MATH 1314

Section 2.2

Applications

Using Modeling to Solve Problems

Step 1: Define variables.

Step 2: Express each unknown quantity in terms of one variable.

Step 3: Write the equation in one variable which models the situation given.

Step 4: Solve the equation.

Step 5: Answer the question(s) posed, including appropriate units

Example 1: Find three consecutive even integers whose sum is 222.

Consecutive integers:

$x,\ x+1,\ x+2$

Consecutive even integers:

$x,\ x+2,\ x+4$

Consecutive odd integers:

$x,\ x+2,\ x+4$

smallest: $x = 72$

middle: $x+2 = 74$

largest: $x+4 = 76$

72, 74, 76

$x + x+2 + x+4 = 222$

$3x + 6 = 222$

$-6 \quad -6$

$\frac{3x}{3} = \frac{216}{3}$

$x = 72$

$\frac{210+6}{3}$

$= 70 + 2$

$= 72$

Example 2: If the first and third of three consecutive odd integers are added, the result is 63 less than five times the second integer. Find the third integer.

first: $x = 19$

second: $x+2 = 21$

third: $x+4 = \boxed{23}$

$x + x+4 = 5(x+2) - 63$

$2x+4 = 5x+10-63$

$2x+4 = 5x-53$

$-2x \quad -2x$

$4 = 3x - 53$

$+53 \quad +53$

$\frac{57}{3} = \frac{3x}{3}$

$19 = x$

$\frac{57}{3} = \frac{30}{3} + \frac{27}{3}$

$= 10 + 9 = 19$

Example 3: The length of a rectangular garden is 20 feet more than the width. The perimeter is 140 feet. What are the dimensions of the garden?

length: $l = w + 20$

width: w

$P = 2l + 2w$

$140 = 2(w + 20) + 2w$

$140 = 2w + 40 + 2w$

$140 = 4w + 40$

$-40 \quad -40$

$\frac{100}{4} = \frac{4w}{4}$

$25 = w$

$l = 25 + 20 = 45$

width: 25 ft

length: 45 ft

Example 4: When the sides of a square are each increased by 2cm, the area increases by 14 cm². Find the length of a side in the original square.

- original square:
  - side: x
  - area: $A = s^2 = x^2$

new square:

side: $x+2$

area: $A = s^2 = (x+2)^2$

$= (x+2)(x+2)$ (FOIL)

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

2 ways of explaining New Area.

area: $A = x^2 + 14$

$x^2 + 4x + 4 = x^2 + 14$

$-x^2 \qquad -x^2$

$4x + 4 = 14$

$-4 \quad -4$

$\frac{4x}{4} = \frac{10}{4}$

$x = \frac{5}{2} = 2.5$

Example 5: Two sides of a triangle have the same length. The third side is 15 cm longer than each of the equal sides. The perimeter is no less than 90 cm. What are the smallest possible lengths of the sides of the triangle.

equal sides: $x = 25$ (two)

third side: $x + 15 = 40$

$P = 90$

$P = s_1 + s_2 + s_3$

$90 = x + x + x + 15$

$90 = 3x + 15$

$-15 \quad -15$

$\frac{75}{3} = \frac{3x}{3}$

$25 = x$

25cm, 25cm, 40cm