

# MATH 1314

Section 2.5

# Other Techniques for Solving Equations

Solving by Factoring:

Factoring can be used to solve many types of equations. Always begin by Factoring Completely. Then, set each factor equal to zero.

Find all solutions to

$$(x^3 + 3x^2) + (2x + 6) = 0$$

$$x^2(x+3) + 2(x+3) = 0$$

$$(x+3)(x^2+2) = 0$$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline \end{array}$$

$$x = -3$$

$$\begin{array}{r} x^2+2=0 \\ -2 \quad -2 \\ \hline \end{array}$$

$$\sqrt{x^2} = \sqrt{2}$$

$$x = \pm\sqrt{2}i$$

$$\{-3, -\sqrt{2}i, +\sqrt{2}i\}$$

Find all the solutions of  $x^3 = x$

$$\begin{array}{r} x^3 = x \\ -x \quad -x \\ \hline x^3 - x = 0 \end{array}$$

$$x(x^2 - 1) = 0$$

$$x(x+1)(x-1) = 0$$

$$x = 0$$

$$\begin{array}{r} x+1 = 0 \\ -1 \quad -1 \\ \hline x = -1 \end{array}$$

$$\begin{array}{r} x-1 = 0 \\ +1 \quad +1 \\ \hline x = 1 \end{array}$$

$$\boxed{\left\{ -1, 0, 1 \right\}}$$

Common mistake:

$$\begin{array}{r} x^3 = x \\ \hline x \quad x \\ \hline \sqrt{x^2 = 1} \end{array}$$

$$x = \pm 1$$

We lost the  $x=0$  answer!

Reason: You cannot divide (or cancel) anything that may equal zero.

## Equations Involving Fractions:

Option 1: Rewrite all fractions so that they have the same denominator, then drop all denominators from the equation.

or

Option 2: Multiply the entire equation by the LCD to clear the fractions.

Then: Solve normally.

*Be advised: If your answer makes any of the original fractions undefined, it must be rejected!!*

$$\frac{4}{x-1} + \frac{3}{x} = \frac{3}{1}$$

LCD:  $x(x-1)$

(cross cancel:

cancel any numerator  
with any denominator  
when multiplying.

$$\frac{4}{\cancel{x-1}} \cdot \frac{x(\cancel{x-1})}{1} + \frac{3}{\cancel{x}} \cdot \frac{x(x-1)}{1} = \frac{3}{1} \cdot \frac{x(x-1)}{1}$$

$$\underline{4x} + \underline{3x} - 3 = 3x^2 - 3x$$

$$\begin{array}{r} 7x - 3 = 3x^2 - 3x \\ -7x + 3 \quad \quad \quad -7x + 3 \\ \hline 0 = 3x^2 - 10x + 3 \end{array}$$

$$3x^2 - 10x + 3 = 0$$

$$3x^2 - 10x + 3 = 0$$

$$(3x^2 - 9x)(-x + 3) = 0$$

$$3x(x-3) - 1(x-3) = 0$$

$$(x-3)(3x-1) = 0$$

$$\begin{array}{r} x-3=0 \\ \hline +3 \quad +3 \\ \hline x=3 \end{array}$$

$$\begin{array}{r} 3x-1=0 \\ \hline +1 \quad +1 \\ \hline 3x=1 \\ \hline \frac{1}{3} \\ \hline x = \frac{1}{3} \end{array}$$

$$\begin{array}{r} (3)(3) = 9 \\ \wedge \\ -9, -1 \end{array}$$

$$\left\{ \frac{1}{3}, 3 \right\}$$

Equations involving radicals:

If an equation involves a square root (also called a radical), you must isolate the radical, square both sides, and solve the remaining equation. Be certain to check your answers!



Find all solutions to  $\sqrt{x+8} - 2 = x$

$$\begin{array}{r} \cancel{+2} \quad +2 \\ \hline (\sqrt{x+8})^2 = (x+2)^2 \end{array}$$

$$x+8 = (x+2)(x+2)$$

$$x+8 = x^2 + 2x + 2x + 4$$

$$x+8 = x^2 + 4x + 4$$

$$\begin{array}{r} x+8 \\ -x-8 \\ \hline 0 \end{array} \quad \begin{array}{r} x^2 + 4x + 4 \\ -x^2 - 4x - 4 \\ \hline 0 \end{array}$$

$$0 = x^2 + 3x - 4$$

$$0 = (x+4)(x-1)$$

$$x+4=0$$

~~reject~~  $x = -4$

$$x-1=0$$
$$\boxed{x=1}$$

Check  $x = -4$

$$\sqrt{-4+8} - 2 = -4$$

$$\sqrt{4} - 2 = -4$$

$$2 - 2 = -4$$

$$0 \neq -4$$

Check  $x = 1$

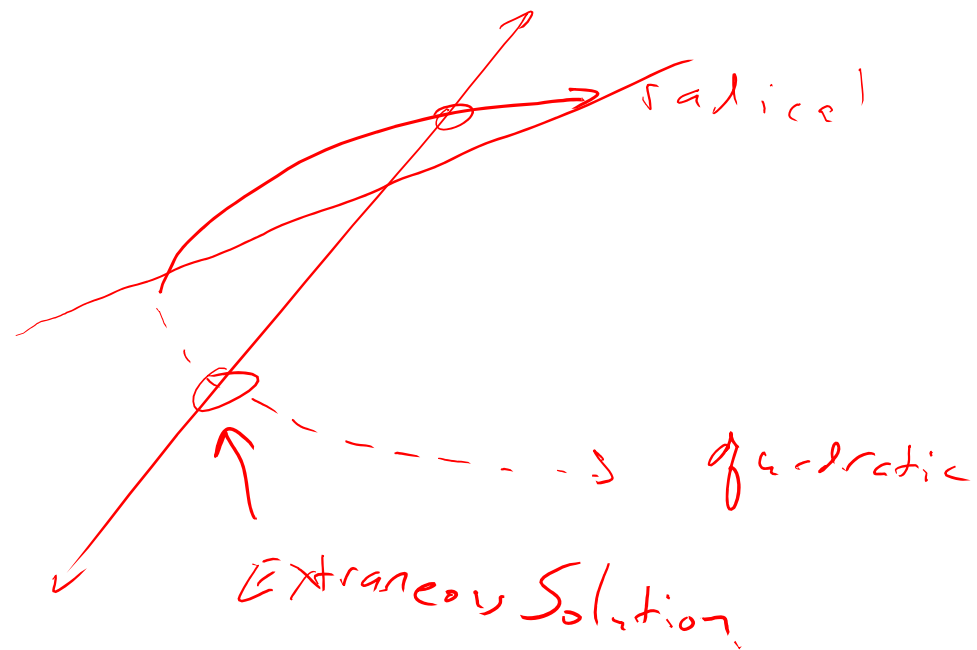
$$\sqrt{1+8} - 2 = 1$$

$$\sqrt{9} - 2 = 1$$

$$3 - 2 = 1$$

$$1 = 1 \checkmark$$

**Extraneous Solutions:** In a radical solution, you may “create” additional answers that are not correct. These must be rejected!



Find all solutions to  $\sqrt{3x+1} - 1 = x$

$$(\sqrt{3x+1})^2 = (x+1)^2$$

$$3x+1 = (x+1)(x+1)$$

$$3x+1 = x^2 + x + x + 1$$

$$3x+1 = x^2 + 2x + 1$$

$$\begin{array}{r} 3x+1 \\ -3x-1 \\ \hline \end{array} = \begin{array}{r} x^2+2x+1 \\ -3x-1 \\ \hline \end{array}$$

$$0 = x^2 - x$$

$$0 = x(x-1)$$

$$x=0$$

$$x-1=0$$

$$x=1$$

Check  $x=0$

$$\sqrt{3(0)+1} - 1 = 0$$

$$\sqrt{1} - 1 = 0$$

$$1 - 1 = 0$$

$$0 = 0 \checkmark$$

Check  $x=1$

$$\sqrt{3(1)+1} - 1 = 1$$

$$\sqrt{4} - 1 = 1$$

$$2 - 1 = 1$$

$$1 = 1 \checkmark$$

$$\{0, 1\}$$

# Popper # 1

1. Solve the following:  $x^3 = 9x$

a. 0, 3

b. 0, 9, -9

c. 0, -3, 3

d. -3, 3

$$x^3 - 9x = 0$$

$$x(x^2 - 9) = 0$$

$$x = 0 \quad x^2 - 9 = 0$$

$$x^2 = 9$$

$$x = \pm 3$$

2. Solve the following:  $x + \sqrt{x+1} = 5$

a. 8, 3

b. 3

c. 8

d. No Answer

$$(\sqrt{x+1})^2 = (x+5)^2$$

$$x+1 = (-x+5)(-x+5)$$

3. Solve the following:  $\frac{8}{x+1} + \frac{3}{x} = 3$

a. 3

b.  $^{-1}/_3$

c. -1

d. 3,  $^{-1}/_3$

$$x+1 = x^2 - 5x - 5x + 25$$

$$x+1 = x^2 - 10x + 25$$

$$x^2 - 11x + 24 = 0$$

$$0 = x^2 - 11x + 24$$

$$0 = (x-3)(x-8)$$

$$x-3=0 \quad x-8=0$$

$$x=3 \quad x=8$$

reject

check  $x=3$

$$3 + \sqrt{3+1} = 5$$

$$3 + \sqrt{4} = 5$$

$$3 + 2 = 5 \checkmark$$

check  $x=8$

$$8 + \sqrt{8+1} = 5$$

$$8 + \sqrt{9} = 5 \rightarrow 8 + 3 \neq 5$$

$$\frac{8}{x+1} + \frac{3}{x} = \frac{3}{1} \quad \text{LCD: } x(x+1)$$

$$\frac{8}{\cancel{x+1}} \cdot \frac{x(\cancel{x+1})}{1} + \frac{3}{\cancel{x}} \cdot \frac{x(\cancel{x+1})}{1} = \frac{3}{1} \cdot \frac{x(x+1)}{1}$$

$$8x + 3x + 3 = 3x^2 + 3x$$

$$\begin{array}{r} 11x + 3 = 3x^2 + 3x \\ -11x \quad -3 \\ \hline 0 = 3x^2 - 8x - 3 \end{array}$$

$$0 = 3x^2 - 8x - 3$$

$$0 = (3x^2 - 9x) + (x - 3)$$

$$0 = 3x(x-3) + 1(x-3)$$

$$0 = (x-3)(3x+1)$$

$$x-3=0$$

$$x=3$$

$$\begin{array}{r} 3x+1=0 \\ -x \quad -1 \\ \hline 2x = -1 \end{array}$$

$$\frac{2x}{2} = \frac{-1}{2}$$

$$x = -\frac{1}{2}$$

$$\left\{ -\frac{1}{2}, 3 \right\}$$