



Midterm: 4/3 from 10:00 - 11:30 AM  
SEC 101

# MATH 1310

Session 6

Test 2 Review

**MATH 1310**

Example 1: The length of a rectangle is twice its width. If the perimeter of the rectangle is 180 feet, find the dimensions of the rectangle.

$$\text{length: } l = 2w$$

$$P = 180$$

$$\text{width: } w$$

$$2l + 2w = 180$$

$$2(2w) + 2w = 180$$

$$4w + 2w = 180$$

$$\frac{6w}{6} = \frac{180}{6}$$

$$w = 30$$

$$l = 2w$$

$$l = 2(30)$$

$$l = 60$$

length: 60ft
width: 30ft

Example 2: Solve the following system of equations for y:

$$4x + y = 47$$

$$(6x - 2y = -10) \div 2$$

Elimination

Method

$$4(x) + y = 47$$

$$4(6) + y = 47$$

$$24 + y = 47$$

$$\begin{array}{r} -24 \\ \hline \end{array}$$

$$\boxed{y = 23}$$

$$\begin{array}{r} 4x + y = 47 \\ \rightarrow + (3x - y = -5) \\ \hline 7x = 42 \\ \hline x = 6 \end{array}$$

Example 3: State all solutions to the equation:

$$x^2 + 40 = 0$$

$$-40 \quad -40$$

$$\sqrt{x^2 = \pm \sqrt{-40}}$$

$$\sqrt{-40}$$

$$x = \pm \sqrt{-40}$$

$$\sqrt{-1} \sqrt{4} \sqrt{10}$$

$$\downarrow \quad \downarrow \quad \downarrow$$
$$i \quad 2 \quad \sqrt{10}$$

$$x = \pm 2i\sqrt{10}$$

Example 4: Solve the following equation:

$$x^2 + 5x - 7 = 0$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad a=1 \quad b=5 \quad c=-7$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-7)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 28}}{2}$$

$$x = \frac{-5 \pm \sqrt{53}}{2}$$

Example 5: Solve the following by completing the square:

$$(x^2 - 8x) + 20 = 0$$

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$$-20 \quad -20$$

$$x^2 - 8x = -20$$

$$b = -8$$

$$\frac{b}{2} = -4$$

$$\left(\frac{b}{2}\right)^2 = 16$$

$$x^2 - 8x + 16 = -20 + 16$$

$$(x-4)^2 = -4$$

$$\sqrt{(x-4)^2} = \sqrt{-4}$$

$$x-4 = \pm 2i$$

$$+4 \quad +4$$

$$\boxed{x = 4 \pm 2i}$$

Example 6: Solve:

common

denominator

LCD:  $30x$

$$\frac{3 \cdot 3}{3 \cdot 10x} - \frac{1 \cdot 5}{6x \cdot 5} = 1$$

$$\frac{9}{30x} - \frac{5}{30x} = 1$$

$$\frac{4}{30x} = 1$$

$$\frac{4}{30} = 30x$$

$$x = \frac{2}{15}$$

$$\frac{3}{10x} - \frac{1}{6x} = 1$$

$$10x \left( \frac{3}{10x} \right) - 10x \left( \frac{1}{6x} \right) = 10x \cdot 1$$

$$3 - \frac{5}{3} = 10x$$

$$3 \cdot 3 - \frac{5}{1} \left( \frac{2}{2} \right) = 3 \cdot 10x$$

$$9 - 5 = 30x$$

$$\frac{4}{30} = \frac{30x}{30}$$

$$x = \frac{2}{15}$$



Example 7: Simplify  $\frac{3-2i}{2+i}$

$$\frac{(3-2i)(2-i)}{(2+i)(2-i)} = \frac{6-3i-4i+2i^2}{4-2i+2i-i^2} = \frac{6-3i-4i+2i^2}{4-2i+2i-i^2} = \frac{4-7i}{5}$$

$\frac{4}{5} - \frac{7}{5}i$

Example 8: Simplify

A.  $(5-4i)(-1-2i)$  FOIL

$$-5 - 10i + 4i + 8i^2 = \boxed{-13 - 6i}$$

*(Note: In the original image, the term  $8i^2$  is circled, and an arrow points from it to the  $-8$  above the equals sign.)*

B.  $(2-3i)-(-1+5i)$

$$2 - 3i + 1 - 5i = \boxed{3 - 8i}$$

Example 9: Solve the following inequality and express your answer in interval notation.

$$-2 < -4x \leq 7$$

$$\frac{-3 \quad -3}{-4 \quad -4}$$

$$\frac{-5 < -4x \leq 4}{-4 \quad -4}$$

$$\left[-1, \frac{5}{4}\right)$$

$$\frac{5}{4} > x \geq -1$$

$$-1 \leq x < \frac{5}{4}$$

Example 10: Solve the following inequality and express your answer in interval notation.

$$|7x + 8| - 4 < -3$$

$$+4 \quad +4$$

$$\underline{|7x + 8| < 1}$$

$$\boxed{(-9/7, -1)}$$

$$-1 < 7x + 8 < 1$$

$$\underline{-8 \quad -8 \quad -8}$$

$$-9 < 7x < -7$$

$$\underline{-7 \quad -7 \quad -7}$$

$$-9/7 < x < -1$$

Example 11: Find the solution of the equation

$$2 + 3|4x - 1| \geq 14.$$

$$-2$$

$$\frac{3}{3} |4x - 1| \geq \frac{12}{3}$$

$$|4x - 1| \geq 4$$

$$4x - 1 \geq 4$$

$$+1 \quad +1$$

$$\frac{4x \geq 5}{4}$$

$$\frac{x \geq 5/4}{4}$$

$$x \geq 5/4$$

or

$$4x - 1 \leq -4$$

$$+1 \quad +1$$

$$\frac{4x \leq -3}{4}$$

$$\frac{x \leq -3/4}{4}$$

$$x \leq -3/4$$

or

$$(-\infty, -3/4] \cup [5/4, \infty)$$

Example 12: Find all solutions to the equation:

$$|3 - 2x| = 6$$

$$\begin{array}{r} 3 - 2x = 6 \\ -3 \quad -3 \\ \hline -2x = 3 \\ -x \quad -2 \\ \hline x = -3/2 \end{array}$$

$$\begin{array}{r} 3 - 2x = -6 \\ -3 \quad -3 \\ \hline -2x = -9 \\ -x \quad -2 \\ \hline x = 9/2 \end{array}$$

$$\left\{ -3/2, 9/2 \right\}$$

Example 13: Tom has a drawer with dimes, nickels and pennies in it. He has an equal number of each kind of coin. Tom counted his money and found that he has a total of \$2.40 in the drawer. How many nickels does Tom have?

15 of each coin

$$d = n = p = x$$

dimes:  $d$

nickels:  $n$

pennies:  $p$

$$100d + .05n + .01p = 2.40 \times 100$$

$$10d + 5n + 1p = 240$$

$$10x + 5x + x = 240$$

$$16x = 240$$

$$x = 240 / 16 = 15$$

$$\begin{array}{r} 15 \\ 16 \overline{)240} \\ \underline{-160} \\ 80 \\ \underline{80} \\ 0 \end{array}$$

Example 14: Solve the following for x:

$$x^6 - 9x^3 - 36 = 0$$

$$u = x^3$$

$$u^2 = x^6$$

$$u^2 - 9u - 36 = 0$$

$$(u+3)(u-12) = 0$$

$$u+3=0 \quad | \quad u-12=0$$

$$u = -3 \quad | \quad u = 12$$

$$x^3 = -3 \quad x^3 = 12$$

$$x = \sqrt[3]{-3} \quad x = \sqrt[3]{12}$$

Example 14: Solve the following for x:  ~~$x^6 - 9x^3 - 36 = 0$~~

-36	
1 -36	-35
2 -18	-16
3 -12	-9
4 -9	-5
6 -6	0



Example 15: Solve the following for x:

$$\sqrt{x+5} + x = 5$$

$$\frac{-x}{-x}$$

$$(\sqrt{x+5})^2 = (-x+5)^2$$

$$\frac{x+5}{-x-5} = \frac{x^2-10x+25}{-x-5}$$

$$0 = x^2 - 11x + 20$$

$$(x+5)(-x+5)$$

$$x^2 - 5x - 5x + 25$$

$$x^2 - 10x + 25$$

$$x = \frac{11 \pm \sqrt{121 - 4(1)(20)}}{2(1)}$$

$$x = \frac{11 \pm \sqrt{121 - 80}}{2}$$

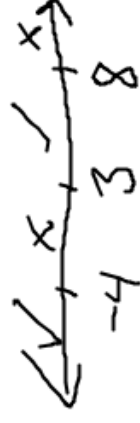
$$x = \frac{11 \pm \sqrt{41}}{2}$$

Example 15: Solve the following for x:

Example 17: Solve the following for x:

$$\frac{(x-8)(x+4)}{x-3} \leq 0$$

$$\boxed{(-\infty, -4] \cup (3, 8]}$$



Denominator:  
 $x-3=0$   
 $x=3$

Numerator:  
 $(x-8)(x+4)=0$   
 $x-8=0$      $x+4=0$   
 $x=8$        $x=-4$

Test  $x=0$   
 $\frac{(-8)(4)}{(-3)} = \text{positive} \neq 0$

Test  $x=5$   
 $\frac{(5-8)(5+4)}{(5-3)} = \text{Neg} \leq 0 \checkmark$

Test  $x=-5$   
 $\frac{(-5-8)(-5+4)}{(-5-3)}$   
 $\frac{(-13)(-1)}{(-8)}$   
 Negative  $\leq 0 \checkmark$

Example 17: Solve the following for x:

$$\frac{(10-8)(10+4)}{10-3} = \text{Pos} \neq 0$$

18. Find the domain:

a.  $f(x) = \frac{x}{7x-14}$

$$7x - 14 = 0$$
$$7x = 14$$
$$x = 2$$

$$(-\infty, 2) \cup (2, \infty)$$

b.  $f(x) = \sqrt{5x-1}$

$$5x - 1 \geq 0$$
$$5x \geq 1$$
$$x \geq 1/5$$

$$[1/5, \infty)$$

c.  $f(x) = \sqrt{5-4x}$

$$5 - 4x \geq 0$$
$$5 \geq \frac{4x}{4}$$

$$\frac{5}{4} \geq x$$

or

$$x \leq \frac{5}{4}$$
$$(-\infty, \frac{5}{4}]$$

19.

a. Calculate  $f(-2)$  if  $f(x) = x^2 + x$

$$f(-2) = (-2)^2 + (-2) = 4 - 2 = 2$$
$$f(-2) = 2$$

b. Calculate  $f(-2)$  if  $f(x) = \begin{cases} x^2 + 2x & x \leq -1 \\ x & x > -1 \end{cases}$ \*

$$-2 < -1$$

$$f(-2) = (-2)^2 + 2(-2) = 4 - 4 = 0$$
$$f(-2) = 0$$

c. Which point below is on the graph of  $f(x)$ .

$$f(x) = \begin{cases} 2 & x < -1 \\ 4 & x = -1 \\ x^2 - 1 & x > -1 \end{cases}$$

~~$(-2, 0)$~~  or  $(1, 0)$

$$f(-2)$$

$-2 < -1$  (First Equation)

$$f(-2) = 2$$

$$f(1)$$

$1 > -1$  (Third Equation)

$$(1)^2 - 1 = 0$$

$$(1, 0)$$

20. Determine which of the following is on the graph.

a.  $f(x) = -\frac{1}{2}x - 3$

~~(-1, 1)~~

$$f(-1) = \frac{-1}{2}(-1) - 3 = \frac{1}{2} - 3 = \frac{1}{2} - \frac{6}{2} = \frac{-5}{2}$$

(0, -3)

$$f(0) = \frac{-1}{2}(0) - 3 = 0 - 3 = -3$$

b.  $f(x) = 2x^2 - 3x - 1$

(1, -2)

$$\begin{aligned} f(1) &= 2(1)^2 - 3(1) - 1 \\ &= 2 - 3 - 1 = -2 \end{aligned} \quad (1, -2)$$

~~(-1, -1)~~

$$\begin{aligned} f(-1) &= 2(-1)^2 - 3(-1) - 1 \\ &= 2 + 3 - 1 = 4 \end{aligned}$$

21. Determine if the following is a function:

a.  $x^2 + y^2 = 25$  No

$$y^2 = -x^2 + 25$$

$$y = \pm \sqrt{-x^2 + 25}$$

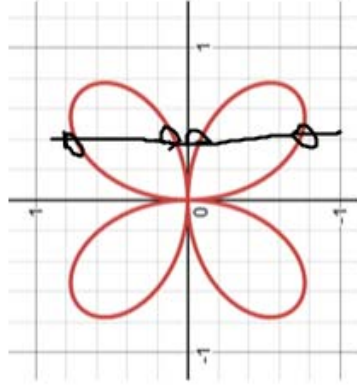
b.  $y = x^3 + 2x^2 + 5x - 1$

Yes

(one x-value gets

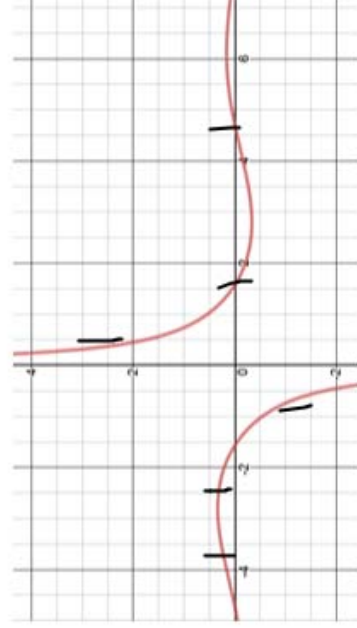
1 answer (only) in  
y.

c. vertical line test



No

Yes



21. Determine if the following is a function:

22.

a. Sketch the graph  $f(x) = -\sqrt{x-1}$

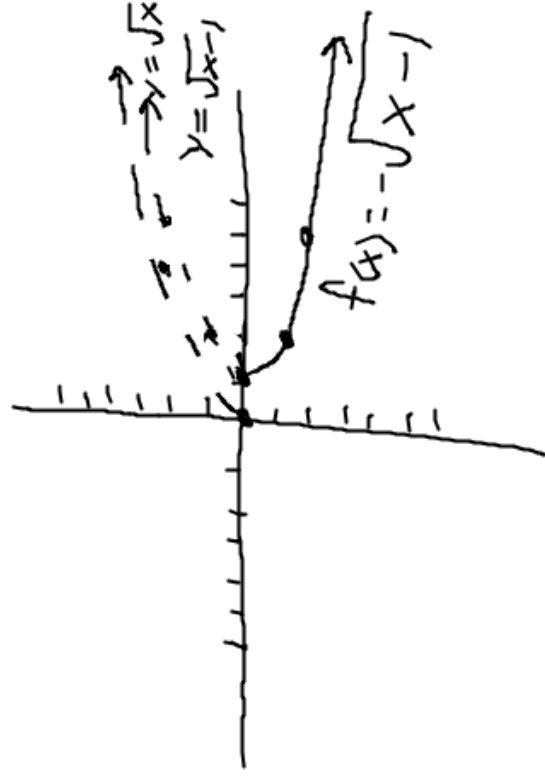
HRA

$$y = \sqrt{x}$$

$y = \sqrt{x-1}$  (Horizontal  
shift 1 right)

$$y = -\sqrt{x-1}$$

(x-axis  
reflection)





b. Sketch the graph  $f(x) = -(x+2)^2 - 1$

$$y = x^2$$

$$y = (x+2)^2$$

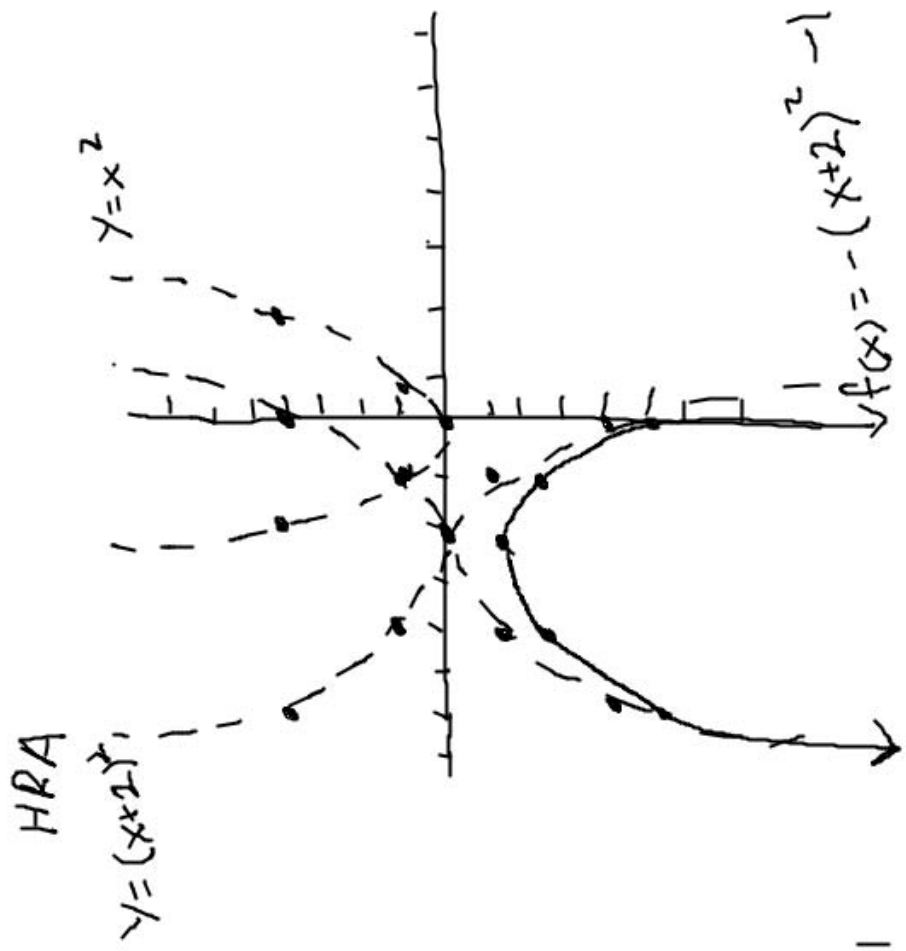
Shift Left  
2 spaces

$$y = -(x+2)^2$$

(x axis  
reflection)

$$y = -(x+2)^2 - 1$$

shift down



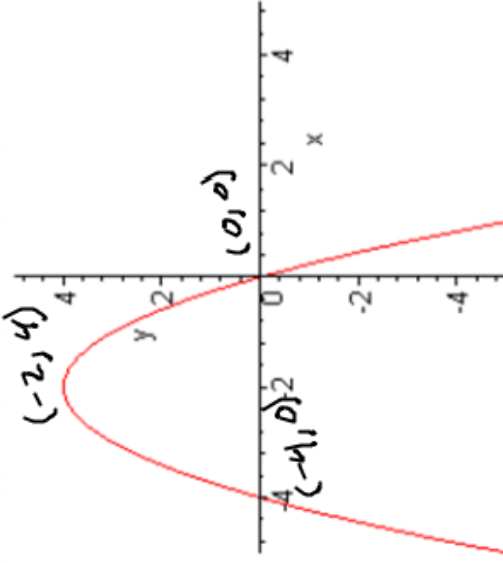
23.

a. What are the necessary transformations  $f(x) = (x + 3)^3 - 2$

shift left 3 spaces ←

shift down 2 spaces ↓

b. What is the function?



$$y = a(x-h)^2 + k$$

$$y = a(x+2)^2 + 4$$

$$0 = a(0+2)^2 + 4$$

$$0 = a(4) + 4$$

$$\frac{-4}{-4} = \frac{4a}{-4}$$

$$1 = -a$$

$$-1 = a$$

$$y = -1(x+2)^2 + 4$$

$$y = -1(x^2 + 4x + 4) + 4$$

$$y = -x^2 - 4x - 4 + 4$$

$$y = -x^2 - 4x$$

check: (-4, 0)

$$y = -x^2 - 4x$$

$$0 = -(-4)^2 - 4(-4)$$

$$-16 + 16 = 0$$

24.

a. Find the vertex  $f(x) = 2x^2 - 4x + 21$

$$x = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = \frac{4}{4} = 1$$

$$(1, 19)$$

$$\begin{aligned} f(1) &= 2(1)^2 - 4(1) + 21 \\ &= 2 - 4 + 21 \\ &= -2 + 21 = 19 \end{aligned}$$

b. Find the maximum or minimum value of the function

$$f(x) = x^2 - 16x + 8$$

$$x = \frac{-b}{2a} = \frac{-(-16)}{2(1)} = \frac{16}{2} = 8$$

$$\text{vertex: } (8, -56)$$

opening up:  $a > 0$   
( $a=1$ )

$$\begin{aligned} f(8) &= 8^2 - 16(8) + 8 = 64 - 128 + 8 = -64 + 8 \\ &= -56 \end{aligned}$$

Minimum:  $-56$

25. Put in standard form  $f(x) = -x^2 - 6x + 2$

$$f(x) = (-x^2 - 6x) + 2$$

$$f(x) = -1(x^2 + 6x) + 2$$

$$b = 6$$

$$\frac{b}{2} = 3$$

$$\left(\frac{b}{2}\right)^2 = 9$$

$$f(x) = -1(x^2 + 6x + 9) - 1(-9) + 2$$

$$f(x) = -1(x+3)^2 + 9 + 2$$

$$f(x) = -(x+3)^2 + 11$$

$$x = \frac{-b}{2a} = \frac{-(-6)}{2(-1)} = \frac{6}{-2} = -3$$

$$f(-3) = (-3)^2 - 6(-3) + 2$$

$$= -9 + 18 + 2$$

$$= 9 + 2 = 11$$

Vertex:  $(-3, 11)$

$$y = a(x-h)^2 + k$$

$$y = -(x+3)^2 + 11$$

$$f(x) = -(x+3)^2 + 11$$

26. If  $f(x) = \sqrt{x+1}$  and  $g(x) = x^2$ , find  $(g \circ f)(x)$  and  $(f \circ g)(-1)$

$$(g \circ f)(x) = g(f(x))$$

$$= g(\sqrt{x+1})$$

$$= (\sqrt{x+1})^2 = x+1$$

$$(g \circ f)(x) = x+1$$

$$(f \circ g)(-1) = f(g(-1)) = f(1)$$

$$g(-1) = (-1)^2 = 1$$

$$f(1) = \sqrt{1+1} = \sqrt{2}$$

$$(f \circ g)(-1) = \sqrt{2}$$

27. If  $f(x) = \frac{1}{2x}$  and  $g(x) = x^2 - 1$ , find  $(f \circ g)(2)$ .

$$(f \circ g)(2) = f(g(2)) = f(3) = \frac{1}{2(3)} = \frac{1}{6}$$

$g(2) = 2^2 - 1 = 4 - 1 = 3$

$(f \circ g)(2) = \frac{1}{6}$

28. If  $f(x) = -2x + 2$  and  $g(x) = x^2 + x$ , find  $(f \circ g)(2)$ .

$$(f \circ g)(2) = f(g(2)) = f(6) = -2(6) + 2 = -12 + 2$$

$$= -10$$

$$g(2) = 2^2 + 2 = 4 + 2 = 6$$

$$(f \circ g)(2) = -10$$



29. Find the inverse:

a.  $f(x) = -2x + 2$

$$y = -2x + 2$$

$$x = \frac{-y + 2}{-2}$$

$$\frac{x-2}{-2} = \frac{-y}{-2}$$

$$\frac{-1}{2}x + 1 = y$$

$$f^{-1}(x) = \frac{-1}{2}x + 1$$

or

$$f^{-1}(x) = \frac{x-2}{-2}$$

b.  $f(x) = \frac{1}{x+2}$

$$\frac{1}{y} = \frac{1}{x+2}$$

$$x = \frac{1}{y} - 2$$

$$x(y+2) = 1$$

$$xy + 2x = 1$$

$$-2x = 1 - 2x$$

$$xy = 1 - 2x$$

$$\frac{xy}{x} = \frac{1-2x}{x}$$

$$y = \frac{1-2x}{x}$$

$$f^{-1}(x) = \frac{1-2x}{x}$$

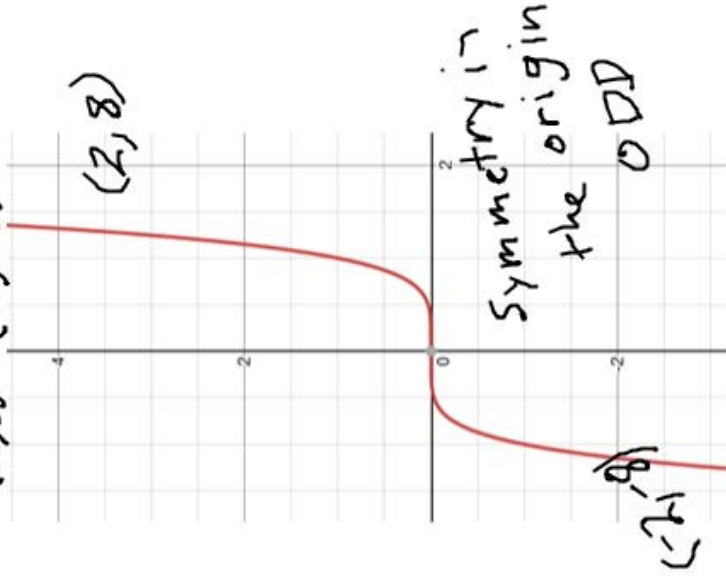
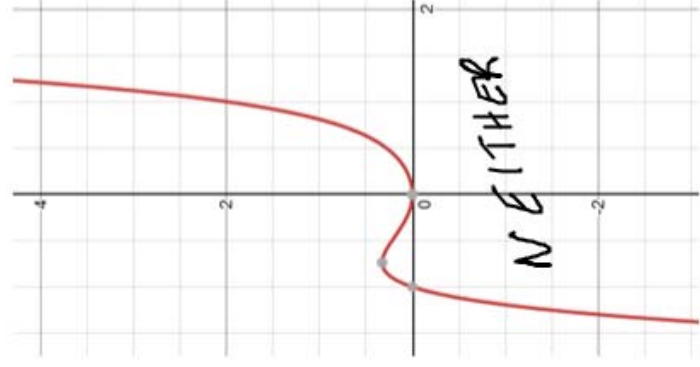
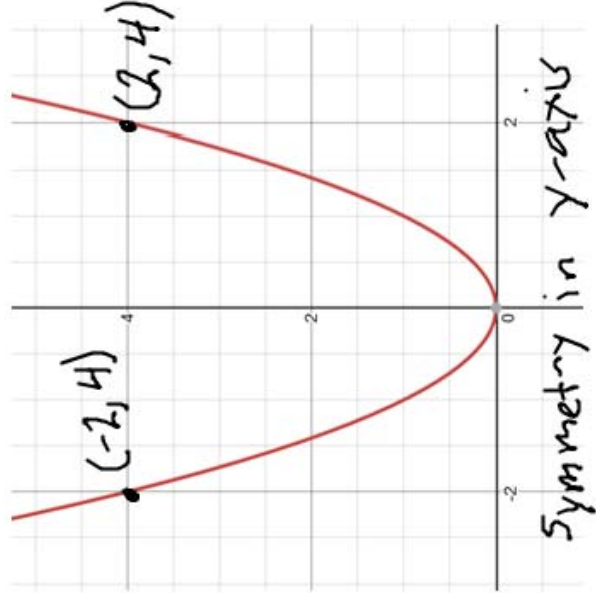
or

$$f^{-1}(x) = \frac{1}{x} - 2$$

30. Classify the function as Even, Odd or Neither:

$$(a,b) \rightarrow (-a,b)$$

$$(a,b) \rightarrow (-a,-b)$$



30. Classify the function as Even, Odd or Neither:

**EVEN**

31. Evaluate the difference quotient for the

$$\text{function: } f(x) = 2x^2 + 8$$

$$(x+h)^2$$

$$\frac{f(x+h) - f(x)}{h} \quad \frac{(x+h)(x+h)}{x^2 + 2xh + h^2}$$

$$f(x+h) = 2(x+h)^2 + 8 = 2(x^2 + 2xh + h^2) + 8 = 2x^2 + 4xh + 2h^2 + 8$$

$$\frac{\cancel{2x^2} + 4xh + 2h^2 + 8 - \cancel{2x^2} - 8}{h} = \frac{4xh + 2h^2}{h} = \cancel{h} \frac{4x + 2h}{\cancel{h}}$$

$$\boxed{4x + 2h}$$

31. Evaluate the difference quotient for the function:  $f(x) = 2x^2 + 8$

## Popper 6

Questions 1 – 5, fill out answer choice B.