MATH 1314

Final Exam Review

30 Multiple Choice Questions (Equally Weighted)

1. Find the slope of the line that passes through the points $(4,6)$ and $(-2,-4)$.

x_1, y_1

x_2, y_2

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-6}{-2-4} = \frac{-10}{-6} = \frac{5}{3}$$

2. Find the x and y intercepts $2x + 8y + 2 = 0$.

x-int $(y=0)$

$2x + \cancel{8(0)} + 2 = 0$

$2x + \cancel{2} = 0$

$\quad\ \ \cancel{-2} \quad -2$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{-2}{2}$

$x = -1$

$(-1, 0)$

y-int: $(x=0)$

$\cancel{2(0)} + 8y + 2 = 0$

$8y + \cancel{2} = 0$

$\quad\ \cancel{-2} \quad -2$

$\frac{\cancel{8}y}{\cancel{8}} = \frac{-2}{8}$

$y = \frac{-1}{4}$

$(0, -1/4)$

3. **Solve for x:** $\frac{2}{3}\frac{x}{1} = \frac{4}{5}$

$$\frac{2x}{3} = \frac{4}{5}$$

$$\frac{\cancel{10}x}{\cancel{10}} = \frac{12}{10}$$

$$x = \frac{6}{5}$$

4. Solve for x: $\frac{1}{2}(x+1)-\frac{1}{3}(x-2)=\frac{4}{1}$ CD: 6

$$\frac{3}{3}\cdot\frac{1}{2}(x+1)-\frac{2}{2}\cdot\frac{1}{3}(x-2)=\frac{4}{1}\cdot\frac{6}{6}$$

$$\frac{3}{\cancel{6}}(x+1)-\frac{2}{\cancel{6}}(x-2)=\frac{24}{\cancel{6}}$$

$$3x+3-2x+4=24$$

$$x+7=24$$

$$-7 \quad -7$$

$$\boxed{x=17}$$

5. The perimeter of a rectangle is 70 m. If the length 4 times its width. Find the length of this rectangle.

$P = 70$

$l = 4 \cdot w$

$2l + 2w = 70$

$2(4w) + 2w = 70$

$8w + 2w = 70$

$\frac{\cancel{10}w}{\cancel{10}} = \frac{70}{10}$

$w = 7$

$l = 4 \cdot w$

$l = 4 \cdot 7 = \boxed{28}$

6. Find three consecutive integers whose sum is 336.

First Integer: x

Second Integer: $x+1$

Third Integer: $x+2$

$$x+(x+1)+(x+2)=336$$

$$3x+\cancel{3}=336$$

$$\quad\ \ -\cancel{3}\quad\ -3$$

$$\frac{\cancel{3}x}{\cancel{3}}=\frac{333}{3}$$

$$x=111$$

First: $x=111$

Second: $x+1=112$

Third: $x+2=113$

7. Solve by factoring: $2x^2 + 5x + 3 = 0$

Leading Coefficient $\neq 1$

$(2)(3) = 6$

2, 3 (Multiply to 6)

(Add to 5: Middle Term)

$$(2x^2+2x)+(3x+3)=0$$

$$2x(x+1)+3(x+1)=0$$

$$(x+1)(2x+3)=0$$

$$x+1=0 \qquad 2x+3=0$$

$$-1 \quad -1 \qquad -3 \quad -3$$

$$x=-1 \qquad \frac{2x}{2}=\frac{-3}{2}$$

$$x=-\frac{3}{2}$$

$$\left\{-\frac{3}{2}, -1\right\}$$

8. Solve ~~by factoring~~: $x^2 + 36 = 0$

$$x^2 + 36 = 0$$

$$-36 \quad -36$$

$$\sqrt{x^2} = \sqrt{-36}$$

$$\boxed{x = \pm 6i}$$

9. **Simplify:** $(2i - 1) - (1 - i)$

$2i - 1 - 1 + i$

$3i - 2$

$-2 + 3i$

10. Simplify: $3i(2-3i)$

$(i^2 = -1)$

$6i - 9i^2$

$6i - 9(-1)$

$6i + 9$

$\boxed{9 + 6i}$

11. Simplify: $\frac{(2+3i)}{(4+i)} \cdot \frac{(4-i)}{(4-i)} = \frac{8-2i+12i-\underbrace{3i^2}_{+3}}{16-\cancel{4i}+\cancel{4i}-\underbrace{i^2}_{+1}}$

conjugate (change sign of imaginary term)

$$= \frac{11+10i}{17} = \boxed{\frac{11}{17}+\frac{10}{17}i}$$

12. Simplify: $\dfrac{1}{(3-i)} \cdot \dfrac{3+i}{(3+i)} = \dfrac{3+i}{9+\cancel{3i}-\cancel{3i}\underbrace{-i^2}_{+1}}$

$$= \frac{3+i}{10} = \frac{3}{10} + \frac{1}{10}i$$

13. Solve for x: $-2 \le \frac{(3x+2)}{3} < 2$

$$3(-2) \le \frac{(3x+2)}{\cancel{3}}\cancel{3} < (2)3$$

$$-6 \le 3x + \cancel{2} < 6$$

$$\underline{-2 \qquad\qquad -\cancel{2} \qquad -2}$$

$$\frac{-8}{3} \le \frac{\cancel{3}x}{\cancel{3}} < \frac{4}{3}$$

$$-8/3 \le x < 4/3$$

$$[-8/3, 4/3)$$

Endpoint included

Endpoint Not Included

14. **Solve of x:** $5 + 2|x + 5| = 7$

$$\begin{aligned} \cancel{5} + 2|x+5| &= 7 \\ -5 \qquad\quad &\;\; -5 \end{aligned}$$

$$\frac{\cancel{2}|x+5|}{\cancel{2}} = \frac{2}{2}$$

$$|x+5| = 1$$

$$\begin{aligned} x + \cancel{5} &= 1 \\ -\cancel{5} &\;\; -5 \\ \hline x &= -4 \end{aligned} \qquad\qquad \begin{aligned} x + \cancel{5} &= -1 \\ -\cancel{5} &\;\; -5 \\ \hline x &= -6 \end{aligned}$$

$$\{-6, -4\}$$

15. Solve for x: $\cancel{-2}|x-1| \le -6$

$$\frac{-\cancel{2}}{} \qquad \frac{}{-2}$$

$$|x-1| \ge 3$$

$|\text{Abs}| > \text{Number}$

$\text{Abs} < -\text{Number}$ or $\text{Abs} > +\text{Number}$

$x \cancel{-1} \le -3$ or $x \cancel{-1} \ge 3$

$\quad +1 \quad +1 \qquad\qquad +1 \quad +1$

$x \le -2$ or $x \ge 4$

$(-\infty, -2] \cup [4, \infty)$

special case:

$|\text{Abs}| \ge \text{Neg}$

$(-\infty, \infty)$

16. Solve for x: $|3x - 4| < 5$

$-5 < 3x - 4 < 5$

$+4 \qquad +4 \qquad +4$

$\frac{-1}{3} < \frac{\cancel{3}x}{\cancel{3}} < \frac{9}{3}$

$-\frac{1}{3} < x < 3$

$(-\frac{1}{3}, 3)$

|Abs| < Number

− Number < Abs < + Number

special Case:

$|Abs| \le Neg$

No Solution

17. **Find the domain:** $f(x) = \dfrac{x+2}{x-1}$

$x - \cancel{1} \neq 0$

$\quad \cancel{+1} \quad +1$

$x \neq 1$

$(-\infty, 1) \cup (1, \infty)$

18. Find the domain: $f(x) = \sqrt{3x+9}$

$$3x + 9 \geq 0$$

$$\underline{\quad -9 \quad -9}$$

$$\frac{\cancel{3}x}{\cancel{3}} \geq \frac{-9}{3}$$

$$x \geq -3$$

$$[-3, \infty)$$

19. Calculate $f(2)$ if $f(x) = -2x^2 + 3x - 2$.

$$f(2) = -2(2)^2 + 3(2) - 2 = -2(4) + 3(2) - 2 = -8 + 6 - 2 = -4$$

20. **Calculate $f(4)$ if** $f(x) = \begin{cases} x-1 & x<2 \\ 3 & x=2 \\ -x & 2<x \end{cases}$

is $4<2$? No

is $4=2$? No

is $2<4$? Yes

$f(x) = -x$

$f(4) = -4$

21. Solve for x using substitution:

$$3x^8 - 14x^4 - 5 = 0$$

$u = x^4$; $u^2 = (x^4)^2 = x^8$

$3u^2 - 14u - 5 = 0$

$(3u^2 - 15u) + (1u - 5) = 0$

$3u(u-5) + 1(u-5) = 0$

$(u-5)(3u+1) = 0$

$u - 5 = 0$
$+5 \quad +5$
$u = 5$

$3u + 1 = 0$
$-1 \quad -1$
$\frac{3u}{3} = \frac{-1}{3}$

$u = -1/3$

$(3)(-5) = -15$

$-15, +1$

$u = 5$

$\sqrt[4]{x^4} = \sqrt[4]{5}$

$x = \pm\sqrt[4]{5}$

check $x = \pm\sqrt[4]{5}$

$(\pm\sqrt[4]{5})^4 = 5$

$5 = 5$ ✓

$u = -1/3$

$\sqrt[4]{x^4} = \sqrt[4]{-1/3}$

$x = \pm\sqrt[4]{-1/3}$

check $x = \pm\sqrt[4]{-1/3}$

$(\pm\sqrt[4]{-1/3})^4 = -1/3$

$-1/3 = -1/3$ ✓

$$\boxed{\begin{array}{l} x = \pm\sqrt[4]{5} \\ x = \pm\sqrt[4]{-1/3} \end{array}}$$

22. What reflections and transformations take $f(x) = |x|$ to the function $f(x) = 3 - |x - 1|$

$f(x) = -|x-1| + 3$

$y = |x|$ Parent Function

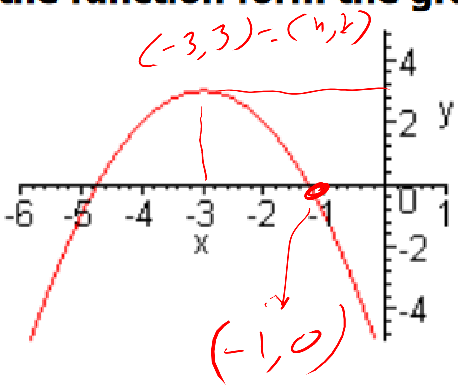
Horizontal Shift: Right 1 $y = |x-1|$

Reflections: x-axis reflection: $y = -|x-1|$

Vertical Shift: up 3 $f(x) = -|x-1| + 3$

Right 1, x-axis reflection, up 3

23. Find the function form the graph.



$$f(x) = a(x-h)^2 + k$$

$$f(x) = a(x+3)^2 + 3 \ *$$

plug in any (x, y) point

$$0 = a(-1+3)^2 + 3$$

$$0 = a(2)^2 + 3$$

$$0 = 4a + 3$$

$$\frac{-3}{4} = \frac{\cancel{4}a}{\cancel{4}}$$

$$-3/4 = a$$

$$\boxed{f(x) = -\frac{3}{4}(x+3)^2 + 3}$$ (First)

$$f(x) = -\frac{3}{4}(x+3)(x+3) + 3$$

$$f(x) = -\frac{3}{4}(x^2 + 3x + 3x + 9) + 3$$

$$f(x) = -\frac{3}{4}(x^2 + 6x + 9) + 3$$

$$f(x) = -\frac{3}{4}x^2 - \frac{18}{4}x - \frac{27}{4} + \frac{3 \cdot 4}{1 \cdot 4}$$

$$f(x) = -\frac{3}{4}x^2 - \frac{9}{2}x - \frac{27}{4} + \frac{12}{4}$$

$$\boxed{f(x) = -\frac{3}{4}x^2 - \frac{9}{2}x - \frac{15}{4}}$$ (second)

24. Find the vertex; $f(x) = x^2 - 14x + 64$

$a=1 \quad b=-14 \quad c=64$

$$x = \frac{-b}{2a} = \frac{14}{2(1)} = \frac{14}{2} = 7 \to h$$

$14(7) = (10+4)(7)$

$70+28 = 98$

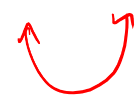
$$f(7) = (7)^2 - 14(7) + 64 = 49 - 14(7) + 64 = 49 - 98 + 64$$

$$= -49 + 64 = 15 \to k$$

vertex: $(7, 15)$

(minimum)

$a>0$, parabola opens up



25. **Find the vertex:** $f(x) = -2x^2 - 8x + 5$

Complete the square

$f(x) = -2(x^2 + 4x) + 5 \longrightarrow f(x) = -2(x^2 + 4x + \overset{(\frac{b}{2})^2}{4}) \underbrace{- (-2)(4)}_{-a(\frac{b}{2})^2} + 5$

$b = 4$

$\frac{b}{2} = 2$

$(\frac{b}{2})^2 = 4$

$f(x) = -2(x + \overset{b/2}{2})^2 + 13$

$a(x - h)^2 + k$

Standard form

vertex: $(h, k) = (-2, 13)$ Maximum

$a < 0$, parabola opens down

26. **Given $f(x) = 2x + 3$ and $g(x) = x^2 + 2x$**

$f(x) = 2x + 3$

a. **Find $(f \circ g)(x)$** $= f(g(x)) = f(x^2+2x) = 2(x^2+2x)+3 = \boxed{2x^2+4x+3}$

$g(x) = x^2 + 2x$

b. **Find $(g \circ f)(-1)$** $= g(f(-1)) = g(1) = (1)^2 + 2(1) = 1 + 2 = \boxed{3}$

$f(-1) = 2(-1) + 3 = -2 + 3 = 1$

27. **Find the inverse of $f(x) = \frac{1}{x-1}$**

$$y = \frac{1}{x-1}$$

$$\frac{x}{1} = \frac{1}{y-1}$$

$$xy - \cancel{x} = 1$$
$$\cancel{+x} \quad +x$$

$$\frac{\cancel{x}y}{\cancel{x}} = \frac{x+1}{x}$$

$$y = \frac{x+1}{x}$$

$$\boxed{f^{-1}(x) = \frac{x+1}{x}}$$

$f(x)$:

VA: $x-1=0$

$x=1$

HA: $\frac{\text{Small Deg}}{\text{Large Deg}}$ $y=0$

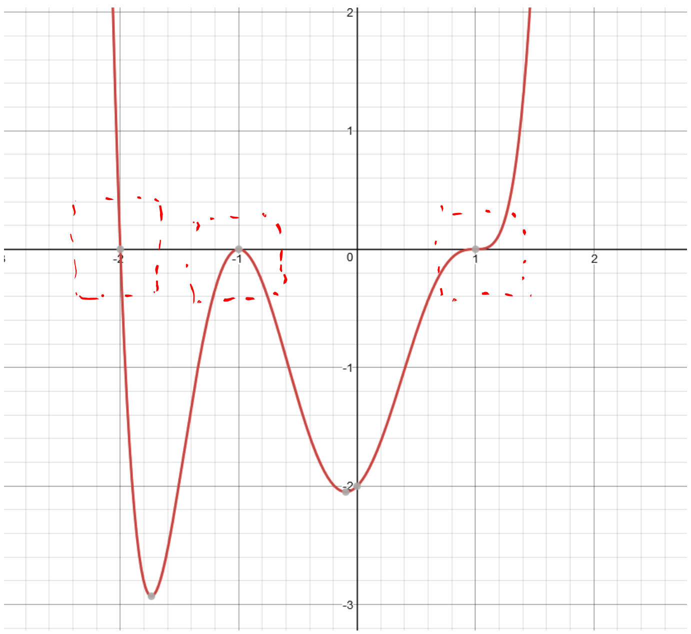
$f^{-1}(x)$

VA: $x=0$

HA: $\frac{\text{Deg }1}{\text{Deg }1}$: $\frac{1x}{1x}$ $y=1$

since they are inverses, the x's and y's swap.

28. The function which corresponds to the graph.



Polynomial:

End Behavior: L: ↑, R: ↑ (Leading Term: Pos, even deg)

x-int: $x = -2$ (Linear) M:1

$x = -1$ (Quadratic) M:2

$x = 1$ (Cubic): M:3

y-int: $y = -2 \longrightarrow (0,-2)$

$p(x) = a(x-\text{root})^m (x-\text{root})^n (x-\text{root})^n \ldots$

$p(x) = a(x+2)^1 (x+1)^2 (x-1)^3$

$-2 = a(0+2)(0+1)^2(0-1)^3$

$-2 = a(2)(1)^2(-1)^3$

$-2 = -2a$

$1 = a$

$\boxed{p(x) = (x+2)(x+1)^2(x-1)^3}$

Check: Leading Term: $x^1 \cdot x^2 \cdot x^3 = x^6$

pos, even degree

29 Find the quotient and remainder $\dfrac{2x^3+13x^2+28x+21}{x^2+3x+1}$

$2x+7 = Q(x)$

$$
\begin{array}{r|l}
 & \quad 2x+7 \\
x^2+3x+1 & 2x^3+13x^2+28x+21 \\
 & -(2x^3+6x^2+2x) \\
 & \quad 7x^2+26x+21 \\
 & -(7x^2+21x+7) \\
 & \quad 5x+14
\end{array}
$$

$5x+14 = R(x)$

$\boxed{Q(x) = 2x+7 \quad R(x) = 5x+14}$

$\boxed{2x+7+\dfrac{5x+14}{x^2+3x+1}}$

$Q(x) + \dfrac{R(x)}{D(x)}$

Long Division

① $x^2 \cdot \square = 2x^3$

$2x$

$2x(x^2+3x+1) = 2x^3+6x^2+2x$

subtract

② $x^2 \cdot \square = 7x^2$

7

$7(x^2+3x+1) = 7x^2+21x+7$

subtract

③ Degree of $5x+14$ is 1

Degree of x^2+3x+1 is 2

$1<2 \rightarrow$ Done

30. Find the quotient and the remainder $\dfrac{-2x^2+14x-16}{x-1}$

Synthetic Division
since $D(x)$ is $x \pm$ Number

$$\begin{array}{r|rrr} 1 & -2 & 14 & -16 \\ & \downarrow & -2 & 12 \\ \hline & -2 & 12 & -4 \end{array}$$

$Q(x) = -2x+12$

$R(x)$

$$\boxed{\begin{array}{l} Q(x) = -2x+12 \\ R(x) = -4 \end{array}} \quad \boxed{-2x+12-\frac{4}{x-1}}$$

31. Find the zeros of a polynomial by factoring:

$f(x) = x^2 - 8x + 16$

zeros
roots
x-intercepts

same thing

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$(x-4)^2 = 0$

$x - 4 = 0$

$\boxed{x = 4}$ (M:2)

32. Given f(x) = 5 - 4ˣ $= -4^x + \underbrace{5}_{HA}$ (Exponential Function)

a. Asymptote?

(HA) $y = 5$

b. Range?

Exponential term is Negative → Below the HA

$(-\infty, 5)$

33. What is the transformation of the key point (1, 0):

$\log_6(x - 2) - 4$

Parent function: $y = \log_6 x$

Key Point

$(1, 0)$

Horizontal Shift: Right 2 $y = \log_6(x-2)$

$(3, 0)$

No Reflection

$(3, 0)$

Vertical shift: Down 4: $y = \log_6(x-2) - 4$

$\boxed{(3, -4)}$

34. **Simplify:** $f(x) = \log_2\left(\frac{1}{2^3}\right) = -3$

$y = \log_2 \frac{1}{2^3}$

$2^y = \frac{1}{2^3}$

issue #1 : Fraction → Negative Exponent

issue #2 : Exponent : 3 → 3

$y = -3$

$2^{-3} = \frac{1}{2^3}$

35. **Solve:** $\log_4(x-1) = 0$

$4^0 = x - 1$

$1 = x - 1$

$+1 \quad +1$

$\boxed{2 = x}$

Check $x = 2$

$x - 1 \rightarrow 2 - 1 = 1$ ✓