MATH 1314

Section 1.2

Lines

In this section, we'll review slope and different equations of lines. We will also talk about x-intercept and y-intercept, parallel and perpendicular lines.

Horizontal Lines: $y =$ Number

Vertical Lines: $x =$ Number

Slope

For every line, the slope stays the same.

Definition: The **slope** of a line measures the steepness of a line or the rate of change of the line.

To find the slope of a line you need two points. You can find the slope of a line between two points (x_1, y_1) and (x_2, y_2) by using this formula.

$$\boxed{slope\ (m) = \frac{y_2 - y_1}{x_2 - x_1}}$$

Note:

-Lines with positive slope rise to the right.

-Line with negative slope fall to the right.

-Lines with slope equal to 0 are horizontal lines.

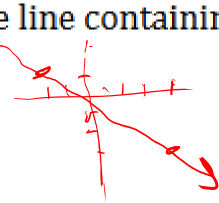
-Lines with undefined slope are vertical lines

$\frac{0}{\text{Non-Zero}} = 0$

$\frac{\text{Non-Zero}}{0} =$ undefined

Example 1: Find the slope of the line containing the following points

a. (4, -3) and (-2, 1)



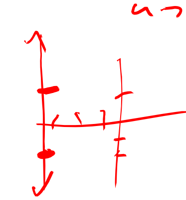
(x_1, y_1) (x_2, y_2)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - -3}{-2 - 4} = \frac{1+3}{-2-4} = \frac{4 \div 2}{-6 \div 2} = -\frac{2}{3}$$

b. (-3, 1) and (-3, -2)

(x_1, y_1) (x_2, y_2)

$$m = \frac{-2-1}{-3+{+3}} = \frac{-3}{0} \rightarrow \text{undefined}$$



Finding the Equation of a Line

Three usual forms:

1. **Point-Slope Form**
   $(y-y_1) = m(x-x_1)$ where (x_1, y_1) is a point on the line and m is the slope.

2. **Slope-Intercept Form**
   $y = mx + b$ where m is the slope and b is the y-intercept of the line.

3. **Standard Form**
   $Ax + By + C = 0$ where A and B are not both equal to 0.

4. Intercept - Intercept
   $\frac{x}{a} + \frac{y}{b} = 1$ a is x-intercept b is y-intercept (Line cannot pass through origin).

Example 2: Write the following equation in slope-intercept form and identify the slope and y-intercept.

$2x - 4y = 5$

$2x - 4y = 5 \rightarrow y = mx + b$

$-2x \qquad -2x$

$\frac{-4y}{-4} = \frac{-2x + 5}{-4} = \frac{-2x}{-4} + \frac{5}{-4}$

$y = \frac{1}{2}x - \frac{5}{4}$

$y = m\,x + b$

$y = \frac{1}{2}x - \frac{5}{4}$

$m = 1/2 \rightarrow$ slope

$b = -5/4 \rightarrow$ y-intercept

$(0, -5/4)$

(0, Number) → y-int.

Example 3: Write an equation of the line that satisfies the given conditions.

a. $m = ½$ and the y-intercept is 3.

slope y-intercept

$y = mx + b$

$y = \frac{1}{2}x + 3$

If needed: convert to standard

$y = \frac{1}{2}x + 3$

$-\frac{1}{2}x \quad -\frac{1}{2}x$

$2 \times \left(-\frac{1}{2}x + y = 3\right) \times 2 \rightarrow \left(-x + 2y = 6\right)$

$x - 2y = -6$

b. m = -3 and the line passes through (-2, 1). $\rightarrow (x_1, y_1)$

slope point

$(y - y_1) = m(x - x_1)$

$(y - 1) = -3(x - -2)$

$(y - 1) = -3(x + 2)$

If needed:

$y - 1 = -3x - 6$

$+1 \qquad +1$

$y = -3x - 5$

$+3x \qquad +3x$

$3x + y = -5$

c. line passes through (-6, 10) and (-2, 2). We need a slope

point point

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-10}{-2+6} = \frac{-8}{4} = -2$$

point-slope formula $(y - y_1) = m(x - x_1)$

use $(-6, 10)$

$(y - 10) = -2(x - -6)$

$(y - 10) = -2(x + 6)$

$y - \cancel{10} = -2x - 12$

$\cancel{+10} \qquad +10$

$y = -2x - 2$

use $(-2, 2)$

$(y - 2) = -2(x - -2)$

$(y - 2) = -2(x + 2)$

$y - \cancel{2} = -2x - 4$

$\cancel{+2} \qquad +2$

$y = -2x - 2$

Parallel and Perpendicular Lines

Definition: Parallel lines are lines with slopes m_1 and m_2 such that they are equal, in other words

$$\left.\begin{aligned} y &= 2x+5 \\ y &= 2x-4 \end{aligned}\right\} \text{Never intersect}$$

$$m_1 = m_2$$

$//$: parallel

Definition: Perpendicular lines are lines in which the product of the slopes equal -1.

$\perp$: perpendicular

$$m_1 m_2 = -1$$

Also known as the negative reciprocal. $m_2 = \frac{-1}{m_1}$

Flip and Negate the slope.

$$\left.\begin{aligned} y &= 2x+5 \\ y &= -\tfrac{1}{2}x-4 \end{aligned}\right\} \text{intersect at right angles}$$

$$\left.\begin{aligned} y &= 2x+5 \\ y &= \tfrac{1}{2}x-4 \end{aligned}\right\} \text{Neither}$$

Example 4: Write an equation of the line that passes through the points (-3, 8) and parallel to $y = -2x + 4$

$\hookrightarrow m = -2$

given line: $m = -2$

parallel line: $m = -2$

$$(y - y_1) = m(x - x_1)$$

$$y - 8 = -2(x - -3)$$

$$y - 8 = -2(x + 3)$$

Example 5: Write an equation of the line that passes through the points (1, 2) and perpendicular to $y = -2x + 4$.

given line: $m = -2 = \frac{-2}{1}$

perpendicular line: $m = \frac{1}{2}$

$$(y - y_1) = m(x - x_1)$$

$$x - 2 = \frac{1}{2}(x - 1)$$

x-intercept and y-intercept

When graphing an equation, it is usually very helpful to find the ***x* intercept(s)** and the ***y* -intercepts** of the graph. An *x* intercept is the first coordinate of the ordered pair of a point where the graph of the equation crosses the *x* axis. To find an *x* intercept, let $y = 0$ and solve the equation for *x*.

The ***y*-intercept** is the second coordinate of the ordered pair of a point where the graph of the equation crosses the *y* axis. To find a *y* intercept, let $x = 0$ and solve the equation for *y*.

x-intercept: set y=0, solve for x
(number, 0)

y-intercept: set x=0, solve for y
(0, number)

Example 5: Find the x and y intercepts of the graph of the equation $3x - 4y = 8$.

x-int: ($y=0$)

$3x - \cancel{4(0)} = 8$

$\frac{3x}{3} = \frac{8}{3}$

$x = 8/3$

$(8/3, 0)$

y-int: ($x=0$)

$\cancel{3(0)} - 4y = 8$

$\frac{-4y}{-4} = \frac{8}{-4}$

$y = -2$

$(0, -2)$

Example 6: Find the x and y intercepts of the graph of the equation $y = x^2 - 9$.

x-int ($y=0$)

$0 = x^2 - 9$

$+9 \qquad +9$

$9 = x^2$

$x^2 = 9 \rightarrow \sqrt{x^2} = \pm\sqrt{9} \rightarrow x = \pm 3$

$(3,0), (-3,0) \rightarrow (\pm 3, 0)$

y-int: ($x=0$)

$y = 0^2 - 9$

$y = 0 - 9 = -9$

$(0, -9)$