MATH 1314

Final Exam Review

30 Multiple Choice Questions (Equally Weighted)

1. Simplify: (7 – i)(2 + i) FOIL

$14 + 7i - 2i - i^2$ $\quad i^2 = -1$

$\underbrace{-i^2}_{-(-1) = +1}$

$14 + 7i - 2i + 1$

$15 + 5i$

2. Simplify: $\dfrac{5-i}{3+i}$

$$\frac{(5-i)}{(3+i)}\cdot\frac{(3-i)}{(3-i)} = \frac{15-5i-3i+\underbrace{i^2}_{\to -1}}{9-\cancel{3i}+\cancel{3i}\underbrace{-i^2}_{\to +1}}$$

$$= \frac{14-8i}{10} = \frac{14^{\div 2}}{10_{\div 2}} - \frac{8^{\div 2}}{10_{\div 2}}i$$

$$\frac{7}{5}-\frac{4}{5}i$$

$a+bi$ form

3. Solve for x: $|5x + 8| < 3$

$|abs| < Num \rightarrow -Num < abs < +Num$

$|abs| > Num \rightarrow abs < -Num \text{ or } abs > +Num$

$$-3 < 5x + \cancel{8} < 3$$
$$-8 \quad\quad \cancel{-8} \quad\quad -8$$

$$-11 < \cancel{5}x < -5$$
$$\frac{-11}{5} \quad \frac{\cancel{5}}{\cancel{5}} \quad \frac{-5}{5}$$

$$-11/5 < x < -1$$

$$(-11/5, -1)$$

4. Find the domain: $f(x) = \frac{\sqrt{x+3}}{x-8}$

Denominator: $x - \cancel{8} \neq 0$

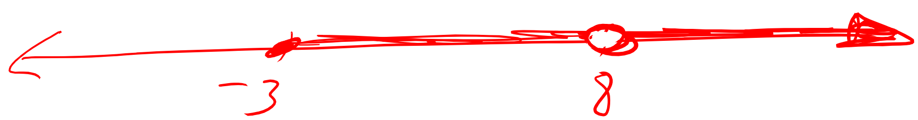
$\quad\quad\quad\quad\; \cancel{+8} \quad +8$

$x \neq 8$

Radical: $x + \cancel{3} \geq 0$

$\quad\quad\quad\; \cancel{-3} \quad -3$

$x \geq -3$



$[-3, 8) \cup (8, \infty)$

5. Calculate f(5) if $f(x) = \begin{cases} x+3, x<0 \\ x^2, 0 \le x \le 5 \\ 2x-3, x>5 \end{cases}$

is 5 < 0 No

is 5 between [0, 5] Yes

is 5 > 5 No

$f(5) = 5^2 = 25$

[middle]

6. Solve for x using substitution:

$$3(x-5) - 11\sqrt{x-5} - 4 = 0$$

$u = \sqrt{x-5}$ $u^2 = (\sqrt{x-5})^2 = x-5$

$3u^2 - 11u - 4 = 0$ $3(-4) = -12$, $-12, +1$

$(3u^2 - 12u) + (1u - 4) = 0$

$3u(u-4) + 1(u-4) = 0$

$(u-4)(3u+1) = 0$

$u - 4 = 0$ $3u + 1 = 0$

$u = 4$ $3u = -1$

$u = -1/3$

$u = 4$

$\sqrt{x-5}^2 = 4^2$

$x - 5 = 16$

$\boxed{x = 21}$

check:

$\sqrt{21-5} = 4$

$\sqrt{16} = 4$

$4 = 4$ ✓

$u = -1/3$

$\sqrt{x-5}^2 = (-1/3)^2$

$x - 5 = 1/9$

$x = 1/9 + 5 = \frac{1}{9} + \frac{45}{9} = \frac{46}{9}$

rejected

check:

$\sqrt{46/9 - 5} = -1/3$

$\sqrt{46/9 - 45/9} = -1/3$

$\sqrt{1/9} = -1/3 \rightarrow 1/3 \neq -1/3$

7. What transformations will take the graph of $f(x) = x^3$ to the graph of $g(x) = 5 – (x + 1)^3$

$g(x) = -(x+1)^3 + 5$ $y = x^3$ iparent function

Horizontal shift: Left 1 $y = (x+1)^3$

Reflections: x-axis reflection: $y = -(x+1)^3$

Vertical shift: up 5: $y = -(x+1)^3 + 5$

Left 1, x-axis reflection, up 5

8. Find the vertex of the following: $f(x) = 2x^2 + 8x + 6$

$a=2 \quad b=8 \quad c=6$

$$x = \frac{-b}{2a} = \frac{-8}{2(2)} = \frac{-8}{4} = -2 \rightarrow h$$

$$f(-2) = 2(-2)^2 + 8(-2) + 6 = 2(4) + 8(-2) + 6 = 8 - 16 + 6 = -2 \rightarrow k$$

vertex: $(-2, -2)$ → Minimum

(h, k)

$a > 0$: opens up

9. If $f(x) = 2x^2 + 1$ and $g(x) = x - 3$, determine $g(f(2))$

$g(f(2)) = g(9) = 9 - 3 = \boxed{6}$

$f(2) = 2(2)^2 + 1 = 2(4) + 1 = 8 + 1 = 9$

10. Determine the inverse of: $f(x) = \frac{x+5}{x-3}$

VA: $x=3$
HA: $y=1$

$y = \frac{x+5}{x-3}$

$\frac{x}{1} = \frac{y+5}{y-3}$

$$\begin{aligned} y+5 &= xy-3x \\ -xy-5 &\quad -xy \quad -5 \end{aligned}$$

$y - xy = -3x - 5$

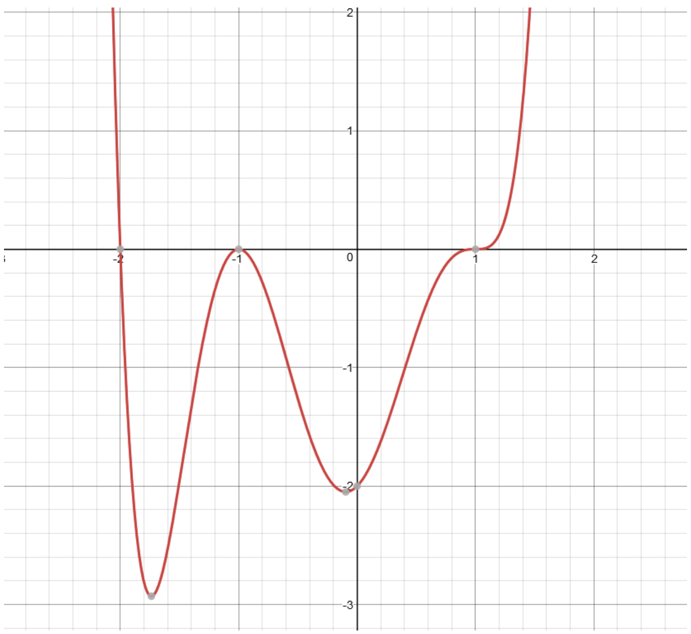
$\frac{y(\cancel{1-x})}{\cancel{1-x}} = \frac{-3x-5}{1-x}$

$y = \frac{(-3x-5)\cdot -1}{(1-x)\cdot -1} = \frac{3x+5}{x-1}$

$f^{-1}(x) = \frac{3x+5}{x-1}$

VA: $x=1$
HA: $y=3$

11. Identify the function corresponding to the graph:



End Behavior: L: ↑ R: ↑ (Even degree, Positive)

x-int: X = -2 (Linear): M: 1

X = -1 (Quadratic): M: 2

X = 1 (Cubic) M: 3

Y int: Y = -2 → (0, -2)

$$P(x) = a\underset{x=-2}{(x+2)^1}\underset{x=-1}{(x+1)^2}\underset{x=1}{(x-1)^3}$$

$$-2 = a(0+2)(0+1)^2(0-1)^3$$

$$-2 = a(2)(1)^2(-1)^3$$

$$-2 = a(2)(1)(-1)$$

$$-2 = a(-2)$$

$$1 = a$$

$$\boxed{P(x) = (x+2)(x+1)^2(x-1)^3}$$

Leading Term:

$$x^1 \cdot (x)^2 \cdot (x)^3 = x^6$$

pos, even deg

12. Identify the quotient and remainder of the following:

$$\frac{3x^3 + 2x - 4}{x + 3} = \frac{3x^3 + 0x^2 + 2x - 4}{x + 3}$$

$$\begin{array}{r|rrrr} & 3 & 0 & 2 & -4 \\ -3 & \downarrow & -9 & 27 & -87 \\ \hline & 3 & -9 & 29 & -91 \end{array}$$

$R(x)$

$$\boxed{\begin{array}{l} Q(x) = 3x^2 - 9x + 29 \\ R(x) = -91 \end{array}}$$

$$\boxed{3x^2 - 9x + 29 - \frac{91}{x+3}}$$

13. Identify the asymptote and range of the following:

$f(x) = 8 - 2^x = -2^x \underbrace{+8}_{HA}$

Asymptote: $\cancel{x =}$ (HA) y = 8

Range: Exp Term is Neg : Below HA

$(-\infty, 8)$

14. Solve $\log_5(x + 3) = 2$

$5^2 = x + 3$

$25 = x + 3$

$\boxed{22 = x}$

Check

$x + 3 \to 22 + 3 = 25$ ✓

15. Solve: $\log_8(x + 3) - \log_8(x - 6) = \log_8 4$

$$\log_8 \frac{x+3}{x-6} = \log_8 4$$

$$\frac{x+3}{x-6} = \frac{4}{1}$$

$$x+3 = 4x-24$$
$$-x \qquad -x$$

$$3 = 3x - 24$$
$$+24 \qquad +24$$

$$\frac{27}{3} = \frac{3x}{3}$$

$$\boxed{9 = x}$$

Check: $x=9$

$x+3 \rightarrow 9+3 = 12$ ✓

$x-6 \rightarrow 9-6 = 3$ ✓

16. Solve the following for x: $\frac{2}{x^2} + \frac{8}{x} = \frac{-8}{1}$

CD: x^2

$$\frac{2}{x^2} + \frac{8}{x} \cdot \frac{x}{x} = \frac{-8}{1} \cdot \frac{x^2}{x^2}$$

$$\frac{2}{\cancel{x^2}} + \frac{8x}{\cancel{x^2}} = \frac{-8x^2}{\cancel{x^2}}$$

$$2 + 8x = -8x^2$$

$$+8x^2 \qquad +8x^2$$

$$\frac{8x^2 + 8x + 2}{2} = \frac{0}{2}$$

$$4x^2 + 4x + 1 = 0$$

Factor: $4(1) = 4$

$2, 2$

$$(4x^2 + 2x) + (2x + 1) = 0$$

$$2x(2x+1) + 1(2x+1) = 0$$

$$(2x+1)(2x+1) = 0$$

$$(2x+1)^2 = 0$$

$$2x + 1 = 0$$

$$2x = -1$$

$$\boxed{x = -1/2}$$

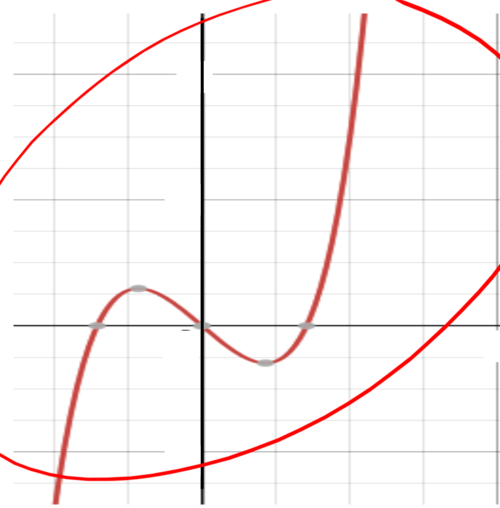
17. The function $f(x)= ax^5 + bx^3 - cx^{1}$ passes through the point (8, -7). What other point must it pass through? (-8, 7)

All odd exponents → odd function

Which is a possible graph of this function:

Even

(a, b) → (-a, b)



Neither

18. Solve this system:

$$5x + 4y = 6$$
$$y = -\frac{5}{4}x + 3$$

Substitution

$5x + 4\left(-\frac{5}{4}x + 3\right) = 6$

$\cancel{5x} - \cancel{5x} + 12 = 6$

$12 = 6$

Not True

No Solution

19. Rewrite the equation of the parabola in standard form: y = 3x^2 + 12x +7

$f(x) = a(x-h)^2+k$

$3 \to a$

$x = \frac{-b}{2a} = \frac{-12}{2(3)} = \frac{-12}{6} = -2 \to h$

$f(-2) = 3(-2)^2 + 12(-2) + 7 = 3(4) + 12(-2) + 7 = 12 - 24 + 7 = -5 \to k$

$f(x) = 3(x+2)^2 - 5$

20. Simplify: $\dfrac{\sqrt{-100}+30}{\sqrt{-4}\cdot\sqrt{-25}}$

$\sqrt{-100} = 10i$

$\sqrt{-4} = 2i$

$\sqrt{-25} = 5i$

$$\frac{10i+30}{(2i)(5i)} = \frac{10i+30}{10i^2} \qquad (i^2 = -1)$$

$$\frac{10i+30}{-10} = \frac{10i}{-10} + \frac{30}{-10}$$

$$-i - 3 = -3 - i$$

21. Find all complex solutions to: $5x^2 = -120$

$$\frac{5x^2}{5} = \frac{-120}{5}$$

$$\sqrt{x^2} = \sqrt{-24}$$

$$x = \pm\sqrt{4}\sqrt{6}\sqrt{-1}$$

$$x = \pm 2i\sqrt{6}$$

$$\frac{120}{5} = \frac{100+20}{5} = 20+4 = 24$$

22. Solve for x: $\sqrt{x-1}+\cancel{7}=x$

$-7 \qquad -7$

$\left(\sqrt{x-1}\right)^2=(x-7)^2$

$(x-7)(x-7)$

$\cancel{x}-\cancel{1}=x^2-14x+49$

$-x+1 \qquad -x+1$

$0=x^2-15x+50$

$0=(x-5)(x-10)$

$x-5=0 \qquad x-10=0$

$x=5$ reject $\qquad \boxed{x=10}$

Check $x=5$

$\sqrt{5-1}+7=5$

$\sqrt{4}+7=5$

$2+7=5$

$9=5$ ✗

Check $x=10$

$\sqrt{10-1}+7=10$

$\sqrt{9}+7=10$

$3+7=10$

$10=10$ ✓

23. Solve the inequality: $7x^2 - 5x - 3 < \cancel{6x^2} + \cancel{3}$

$$-6x^2 \qquad -3 \qquad -\cancel{6x^2} \qquad \cancel{-3}$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x - 6 = 0 \qquad x + 1 = 0$$

$$x = 6 \qquad x = -1$$

No Yes No

-1 6

Test $x = 0$

$$7(0)^2 - 5(0) - 3 < 6(0)^2 + 3$$

$$-3 < 3 \quad \text{True}$$

$$\boxed{(-1, 6)}$$

24. Solve the inequality: $\dfrac{(x-3)(x+5)}{x+10} \geq 0$

Numerator:

$x-3=0 \Rightarrow x=3$

$x+5=0 \Rightarrow x=-5$

Denominator:

$x+10=0 \Rightarrow x=-10$

$(-10,-5] \cup [3,\infty)$

Den Values

inf

Test $x=-15$:

$\dfrac{(-15-3)(-15+5)}{(-15+10)} = \dfrac{N \cdot N}{N} = N < 0$ No

Test $x=-7$

$\dfrac{(-7-3)(-7+5)}{(-7+10)} = \dfrac{N \cdot N}{P} = P > 0$ Yes

Test $x=0$:

$\dfrac{(0-3)(0+5)}{(0+10)} = \dfrac{N \cdot P}{P} = N < 0$ No

Test $x=5$ $\dfrac{(5-3)(5+5)}{(5+10)} = \dfrac{P \cdot P}{P} = P > 0$ Yes

25. Write the polynomial function with roots of 3i and 5, with an y-intercept of 90.

$-3i$ $(x - \text{root})$

$$P(x) = a(x-5)\underbrace{(x-3i)(x+3i)}$$

$$P(x) = a(x-5)(x^2 \cancel{+3ix} \cancel{-3ix} - \underbrace{9i^2}_{\to +9})$$

$$P(x) = a(x-5)(x^2+9)$$

$$P(x) = a(x^3+9x-5x^2-45) = a(x^3-5x^2+9x-45)$$

current y-int: -45

$$-45 \cdot \boxed{} = +90 \; \downarrow \; a = -2$$

$$P(x) = -2x^3+10x^2-18x+90$$

26. Find the vertical asymptotes, horizontal asymptotes, and hole of the following:

$$f(x) = \frac{x^2+16x+60}{2x^2-72} = \frac{(x+10)(x+6)}{2(x+6)(x-6)}$$

$2(x^2-36)$

VA: (Den Only) : $x-6=0 \rightarrow x=6$

HA: (Compare Degrees) : $\frac{\text{Deg } 2}{\text{Deg } 2} \rightarrow \frac{1x^2}{2x^2}$ HA: $y=\frac{1}{2}$

$\frac{\text{Large Deg}}{\text{Small Deg}}$: No HA

$\frac{\text{Small Deg}}{\text{Large Deg}}$: HA: $y=0$

Hole (Num & Den) : $x+6=0 \rightarrow x=-6$

x-int (Num Only) : $x+10=0 \rightarrow x=-10$

y-int: $f(0)=\frac{60}{-72}=-\frac{5}{6}$

27. Expand the logarithmic expression:

$$\log_6\left(\frac{(x+3)^8}{x^4\sqrt{x-5}}\right)$$

all numerator factors → pos logs

all denominator factors → neg logs

$$\log_6 (x+3)^8 - \log_6 x^4 - \log_6 \sqrt{x-5}$$

$$8\log_6 (x+3) - 4\log_6 x - \frac{1}{2}\log_6 (x-5)$$

28. Simplify: $\log_3\left(\frac{12}{\sqrt{27}}\right) - \log_3(4)$

$\log_3(12) - \log_3\sqrt{27} - \log_3 4$

$\log_3(3\cdot 4) - \log_3\sqrt{3^3} - \log_3 4$ $\qquad \sqrt{3^3} = (3^3)^{1/2} = 3^{3/2}$

$\underbrace{\log_3 3} + \log_3 4 - \frac{3}{2}\log_3 3 - \log_3 4$

$1 + \cancel{\log_3 4} - \frac{3}{2}\cdot 1 - \cancel{\log_3 4}$

$1 - \frac{3}{2} = \frac{2}{2} - \frac{3}{2} = \boxed{\frac{-1}{2}}$

29. The polynomial,
$p(x) = x^4 - 10x^3 + 19x^2 - 30x + 48$ has a root located at (8,0). Determine all roots of the polynomial.

Factor of $(x-8)$

$$\begin{array}{r|rrrrr} & 1 & -10 & 19 & -30 & 48 \\ 8 & \downarrow & 8 & -16 & 24 & -48 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array}$$

$0 \rightarrow R(x)$

$$(x^3 - 2x^2) + (3x - 6) = 0$$

$$x^2(x-2) + 3(x-2) = 0$$

$$(x-2)(x^2+3) = 0$$

$x - 2 = 0$, $x = 2$

$x^2 + 3 = 0$, $x^2 = -3$, $x = \pm\sqrt{-3} = \pm\sqrt{3}\,i$

$\{2, 8, \pm\sqrt{3}\,i\}$

1, 2, 3, 4