MATH 1310

Session 8

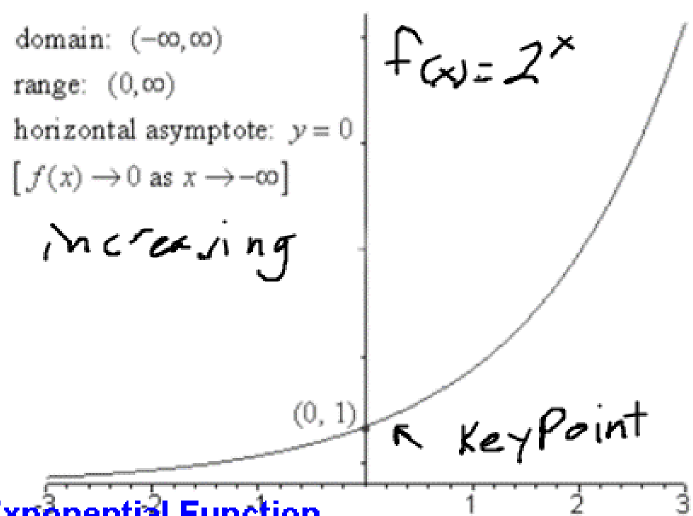
The Exponential Function

An exponential function is defined as a function of the form:

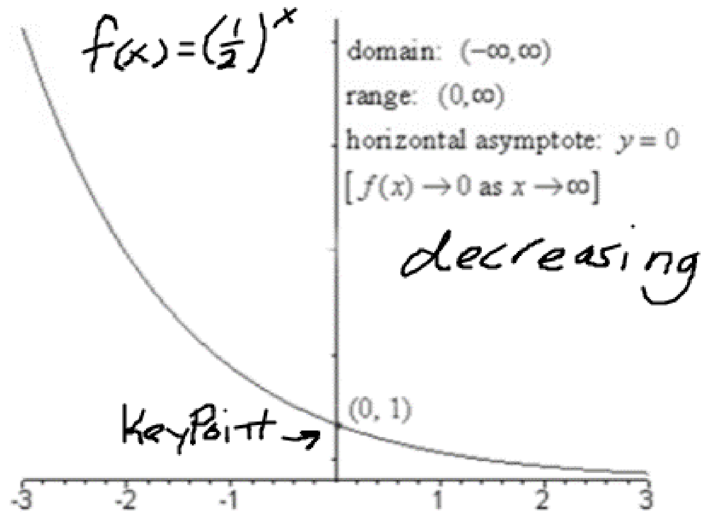
$$f(x) = a^x \qquad a \neq 1$$

where $\underline{a > 0}$. This is considered an exponential function with base a.

The graph of $f(x) = a^x$ for $a > 1$ has the following shape:



The graph of $f(x) = a^x$ for $0 < a < 1$ has the following shape:



The Exponential Function

Example Problem 1:

Sketch the graph of the function $f(x) = -2^x$ by starting from the graph of $y = 2^x$.

State the domain, range, and asymptote.

$Y = 2^x \qquad a > 1$

$(0, 1) \quad [Y = 2^0 = 1]$

$(1, 2) \quad [Y = 2^1 = 2]$

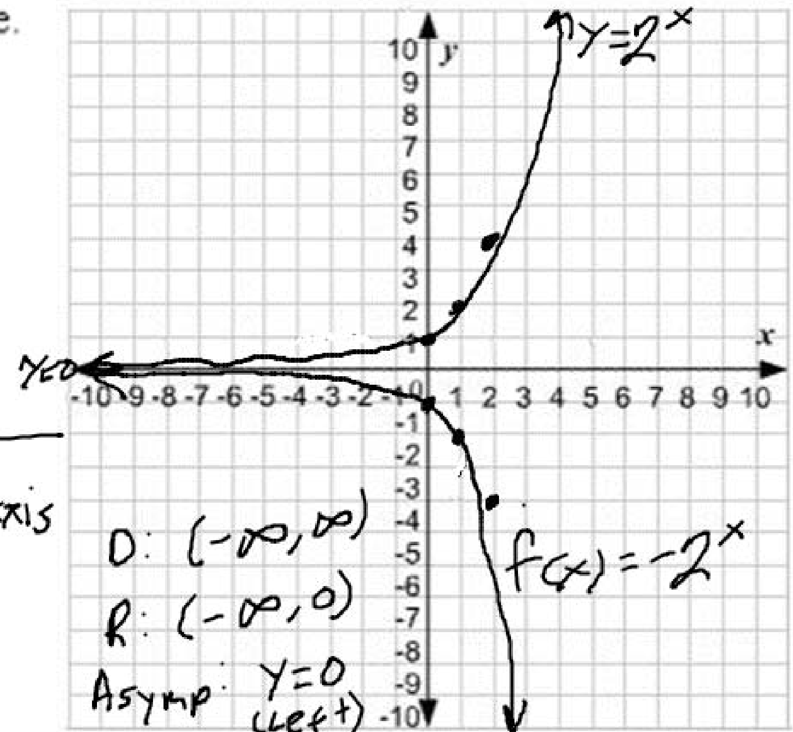
$(2, 4) \quad [Y = 2^2 = 4$

$Y = -2^x$ Reflection in x-axis

$(0, 1) \rightarrow (0, -1)$

$(1, 2) \rightarrow (1, -2)$

$(2, 4) \rightarrow (2, -4)$



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Example Problem 2:

Sketch the graph of the function $f(x) = 2^{x-1}$ by starting from the graph of $y = 2^x$.

State the domain, range, and asymptote.

$y = 2^x \xrightarrow[\text{right 1}]{\text{shift}} y = 2^{x-1}$

$(0,1) \longrightarrow (1,1)$

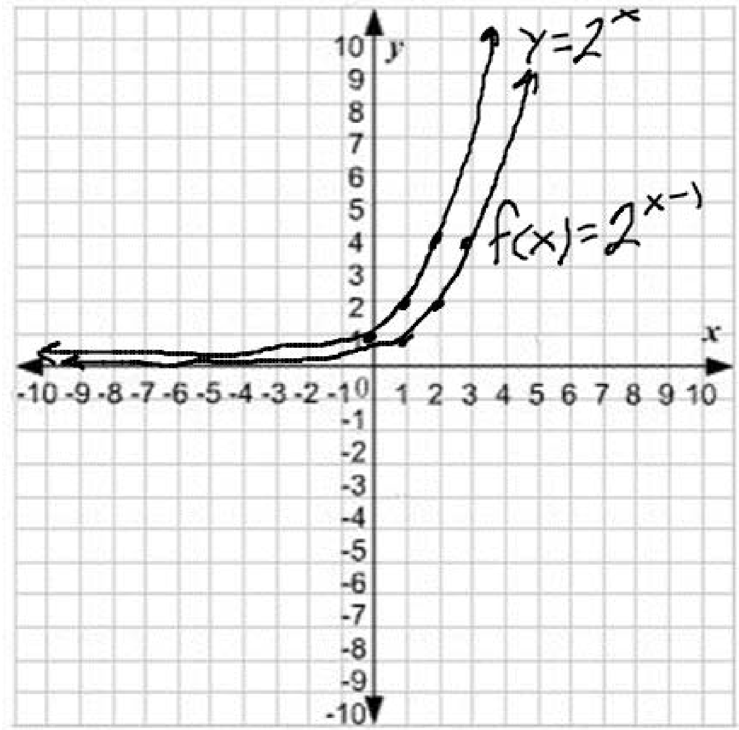
$(1,2) \longrightarrow (2,2)$

$(2,4) \longrightarrow (3,4)$

D: $(-\infty, \infty)$

R: $(0, \infty)$

Asymptote: $y = 0$



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Example 3:

Sketch the graph of the function $f(x) = -4^x + 10$. Do not plot points, but instead apply transformations to the graph of the function $y = 4^x$. Identify the domain, range, and asymptote.

HA: $Y=10$

The -4^x mean we are subtracting from 10.

Reflection in x-axis

$Y = -4^x$

$(0,1) \to (0,-1)$

$(1,4) \to (1,-4)$

Shift up 10

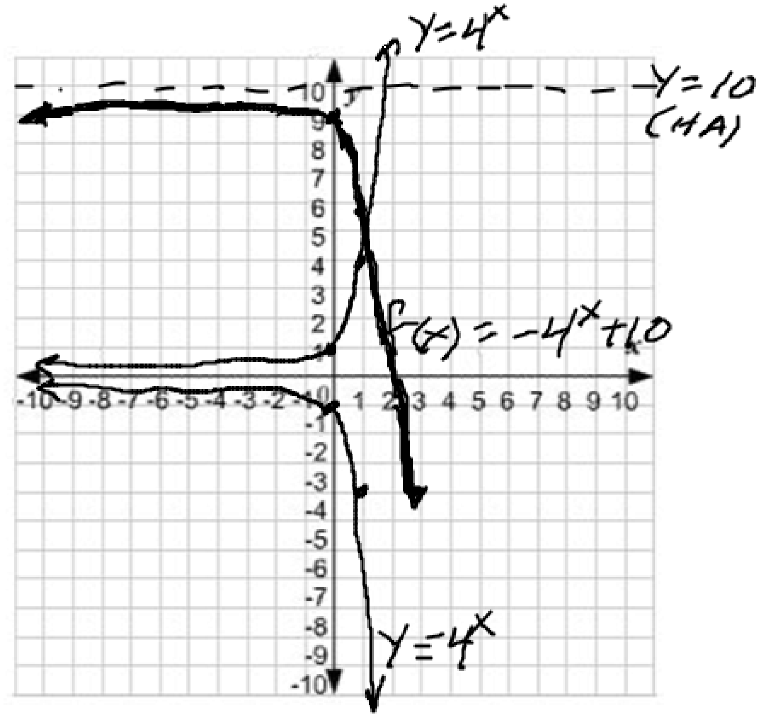
$(0,-1) \to (0,9)$

$(1,-4) \to (1,6)$

D: $(-\infty, \infty)$

R: $(-\infty, 10)$

HA: $Y=10$



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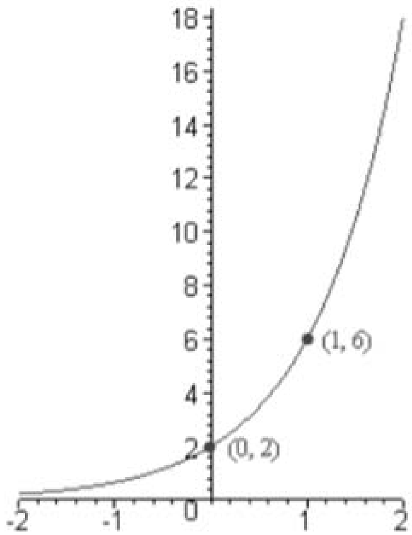
Example 4:

Find the function $f(x) = Ca^x$ whose graph is shown below.



$f(x) = Ca^x$

plug in $(0,2)$

$2 = C \cdot a^0$

$2 = C(1)$

$2 = C$

$f(x) = 2a^x$

plug in $(1,6)$

$6 = 2 \cdot a^1$

$\frac{6}{2} = \frac{\cancel{2}a}{\cancel{2}}$

$3 = a$

$f(x) = 2 \cdot 3^x$

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The number “e.”

More on transformations of the exponential function $f(x) = a^x$, but with $a = e$ *(the natural base)*.

Definition: e is the “limiting value” of $\left(1 + \frac{1}{x}\right)^x$ as x grows to infinity.

* $e \approx 2.718281282459$. It is an irrational number, like π. This means it cannot be written as a fraction nor as a terminating or repeating decimal.

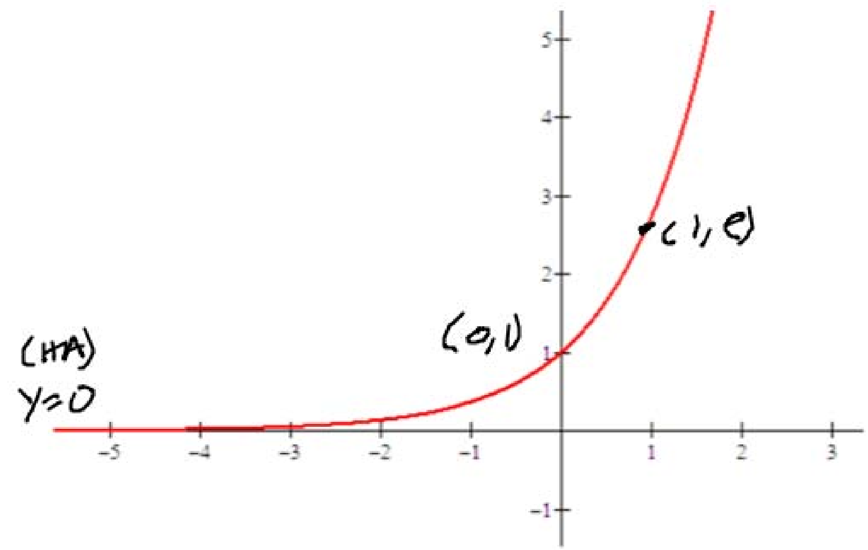
$f(x) = e^x$

side note

In case you were wondering, the letter “e” is used for this particular irrational number because of the mathematician Euclid used this constant extensively in his work.

Since $e > 1$, e can be the base of an exponential function. So everything we learned in Section 5.1 about graphing exponential functions will apply to graphing the function $f(x) = e^x$

The graph of $f(x) = e^x$ will have the following features:
Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Key point: $(0, 1)$
Horizontal asymptote: $y = 0$ since $y \to 0$ as $x \to -\infty$
(Left)



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Example 1: Sketch the graph of the function of $f(x) = -e^{x+2} + 2$ using transformations. State the domain, range, asymptote and translation of the key point.

HA

Horizontal Shift

$Y = e^{x+2}$ (Left 2)

Reflection:

$Y = -e^{x+2}$ (x-axis)

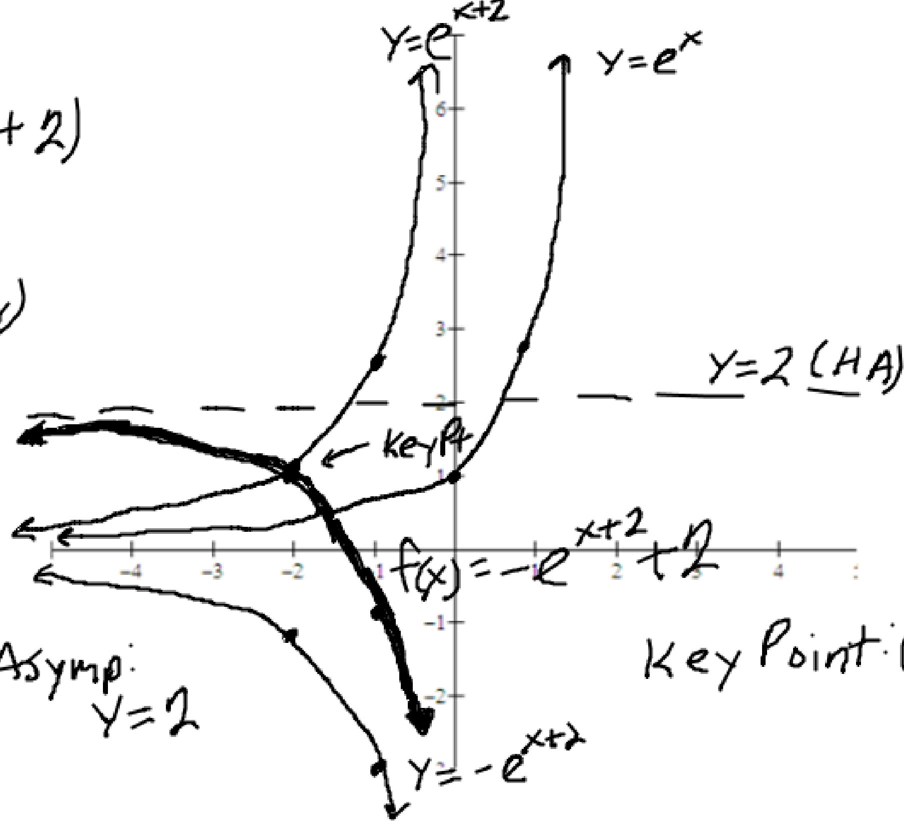
vertical shift

$Y = -e^{x+2} + 2$

(up 2)

D: $(-\infty, \infty)$

R: $(-\infty, -2)$



Asymp:
$Y=2$

Key Point: (-2,1)

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Example 2: Sketch the graph of the function of $f(x) = e^{-x-1} - \overbrace{1}^{HA}$ using transformations. State the domain, range, asymptote and translation of the key point.

Horizontal Shift

$Y = e^{x-1}$ (Right 1)

Reflection

$Y = e^{-x-1}$ (Y-axis)

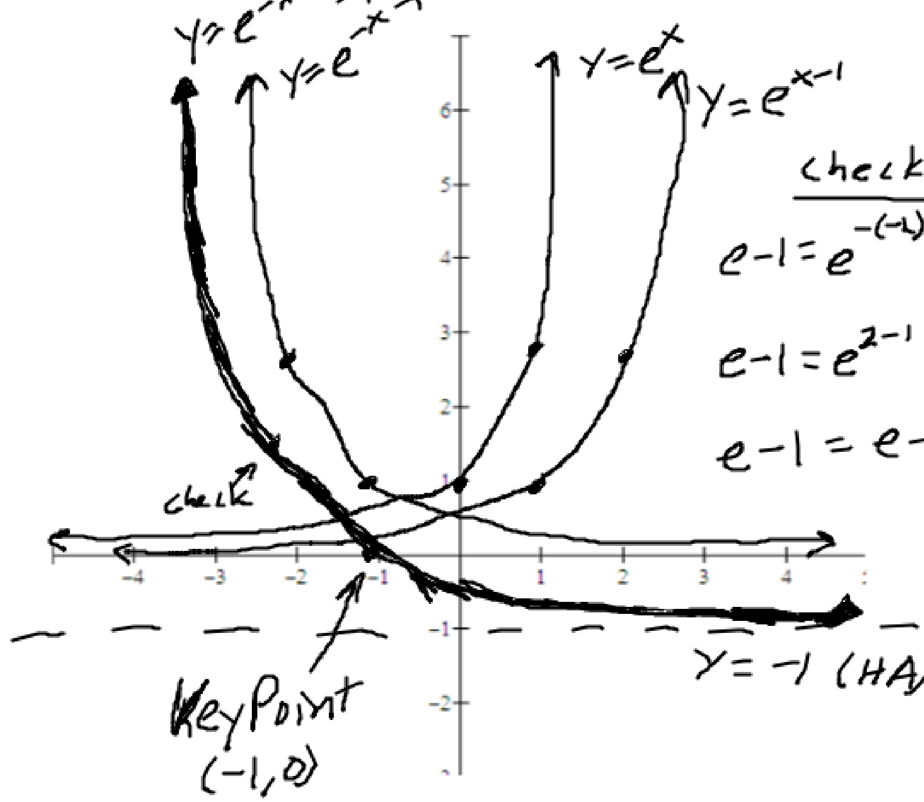
Vertical Shift

$Y = e^{-x-1} - 1$ (Down 1)

D: $(-\infty, \infty)$

R: $(-1, \infty)$

Asympt: $Y = -1$



check $(-2, e-1)$

$e - 1 = e^{-(-2)-1} - 1$

$e - 1 = e^{2-1} - 1$

$e - 1 = e - 1$ ✓

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Example 3:

Base e

Write the equation of a natural exponential function that has been shifted left 3 units, down 1 unit and reflected in the x-axis.

$Y = e^x$: natural exp.

$Y = e^{x+3}$: left 3

$Y = -e^{x+3}$: x axis reflection

$Y = -e^{x+3} - 1$: Down 1

or

$Y = -1 - e^{x+3}$

Example 3:

Popper 8:

Consider the function: $f(x) = -e^{x-2} + 3$.

1. Determine the domain of the function:

a. $(-\infty, \infty)$ b. $(3, \infty)$ c. $[3, \infty)$ d. $(-2, \infty)$

2. Determine the range of the function:

a. $(-\infty, \infty)$ b. $(-3, \infty)$ c. $(-\infty, 3)$ d. $(-\infty, 2)$

3. Determine the y-intercept of the function:

a. 1 b. $e^{-2} + 3$ c. $3 - e^2$ d. $\frac{3e^2-1}{e^2}$

4. Determine the asymptote of the function:

a. $y = e$ b. $x = e$ c. $y = 3$ d. $x = 3$

5. Which transformation did <u>not</u> take place from $g(x) = e^x$?

a. Horizontal Shift b. Vertical Shift c. Horizontal Reflection D. Vertical Reflection

Popper 8:

Logarithmic Functions

The exponential function is 1-1; therefore, it has an inverse function. The inverse function of the exponential function with base a is called the **logarithmic function with base *a***.

For $x > 0$ and $a > 0$ and a not equal to 1, $y = \log_a x$ is equivalent $a^y = x$

The function $f(x) = \log_a x$ is the **logarithmic function with base *a***

$Y = 2^x$

$Y^{-1} = \log_2 x$

Inverses

* The **common logarithm** is the logarithm with base 10. We denote this as $\log_{10} x = \log x$
* The **natural logarithm** is the logarithm with base e. We denote this as $\log_e x = \ln x$

You will find both of these logarithms on a scientific calculator.

Note: We do not typically write either $\log_{10} x$ or $\log_e x$.

$\log x$ $\ln x$

Logarithmic Functions

Example 1: Write each equation in its equivalent exponential form.

$a^b = c \Rightarrow \log_a c = b$

a. $3 = \log_6 x$

$6^3 = x$

b. $2 = \log_a 64$

$a^2 = 64$

c. $\log_3 27 = 3$

$3^3 = 27$

d. $\log 100000 = 5$ (Assume base 10)

$10^5 = 100000$

e. $\ln \frac{1}{e^2} = -2$ (Assume base e)

$e^{-2} = \frac{1}{e^2}$

Example 2: Write each equation in its equivalent logarithmic form.

Answer to log is the exponent of the exponential

a. $4^3 = 64$ $\quad \log_4 64 = 3$

b. $2^6 = 64$ $\quad \log_2 64 = 6$

c. $e^x = 25$ $\quad \log_e 25 = x \rightarrow \ln 25 = x$

d. $10^x = 1000$ $\quad \log_{10} 1000 = x \rightarrow \log 1000 = x$

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Example 3: Evaluate, if possible. convert to exponentials

$\log_6 36 = x$

$6^x = 36$

$x = 2$

$\log_2 \frac{1}{8} = x$

$2^x = \frac{1}{8} = \frac{1}{2^3} = 2^{-3}$

$x = -3$

$\log_5 125 = x$

$5^x = 125$

$x = 3$

$\log_3\left(\sqrt[3]{81}\right) = x$

$3^x = \sqrt[3]{81} = \sqrt[3]{3^4} = 3^{4/3}$ (power/root)

$x = 4/3$

$\log_5 \sqrt[4]{125} = x$

$5^x = \sqrt[4]{125} = \sqrt[4]{5^3} = 5^{3/4}$

$x = 3/4$

$\log 100 = \log_{10} 100 = x$

$10^x = 100 \qquad x = 2$

$\log_4 2 = x$

$4^x = 2 = \sqrt{4} = 4^{1/2} \qquad x = 1/2$

$\log 0.001 = \log_{10} 0.001 = \log_{10} \frac{1}{1000} = \log_{10} 10^{-3} = x$

$10^x = 10^{-3} \qquad x = -3$

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Inverse Property of Logarithms

For $a > 0$ and $a \neq 1$

1. $\log_a a^x = x$

2. $a^{\log_a x} = x$

Base of log must match the base of exponential

$\log_7 7^{5x} = 5x$

$\ln e^{x+2} = x+2$

$5^{\log_5 x^2} = x^2$

$10^{\log 2x} = 2x$

Example 4: Evaluate.

a. $\log_{14} 14^3 = 3$

b. $5^{\log_5 34} = 34$

c. $e^{\ln 32} = 32$

d. $\log_{47} 47^{\pi} = \pi$

Recall that for $x > 0$ (and $a > 0$ and a not equal to 1), we have $f(x) = \log_a x$. So the domain of $f(x) = \log_a x$ consist of all x for which $x > 0$.

Domain of $y = \log_2 (5x - 15)$

$5x - 15 > 0$

$5x > 15$

$x > 3$

Inside part set greater than zero.

Popper 8...continued

Example 5: Find the domain.

6. $f(x) = \log_2(x - 2)$

   a. (2, ∞) b. [2, ∞) c. (-∞, 2) d. (-∞, ∞)

7. $f(x) = \ln(7 - 2x)$

   a. (3.5, ∞) b. [3.5, ∞) c. (-∞, 3.5) d. (-∞, ∞)

8. $f(x) = \log(x^2 + 1)$

   a. (1, ∞) b. (-∞, -1) U (1, ∞) c. (-∞, -1) d. (-∞, ∞)

Popper 8…continued

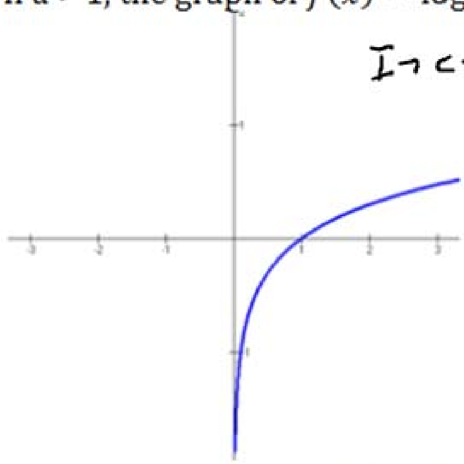
Characteristics of the Graphs of Logarithmic Functions of the Form $f(x) = \log_a x$

key point

1. The x-intercept is $(1, 0)$ and there is no y-intercept.
2. The y-axis is a vertical asymptote. $x=0$
3. The domain is all positive real numbers.
4. The range is all real numbers.

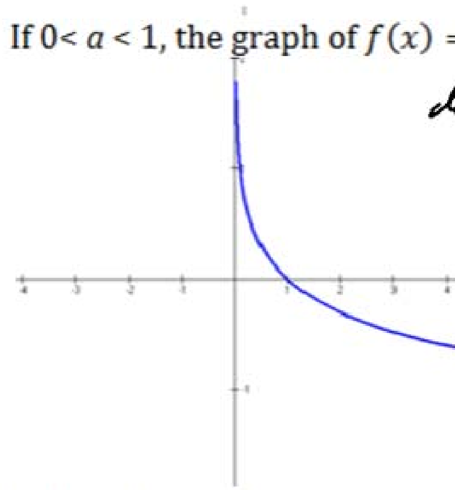
If $a > 1$, the graph of $f(x) = \log_a x$ looks like:

Increasing



If $0 < a < 1$, the graph of $f(x) = \log_a x$ looks like:

decreasing



Note: If a logarithmic function is translated to the left or to the right, the vertical asymptote is shifted by the amount of the horizontal shift.

Example 6: Sketch the graph of $f(x) = \log_4(x+2)$. State the domain, range, asymptote and key point.

$Y = \log_4 x$

$(1,0) \quad (4,1)$

$Y = \log_4 4 \qquad VA: X=0$

$4^Y = 4$

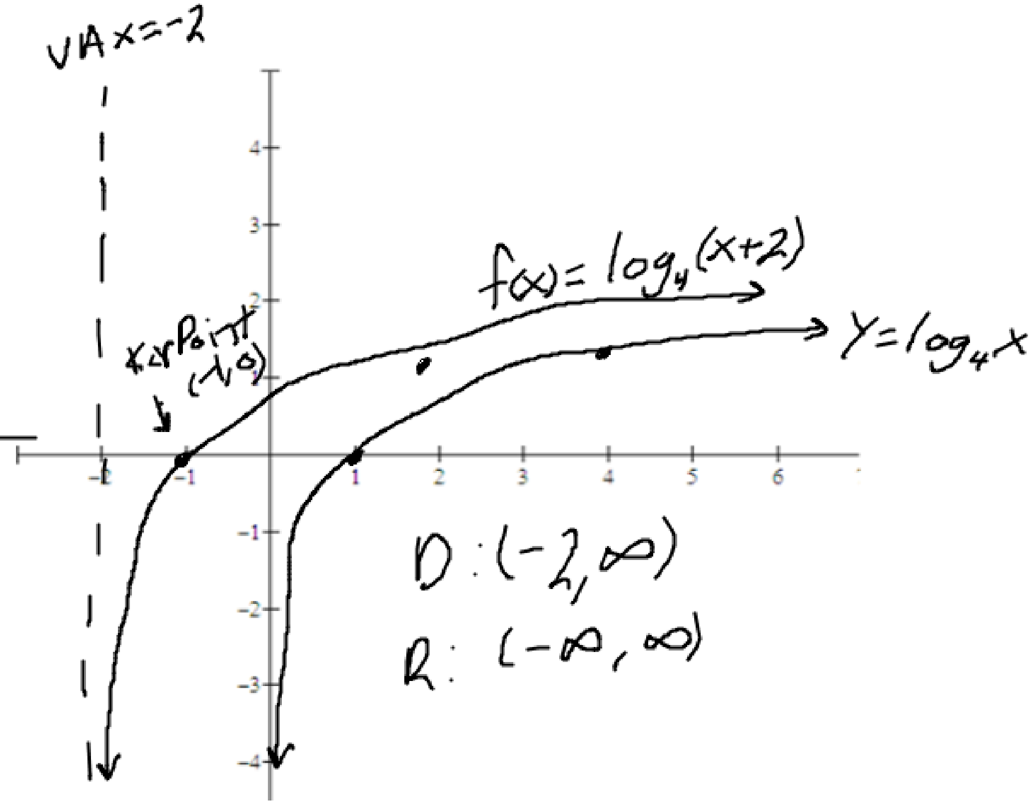
$Y = 1$

$f(x) = \log_4(x+2)$

Left 2 $\qquad$ VA:

$(1,0) \rightarrow (-1,0) \quad x=-2$

$(4,1) \rightarrow (2,1)$



$D: (-2, \infty)$

$R: (-\infty, \infty)$

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Example 7: Sketch the graph of $f(x) = -\ln(x-1) + 1$. State the domain, range, asymptote and key point.

$f(x) = -\ln(x-1)+1$

$y = \ln x$: parent

$y = \ln(x-1)$: right 1

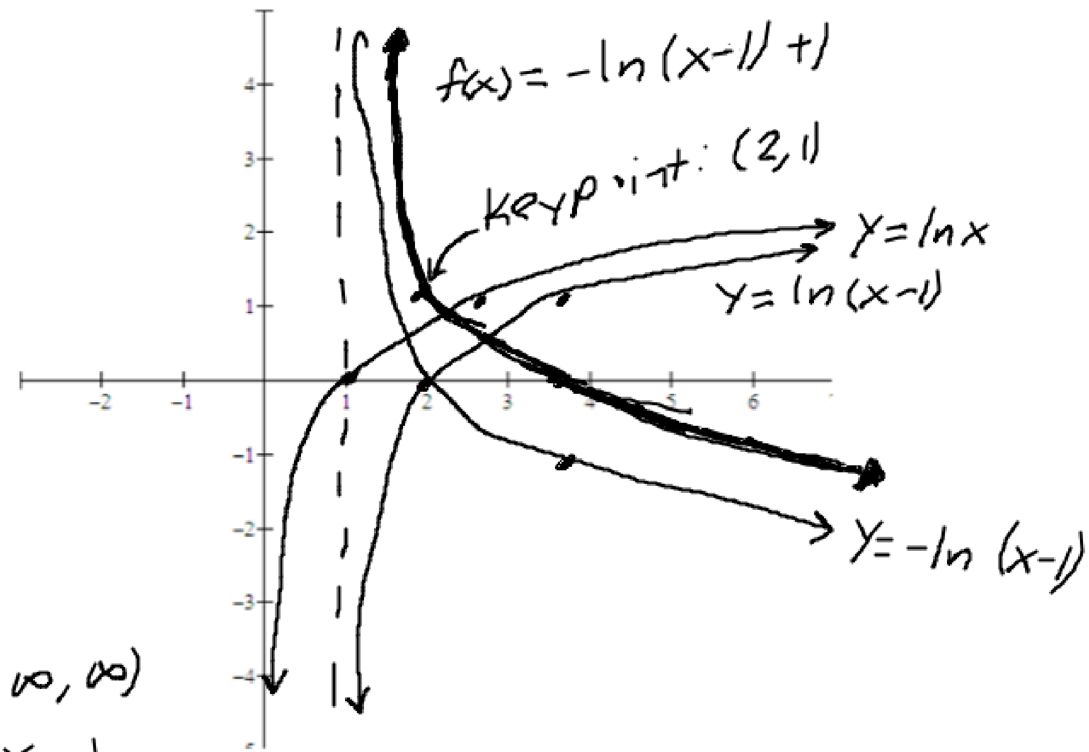
$y = -\ln(x-1)$:
x axis refl.

$f(x) = -\ln(x-1)+1$
up 1

D: $x-1>0$
$x>1$ $(1, \infty)$

R: $(-\infty, \infty)$

VA: $x=1$



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Properties of Logarithms

#8. $\log_5 7 = \frac{\ln 7}{\ln 5}$

You pick the new base.

You will sometimes be asked to rewrite logarithmic expressions in either an expanded or contracted form. To do this, you will use the Laws of Logarithms. You must know all 8 of the Laws of Logarithms.

Laws of Logarithms

If m, n and a are positive numbers, $a \neq 1$, then

1. $\log_a mn = \log_a m + \log_a n$
2. $\log_a \frac{m}{n} = \log_a m - \log_a n$
3. $\log_a m^n = n\log_a m$
4. $\log_a 1 = 0$
5. $\log_a a = 1$ $\quad \log_2 2 = 1$
6. $\log_a a^x = x$ $\quad \log_2 2^x = x$
7. $a^{\log_a x} = x$ $\quad 2^{\log_2 x} = x$
8. $\log_a m = \frac{\log m}{\log a}$ (change of bases formula)

These properties are true for logs of any base, including common logs and natural logs.

Properties of Logarithms

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power. Anything in a denominator should have a neg log.

a. $\log\left(\frac{3x}{y}\right) = \log(3x) - \log y = \log 3 + \log x - \log y$

b. $\ln(a^3bc) = \ln a^3 + \ln b + \ln c = 3\ln a + \ln b + \ln c$

c. $\log_4\left(\frac{4}{x}\right) = \log_4 4 - \log_4 x = 1 - \log_4 x$

Example 1: Rewrite each of the following expressions in a form that has no logarithms of a product, quotient or power.

d. $\log\sqrt{xy} = \log(xy)^{1/2} = \frac{1}{2}\log(xy) = \frac{1}{2}[\log x + \log y] = \frac{1}{2}\log x + \frac{1}{2}\log y$

e. $\ln\left(\frac{\sqrt[3]{x+4}}{(x+2)^4(x-5)^3}\right) = \ln(x+4)^{1/3} - \ln(x+2)^4 - \ln(x-5)^3$

$= \frac{1}{3}\ln(x+4) - 4\ln(x+2) - 3\ln(x-5)$

f. $\log\sqrt{\frac{(x-1)(x+2)^3}{x^2(x-2)}} = \frac{1}{2}\left[\log(x-1) + \log(x+2)^3 - \log x^2 - \log(x-2)\right]$

$= \frac{1}{2}\left[\log(x-1) + 3\log(x+2) - 2\log x - \log(x-2)\right]$

$= \frac{1}{2}\log(x-1) + \frac{3}{2}\log(x+2) - \log x - \frac{1}{2}\log(x-2)$

Example 2: Express each as a single logarithm:

1. $\log x - 3\log y = \log x - \cancel{3}\log y^3 = \log \frac{x}{y^3}$

a. $\log(x - y^3)$ $\quad$ $\boxed{\text{b. } \log\left(\frac{x}{y^3}\right)}$ $\quad$ c. $\log_3(x - y)$ $\quad$ d. $\log\left(\frac{x}{y}\right)^3$

2. $2\ln x + 3\ln(x+2) - 5\ln y = \ln x^2 + \ln(x+2)^3 - \ln y^5 = \ln\left(\frac{x^2(x+2)^3}{y^5}\right)$

a. $\ln\left(\frac{x^5+8x^2}{y^5}\right)$ $\quad$ b. $\ln\left(\frac{(x^2+2x)^5}{5y}\right)$ $\quad$ c. $\ln\left(\frac{x+2}{xy}\right)^5$ $\quad$ $\boxed{\text{d. } \ln\left(\frac{x^2(x+2)^3}{y^5}\right)}$

3. $\frac{1}{2}\ln(x+2) - 3\ln(x^3+1) = \ln\sqrt{x+2} - \ln(x^3+1)^3 = \ln \frac{\sqrt{x+2}}{(x^3+1)^3}$

$\boxed{\text{a. } \ln\left(\frac{\sqrt{x+2}}{(x^3+1)^3}\right)}$ $\quad$ b. $\ln\sqrt{\frac{x+2}{(x^3+1)^3}}$ $\quad$ c. $\ln\left(\frac{\sqrt{x+2}}{x^9+1}\right)$ $\quad$ d. $\ln\left(\frac{x+2}{2(x^3+1)^3}\right)$

Example 2: Express each as a single logarithm:

d. $-2\log_4(x-5) - \log_4(x+1) + 2\log_4(x^2+1)$

$$-\log_4 (x-5)^2 - \log_4(x+1) + \log_4 (x^2+1) = \log_4 \frac{(x^2+1)}{(x+5)^2(x+1)}$$

e. $\ln 18 - \ln 2 = \ln \frac{18}{2} = \ln 9$

Example 3: Rewrite each as sums so that each logarithm contains a prime number.

a. $\ln 72 = \ln(2^3 \cdot 3^2)$

$= \ln 2^3 + \ln 3^2 = \boxed{3\ln 2 + 2\ln 3}$

$72 = 2^3 \cdot 3^2$

72

$2 \cdot 36$

$6 \cdot 6$

$3 \cdot 2 \cdot 3 \cdot 2$

b. $\log_2 96$

$\log_2(2^5 \cdot 3) = \log_2 2^5 + \log_2 3$

$= \boxed{5 + \log_2 3}$

$96 = 2^5 \cdot 3$

96

$24 \cdot 4$

$4 \cdot 6 \cdot 2 \cdot 2$

$2 \cdot 2 \cdot 3 \cdot 2$

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Example 4: Use the change of bases formula to solve $\log_8 12 = x$and write in simplest form. Then use a calculator to evaluate to the nearest thousandth.

$$\log_8 12 = x$$

common log: $\dfrac{\log 12}{\log 8} = \dfrac{1.079}{0.903} = 1.195$

natural log: $\dfrac{\ln 12}{\ln 8} = \dfrac{2.485}{2.079} = 1.195$

same answer

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Example 5: Simplify each.

a. $\log_4 16^3 = \log_4 4^6 = \boxed{6}$

$16^3 = (4^2)^3 = 4^6$

b. $7^{3\log_7 3}$

$3\log_7 3 = \log_7 3^3 = \log_7 27$

$7^{\log_7 27} = \boxed{27}$

Solving Exponential and Logarithmic Equations

We'll start with exponential equations.
An exponential equation is an equation in which the variable appears in the exponent. To solve these equations, isolate the exponential expression on one side of the equation, then take the logarithm of both sides of the equation to solve for the variable. You can use either natural logarithms or common logarithms. Read the directions carefully, as theymay instruct you as to which to use.

$$2^{x+5} \cancel{-8} = 4$$
$$\cancel{+8} \qquad +8$$
$$2^{x+5} = 12$$

$$\ln 2^{x+5} = \ln 12$$
$$\frac{(x+5)\cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 12}{\ln 2}$$
$$x+5 = \frac{\ln 12}{\ln 2}$$
$$\boxed{x = \frac{\ln 12}{\ln 2} - 5 = \log_2 12 - 5}$$

Solving Exponential and Logarithmic Equations

Example 1: Solve for x: $5^{3x} = 9$. (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

$$5^{3x} = 9$$

$$\ln 5^{3x} = \ln 9$$

$$\frac{\cancel{3}x \cancel{\ln 5}}{\cancel{3} \cancel{\ln 5}} = \frac{\ln 9}{3 \ln 5}$$

$$x = \underset{(a)}{\frac{\ln 9}{3 \ln 5}} = \frac{2.197}{3(1.609)} = \underset{(b)}{0.455}$$

c) $$\frac{\ln 9}{3 \ln 5} = \frac{\ln 3^2}{3 \ln 5} = \boxed{\begin{aligned}&\frac{2 \ln 3}{3 \ln 5}\\ &= \frac{2}{3} \log_5 3\end{aligned}}$$

Apply change of base formula

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Popper 8:

Example 2: Solve for x: $4e^{(x+5)} + 5 = 7$. (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

9. Simplify the equation:
   a. $e^{(x+5)} = 2$ b. $e^{(x+5)} = ½$ c. $e^{(x+5)} = 3$ d. $e^{(x+5)} = 48$

10. Solve the equation:
    a. $x = \ln(½) + 5$ b. $x = \log(½) – 5$ c. $x = \ln(½) – 5$ d. $x = \ln(½)$

11. Approximate the answer (Calculator):
    a. $x \approx 4.317$ b. $x \approx -5.301$ c. $x \approx -5.693$ d. $x \approx -0.693$

12. Simplify the logarithmic answer (Question 2):
    a. $x = \ln(-3)$ b. $x = \ln(-4.5)$ c. $x = \ln(½) – 5$ d. $x = -\ln(2) – 5$

Example 3: Solve for x: $e^{2x} - 9e^{x} + 20 = 0$. (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

$$e^{2x} - 9e^{x} + 20 = 0$$

$* \; u = e^{x}$

$u^2 = e^{2x}$

$$u^2 - 9u + 20 = 0$$

$$(u-4)(u-5) = 0$$

$u - 4 = 0$ | $u - 5 = 0$

$u = 4$ | $u = 5$

$u = e^{x}$

$4 = e^{x}$ $\qquad$ $5 = e^{x}$

$\ln 4 = \ln e^{x}$ $\qquad$ $\ln 5 = \ln e^{x}$

(a) $\boxed{\ln 4 = x}$ $\qquad$ $\boxed{\ln 5 = x}$

(b) $\boxed{x = 1.386}$ $\qquad$ $\boxed{x = 1.609}$

(c) $x = \ln 4 = \ln 2^2$

$\boxed{x = 2\ln 2}$ $\qquad$ $\boxed{x = \ln 5}$

Example 4: Solve for x: $25^{3x-2} = \frac{1}{(\sqrt{125})^x}$ (a) Give the exact value using natural logarithms. (b) Use a calculator to approximate the solution to the nearest thousandth. (c) Rewrite the results from part (a) so that each logarithm contains a prime number.

$$25^{3x-2} = \frac{1}{\sqrt{125}^x} = \left(\frac{1}{\sqrt{125}}\right)^x$$

$$\ln 25^{3x-2} = \ln\left(\frac{1}{\sqrt{125}}\right)^x$$

$$(3x-2)\ln 25 = x\ln\left(\frac{1}{\sqrt{125}}\right)$$

$$(3x-2) = x\,\frac{\ln(1/\sqrt{125})}{\ln 25} = x \cdot \frac{-\frac{3}{2}\ln 5}{2\ln 5}$$

$$\ln\frac{1}{\sqrt{125}} = \ln(\sqrt{125})^{-1} = \ln(5)^{-3/2} = -\frac{3}{2}\ln 5$$

$$\ln 25 = \ln 5^2 = 2\ln 5$$

$$3x-2 = \frac{-3}{4}x$$

$$\cancel{12x} - 8 = -3x$$

$$\cancel{-12x} \qquad -12x$$

$$-8 = -15x$$

$$\boxed{\frac{8}{15} = x}$$ (a, b, c)

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$$25^{3x-2} = \frac{1}{(\sqrt{125})^{x}}$$

$$(5^{2})^{3x-2} = 5^{-3/2\,x}$$

$$5^{6x-4} = 5^{-3x/2}$$

$$6x-4 = \frac{-3x}{2}$$

$$12x-8 = -3x$$

$$-8 = -15x$$

$$\boxed{8/15 = x}$$

An equation in which the logarithm of the variable occurs is called a logarithmic equation. To solve such an equation, isolate the logarithmic expression on one side of the equation. Then write the equation in exponential form and solve for the variable, or use the law of logarithms to simplify and solve for the variable.

Example 5: Solve for x: $\log_3(x-4) = 2$

$$\log_3 (x-4) = 2$$

$$3^2 = x-4$$

$$9 = x-4$$

$$\boxed{13 = x}$$

Be sure to check answers for undefined values.

Cannot have logs of negatives.

Example 6: Solve for x: $\log_6(x) + \log_6(x+1) = \log_6 2$

$$\log_6(x) + \log_6(x+1) = \log_6 2$$

$$\log_6\big(x(x+1)\big) = \log_6 2$$

$$\log_6(x^2+x) = \log_6 2$$

$$x^2+x = 2$$

$$x^2+x-2 = 0$$

$$(x+2)(x-1) = 0$$

$$(x+2)(x-1) = 0$$

$x+2=0$ | $x-1=0$

$x=-2$ | $\boxed{x=1}$

reject

Check original logarithms:

$\log(x) \to \cancel{\log(-2)}$ No

$\log(1)$ Yes

$\log(x+1) \to \log(1+1)$

$\log(2)$ Yes

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Example 7: Solve for x: $\log_6 x + \log_6(x+5) = 2$

$$\log_6(x^2+5x) = 2$$

$$6^2 = x^2+5x$$

$$36 = x^2+5x$$

$$x^2+5x-36=0$$

$$(x+9)(x-4)=0$$

$x+9=0$ | $x-4=0$

$x=-9$ reject | $\boxed{x=4}$

Check:

$\log(-9)+\log(-4)$

reject $x=-9$

$\log(4)+\log(9)$

Good.

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Example 8: Solve for x: $\ln(x^2 + 4) = 2$

$\ln(x^2+4) = 2$ (Base e)

$e^2 = x^2+4$

remember: $e \approx 2.7$

so $e^2-4 > 0$

$e^2-4 = x^2$

x^2+4 is always positive.

$$\boxed{\pm\sqrt{e^2-4} = x}$$