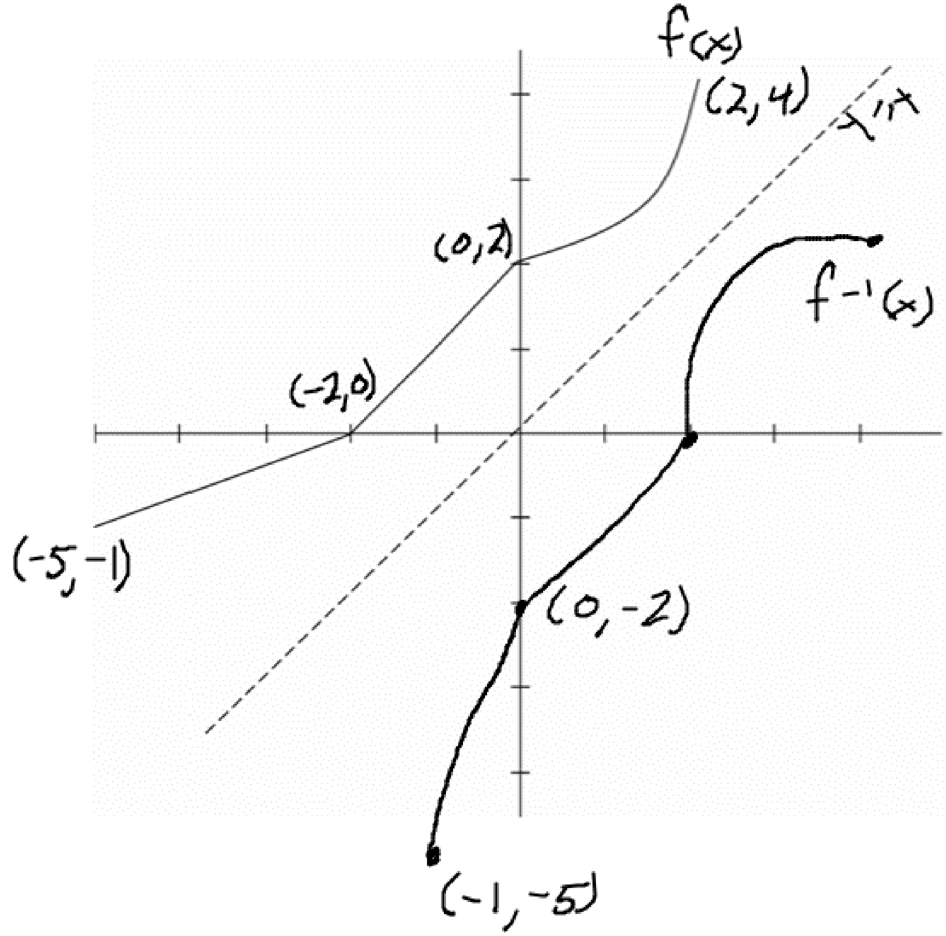

f	$\rightarrow f^{-1}$
$(-5,-1)$	$\rightarrow (-1,-5)$
$(-2,0)$	$\rightarrow (0,-2)$
$(0,2)$	$\rightarrow (2,0)$
$(2,4)$	$\rightarrow (4,2)$

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