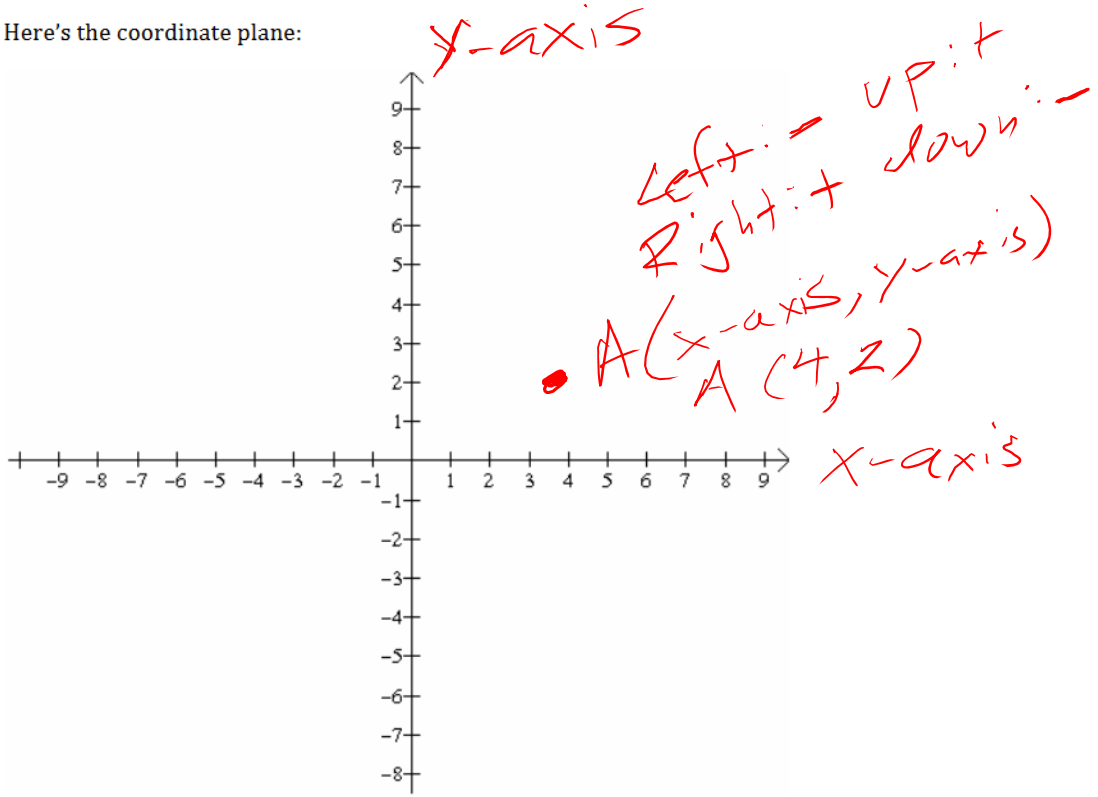
MATH 1314

Section 1.1

Section 1.1: Points, Regions, Distance and Midpoints

Graphing Points and Regions

Here's the coordinate plane:



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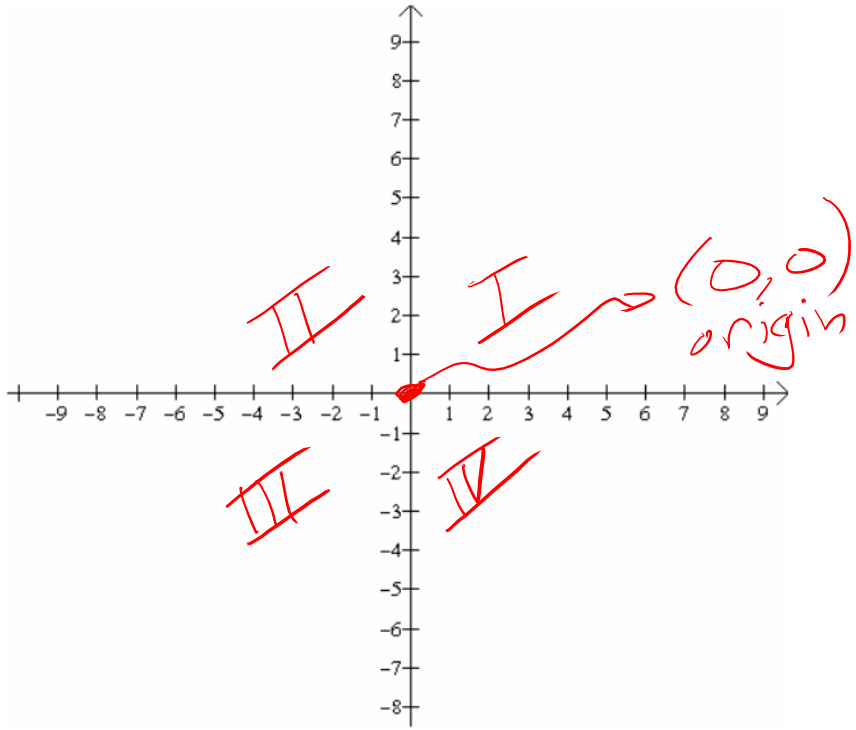


As we see the plane consists of two perpendicular lines, the x-axis and the y-axis. These two lines separate them into four regions, or quadrants.

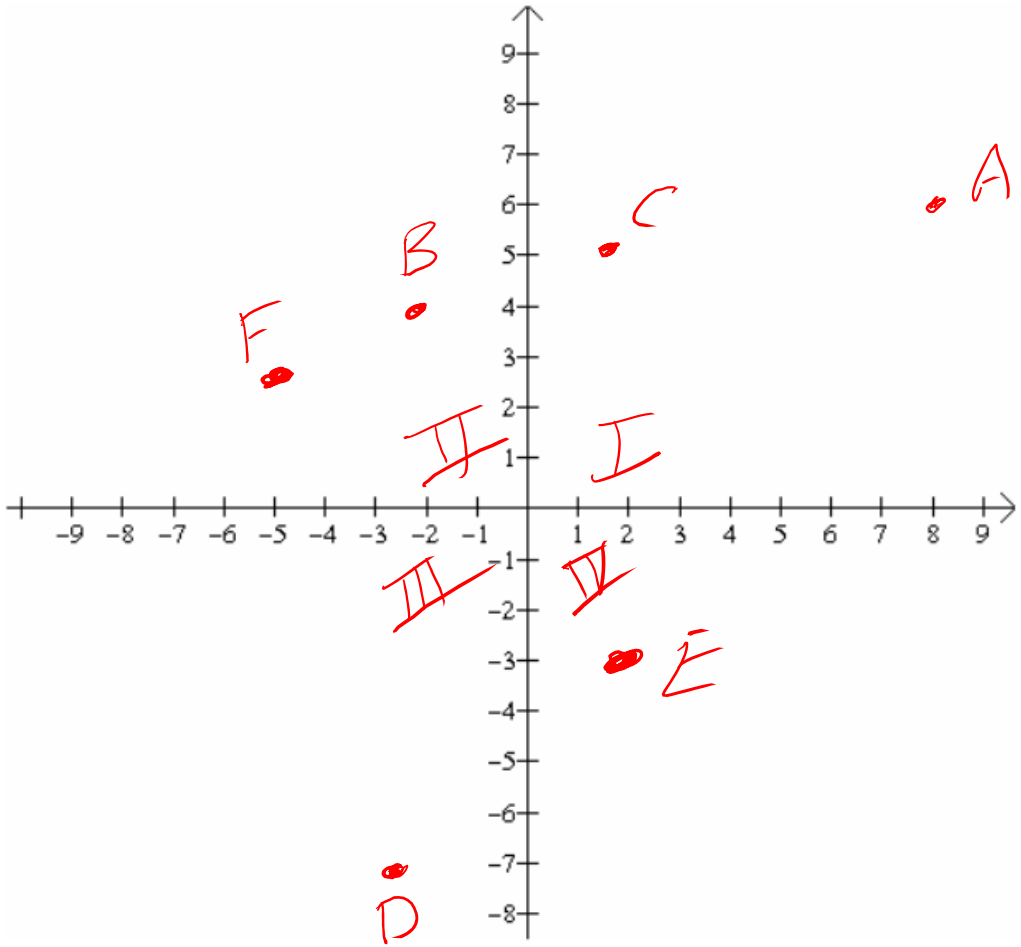
The pair, (x, y), is called an ordered pair. It corresponds to a single unique point in the coordinate plane. The first number is called the x coordinate, and the second number is called the y coordinate.

The ordered pair (0, 0) is referred to as the origin.

The x coordinate tells us the horizontal distance a point is from the origin. The y coordinate tells us the vertical distance a point is from the origin. You'll move right or up for positive coordinates and left or down for negative coordinates.

Example: Plot the following points.

		Quadrant
A.	(8,6)	I
B.	(-2,4)	II
C.	(2,5)	I
D.	(-3,-7)	III
E.	(2,-3)	IV
F.	(-5,3)	II



(x, y)

(+, +) → QI

(−, +) → QII

(−, −) → QIII

(+, −) → QIV

Graphing Regions in the Coordinate Plane

The set of all points in the coordinate plane with y coordinate k is the **horizontal line $y = k$**.
The set of all points in the coordinate plane with x coordinate k is the **vertical line $x = k$**.

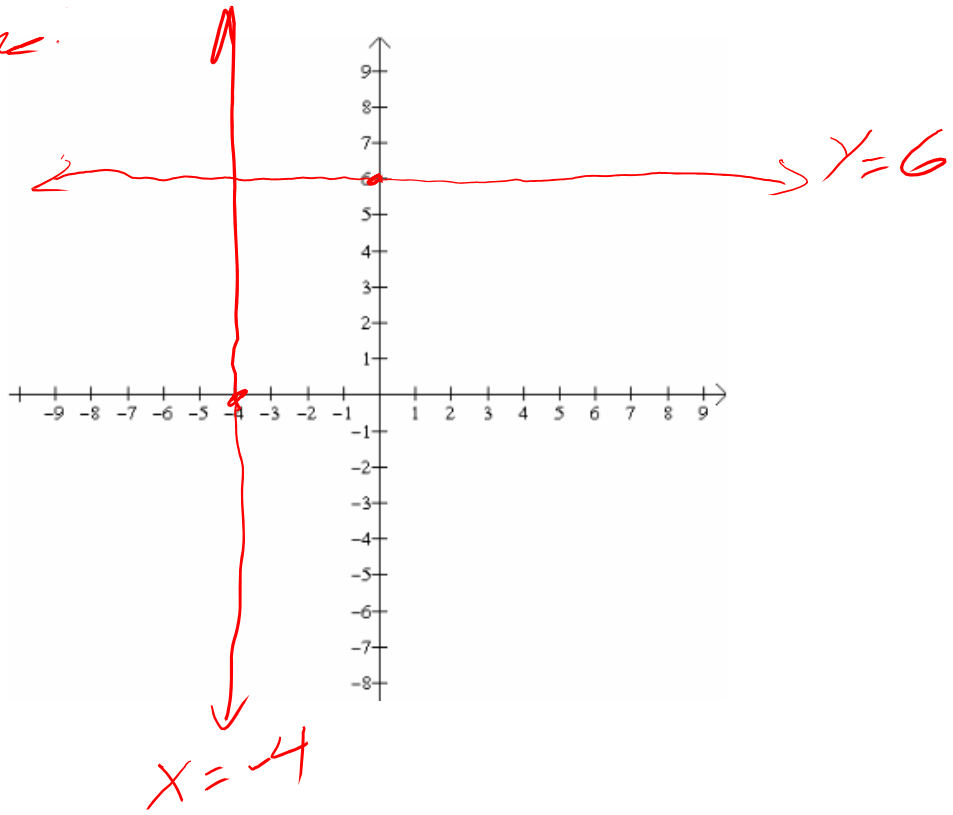
Horizontal Line:

$y =$ Number

↓

where the line crosses y-axis

$y = 6$



Vertical Line:

$x =$ Number

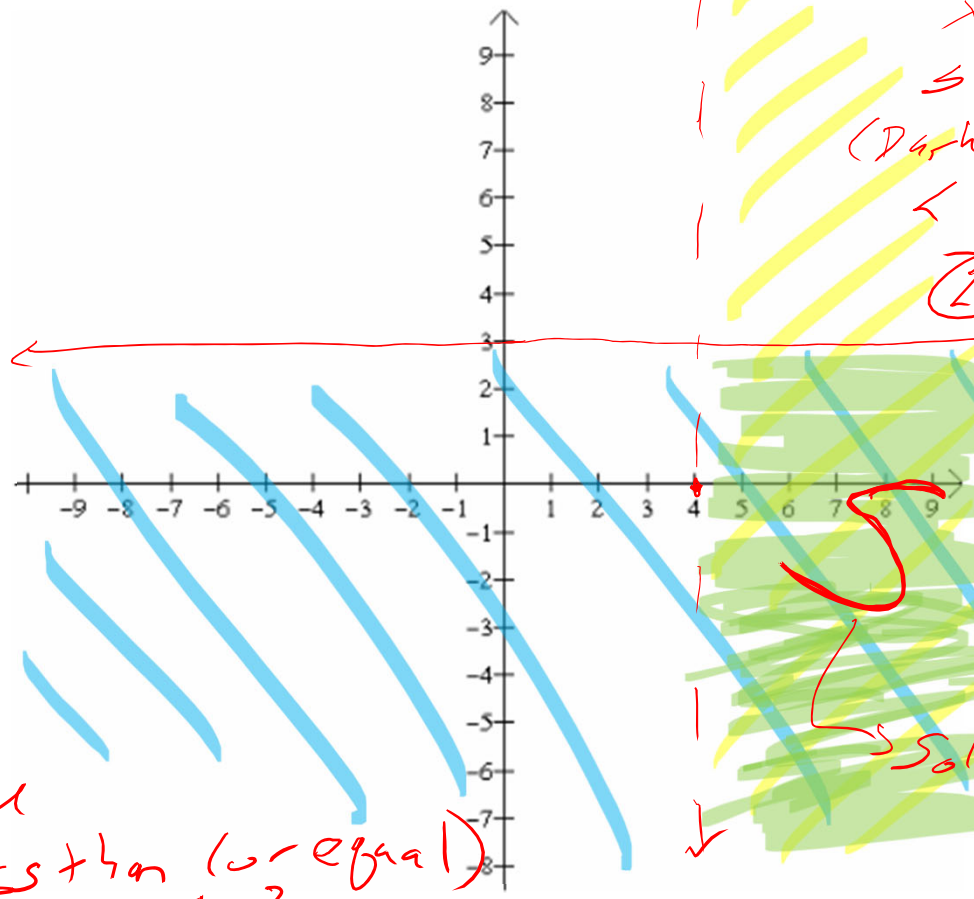
↓

where the line crosses the x-axis

$x = -4$

Graphing Regions in the Coordinate Plane

Example: Graph $\{(x, y) \mid x > 4 \text{ and } y \leq 3\}$.



Set builder Notation:

$\{$ what we graph $\mid$ conditions $\}$

such that

$\{(x, y) \mid x > 4 \text{ and } y \leq 3\}$

We are graphing all (x, y) coordinate points with x-values greater than 4 and y-values less than (or equal) to 3

① $x > 4$: $x = 4$ shade greater (Dashed Line: $<$ or $>$)

② $y \leq 3$: $y = 3$ shade lesser (Solid line: $\leq$ or $\geq$)

③ "and" overlap

Solution Space

The Distance Formula

For any two points (x_1, y_1) and (x_2, y_2), the distance between them is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

(It does not matter which point is (x_1, y_1) or (x_2, y_2))

Example: Find the distance between the following pair of points.

a) $(-3,1)$ & $(1,3)$

(x_1, y_1) (x_2, y_2)

$$d = \sqrt{(1 - -3)^2 + (3 - 1)^2}$$

$$d = \sqrt{(1 + 3)^2 + (3 - 1)^2}$$

Simplify

Square root

$$d = \sqrt{4^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20}$$

$$\sqrt{20} = \sqrt{4} \cdot \sqrt{5} = \boxed{2\sqrt{5}}$$

($4 \cdot 5 = 20$, 4 is perf. sq)

b) $(2\sqrt{3}, 5\sqrt{6})$ & $(-\sqrt{3}, \sqrt{6})$

(x_1, y_1) (x_2, y_2)

$$d = \sqrt{(-\sqrt{3} - 2\sqrt{3})^2 + (\sqrt{6} - 5\sqrt{6})^2}$$

$$d = \sqrt{(-3\sqrt{3})^2 + (-4\sqrt{6})^2}$$

$$d = \sqrt{(-3)^2(\sqrt{3})^2 + (-4)^2(\sqrt{6})^2}$$

$$d = \sqrt{9 \cdot 3 + 16 \cdot 6}$$

$$d = \sqrt{27 + 96} = \boxed{\sqrt{123}}$$

$16 \cdot 6$

$(10+6) \cdot 6$

$60 + 36 = 96$

The Midpoint Formula

Midpoint Formula

The midpoint of the line segment joining the two points (x_1, y_1) and (x_2, y_2) is given by

$$M = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$$

→ (Average x, Average y)

Example: Find the midpoint between the following pair of points.

a) $(-3,1)$ & $(1,3)$

(x_1, y_1) (x_2, y_2)

$M = \left(\frac{1+-3}{2}, \frac{3+1}{2}\right) = \left(\frac{1-3}{2}, \frac{3+1}{2}\right)$

$M = \left(\frac{-2}{2}, \frac{4}{2}\right) = \boxed{(-1, 2)}$

b) $(2\sqrt{3}, 5\sqrt{6})$ & $(-\sqrt{3}, \sqrt{6})$

(x_1, y_1) (x_2, y_2)

$$M = \left(\frac{-\sqrt{3} + 2\sqrt{3}}{2}, \frac{\sqrt{6} + 5\sqrt{6}}{2} \right)$$

$$M = \left(\frac{\sqrt{3}}{2}, \frac{6\sqrt{6}}{2} \right)$$

$$M = \boxed{\left(\frac{\sqrt{3}}{2}, 3\sqrt{6} \right)}$$